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MECHANICS
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Experimental Analysis of the Kinematics of Motion of Floating Elements in Rotor Mechanisms

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Abstract—The kinematics of the motion of floating elements (ring) of rotor mechanisms under steady state rotation modes is examined experimentally. The ring is placed with a gap with respect to the drive cylindrical link and touches it without losing contact. It is shown that the ring motion is a direct asynchronous precession and its points circumscribe epitrochoids, if the driving motion is "lunar."

Keywords: rotor, mechanism, floating rung, contact, ring precession, trajectories.

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Problem definition. Sometimes in rotor mechanisms the rotating rotor makes contact with elements of the rotor system. We are able to distinguish two reasons for this fact: (1) the rotor makes contact with an immovable or elastically fixed rigid stator and (2) the rotor makes contact with movable elements (floating or fingerlike seals, hydrostatic supports with floating bushes, or inertial vibrating crushers). Floating elements can also make contact with the stator or other floating elements.

In these two cases, the contact modes differ significantly.

In the first case, the rotor moves over the stator inside it and it is accompanied by slippage of the rotor. As a result bearings accumulate wear and an emergency situation develops. The rotor under its motion performs asynchronous precession towards the direction opposite to the direction of rotation (retrograde precession). In this case the precession velocity is very high and the pressure on the stator can be higher by a dozen times than the weight of the rotor [1–6] since the precession opposite to the trajectory of the rotor points is hypotrochoidal.

The second type of contact modes is rotation of the rotor inside elements of the light movable system, for example, inside bearings with a floating bush or inside floating seals [7–10]. The floating seals secure proper sealing. Such modes under low rotation velocities are maintained specially in inertial vibration crushers [2]. The contact modes can cause damage to floating elements and even faults in it. In this case the floating ring protects the rotor from outside under direct precession [3]. Hereinafter (in contrast to the modes of rotor motion along an immovable basis), we call such modes the running round rotor of the ring. The systems of such a class are related to the higher kimematic pairs.

In the paper we examine the "rotor–floating ring" system. We analyze the trajectories obtained at an experimental test facility for simulating the process of a ring running round. The running round process is a sufficiently complicated multifrequency mode. Therefore, first of all it is necessary to investigate motion kinematics of the ring under small rotation velocities and to determine its trajectories and to find the main regularities by using the simulation test facility. We determine experimentally the trajectories of the ring motion under different parameters of the system that make it possible to develop and to specify the results of theoretical investigation [11].

DESCRIPTION OF THE TEST FACILITY

For simulating the running round process, we designed and manufactured an experimental test facility, a kinematic block-diagram and general view of which are presented in Figs. 1a and 1b. Motor *2* is placed on basis *1* (Fig. 1b). The motor rotates the drive shaft of the model with the lever *3* hinge connected with axis *4* at which washer *5* is fixed. The washer simulates the rotor and one of the gears of the toothed belt transmission *8.* At the other end of lever *3,* the counterbalance *12* is placed. It balances lever *3* with load-

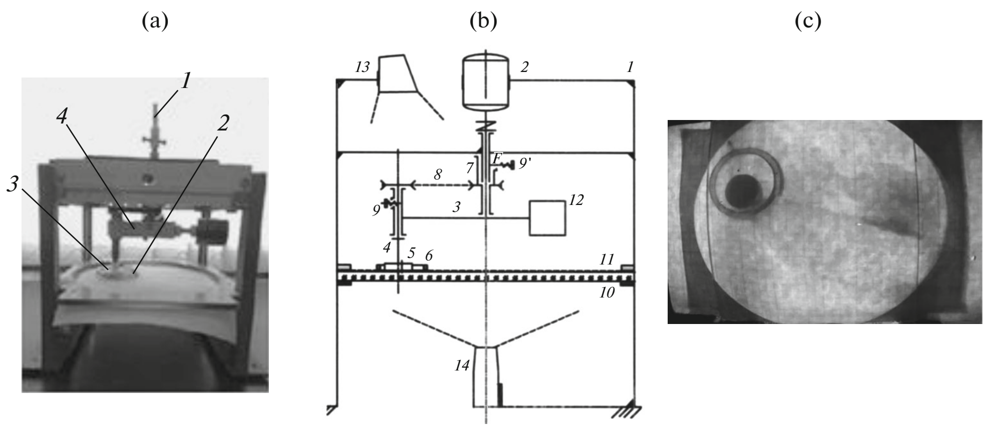

Fig. 1. (a) The general view of the experimental test facility for kinematic analysis of the trajectory of the ring under the running around mode: *1* is the motor; *2* is the ring; *3* is the washer; *4* is the lever transmitting rotation from the motor to the washer *3*; (b) the kinematic block-diagram of the experimental test facility; (c) the video filming frame for the rotation of the ring for it running around the rotor.

carrying elements with respect to the axis of its rotation. The ring *6* enveloping washer *5* with a gap lies on paper sheet *11* with the glass sublayer *10.* The plane of the glass sublayer and, respectively, of the paper sheet is normal to the axes of rotation of leader *3* and washer *5.* The gap between the bottom plane of the washer *5* and the sheet of paper *11* is not higher than 0.5 mm.

The hub of gear *7* of toothed belt transmission *8* is placed either with the possibility to rotate jointly with the drive shaft of the model with lever *3* or to be fixed with respect to immovable hub at basis *1.*

For simulating the "lunar" rotation of the model rotor, axis *4* with washer *5* and the first gear of toothed belt transmission *8* are fixed with respect to lever *3* by screw *9.* If we place screw *9* in position *9'* by fixing the rotation of second gear *7* with respect to basis *1,* we transform the rotations of washer *5* into circular translatory motion. In this case washer *5* and ring *6* form a system of hula hoop type. In the present paper, we investigate the kinematics of the rotor system with the lunar motion of the rotor. The test facility is equipped with lighting devices *13* placed from above and by video camera *14* placed from below. There are marks at ring *6*, which are seen in the video filming frames. The trajectory of ring motion is generated by frame-by-frame processing.

The parameters of the test facility are as follows: the length of the lever is $a = 40$ mm; the radius of the rotating washer is $r = 10$ mm; the internal radii of the replacing rings performing the running round motion are $R = 11.5$ mm, 19 mm, and 35 mm.

Under the relations between parameters a and r mentioned, this test facility differs from the traditional "rotor-floating ring" systems. The velocities also differ. In the test facility, they are much lower than the first critical rotation velocity of rotor systems. But it is characterized by a special feature of rotor systems, in particular, by the lunar motion of a not-balanced rotor. Under steady-state motion, each point of the rotor keeps its position with respect to the axis of rotation since the frequencies of the forced oscillations of the rotor are equal to precession frequency. Thus, we see direct synchronized precession of the rotor [13].

SOME FEATURES OF THE RUNNING ROUND MOTION IN THE EXPERIMENTAL TEST FACILITY

Before describing the kinematics model of the system "rotating washer—floating ring," let us estimate the effect of centrifugal and friction forces acting on the ring under the steady-state mode of motion. In the model the axis of the motor is vertical and the ring slides over the horizontal plane and over the cylindrical surface relative to the washer.

Under the steady-state mode of motion under a constant rotation velocity of the ring with respect to its center of mass, the moment of the inertial forces of the ring is equal to zero.

The dry friction forces of the ring under its sliding along the horizontal surface under low rotation velocities are not symmetrical with respect to the velocity vector of center of mass of the ring. The friction

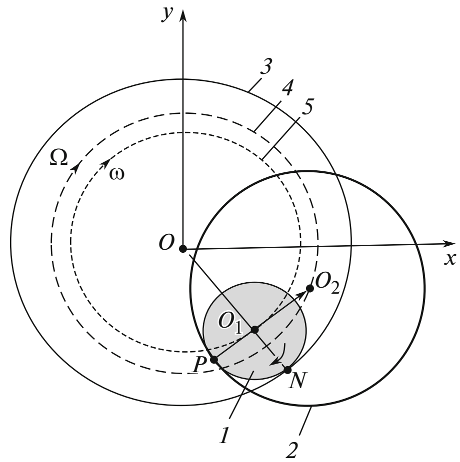

Fig. 2. Centers disposition: O is center of rotation of the system; (*1*) washer; (*2*) floating ring; (*3*) trajectory of point N of the washer, the most distant from the axis of rotation; (*4*) trajectory of center O_2 of the ring; (*5*) trajectory of center O_1 of the washer.

forces on the part of the ring with a lower sliding velocity (i.e., placed closer to the axis of rotation of the ring) are higher than the friction force for the portion of the ring with a higher sliding velocity.

The first become close to static friction forces, and the second become close to the kinetic friction force. Hypothetically, such asymmetry of the friction forces makes it possible to present them in the form of a vector of the friction forces equal to the sum of the friction forces reduced to the center of mass of the ring and directed tangentially to the trajectory of its motion and of the moment of the friction forces equal to the product of the difference of the friction forces at the mentioned sections of the ring and the mean radius of the ring. Such a situation can be due to the difference in the technological resistances of the substance in which the ring moves, and it is difficult to simulate it during the experiment.

The sum of friction force vectors and of the centrifugal forces determines the point of rotor and ring contact and the value of the mutual pressure in it. The rotation velocity of the ring depends on the ratio between the friction force generated by this pressure and the force generated by the moment of resistance at the ring.

For estimating the effect of the force of friction, we conducted an experiment in which a ring made of abrasive paper is glued to the horizontal sublayer. In this case the rotation velocity of the ring increases, but does not run up to the precession velocity. The features mentioned are related only to this experimental facility. As for the rotary systems, they have no horizontal sublayer, and as a result, there is no dry friction between the horizontal surface and the ring.

THE KINEMATIC MOTION MODEL FOR THE SYSTEM "ROTATING WASHER–FLOATING RING" UNDER THE RUNNING ROUND MODE

Figure 2 depicts the mutual disposition of the geometrical center for the washer and ring under the ratio of the parameters presented above. The geometrical center O_1 of the washer reforms the circular motion with amplitude a around axis of rotation O (the circumference in Fig. 2). The following designations are used in Fig. 2: $OO_1 = a$, $O_1N = r$ is the radius of the washer, $O_1O_2 = \delta$, $PO_2 = R$ is the radius of the ring. Point N of the orbit of the washer is the most distant from the center of rotation. Under rotation this point describes the circumference with radius $r + a$, the center of which is at the axis of rotation O (Fig. 2, circumference *3*). Rotation of lever *4* simulates the center motion of the rotor. The length of lever *4* is proportional to the misbalance value, and washer *3* performs a lunar motion and simulates the direct synchronous precession of a misbalanced rotor [12]. This set-up makes possible to suppose that the contact point at the washer is the same.

In actuality, the experiments show that under limited friction between the ring and the basis plane and under a relatively small rotation velocity of the rotor, when the inertia of the ring is not high, the ring contact takes place at point P of the washer so that the angle PO_1N is equal to approximately $\pi/2$. Under steady-state motion, point P on the washer is practically the same (Fig. 2), while the contact points with

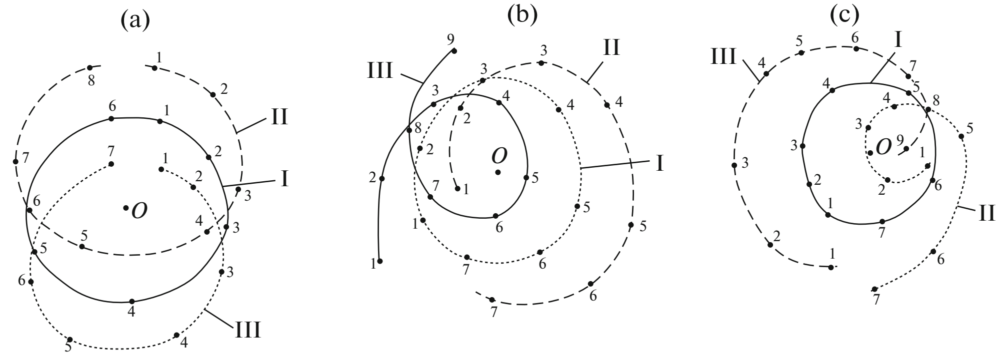

Fig. 3. Experimental trajectories of the points of the ring under the running around mode: I is the trajectory of the center of the ring; II and III are the trajectories of opposite points of the ring: (a) $R/r = 1.1$, (b) $R/r = 1.9$, and (c) $R/r = 3.5$.

the ring are different. The ring rolls the washer without liftoff, and in this case the geometrical center O_2 of the ring also describes the circumference (circumference *4*) concentrically to circumference *3*. The radius of this circumference is determined from triangle OO_1O_2 (Fig. 2) and is equal to $\delta = (a^2 + (R - r)^2)^{1/2}$, where R is the radius of the ring.

According to the experiments, the precession velocity of geometrical center of the ring coincides with the precession velocity of geometrical center of the washer according to the value and direction, i.e., $\omega = \Omega$. It can be explained as follows: the straight line that connects the centers of the washer and the ring is normal to the tangent line at the point of contact. Therefore, when the contact point at the “rotor” is constant, the rotation velocity of this straight line coincides with the rotation velocity of the point of contact. Since the washer performs direct synchronous precession with velocity ω round the center of rotation O, it impacts the ring with the precession velocity $\Omega = \omega$ round the same center O.

Each point of the floating ring participates in two motions: rotation around the center of the ring with velocity ω_1 and precession Ω of the center of the ring in the same direction. The equation for the trajectory of the points of the ring can be written as follows:

$$x(t) = R\sin\omega_1 t + \delta\sin\Omega t, \qquad y(t) = R\cos\omega_1 t + \delta\cos\Omega t. \tag{1}$$

The rotation velocity of the ring can be determined from the following conditions. The radius of the ring is greater than radius of the washer, and during the time within which the washer performs a complete revolution inside the ring, it turns around its center by an angle equal to $\beta = 2\pi(r/R)$. Therefore, the rotation velocity ω_1 of the ring is

$$\omega_1 = \omega r/R. \tag{2}$$

Equation (1) in the case of direct precession of the ring describes epitrochoids, the number of loops of which n is determined by the ratio between the rotation velocity of the ring and the precession velocity of its center $n = 1 + \omega_1/\Omega$.

If the rotor rotates with a rapid velocity, it is necessary to investigate the dynamic processes by considering the hydrodynamic forces in the gap and by considering dry friction forces [10].

EXPERIMENTAL TRAJECTORIES OF POINTS OF THE RING UNDER RUNNING AROUND

There are some difficulties for fixing the point position of the ring for the trajectory while processing the experimental data. The use of recording devices causes additional damping between the ring and the diagram, which disturbs greatly the running around process. Due to this fact, we mark the point position of the ring in the diagram without contact with the ring under different angles of rotation of the lever (“rotor”). In each frame we fix the positions of two opposite points of the ring and it make it possible to determine the trajectory of the center of the ring and of each precession velocity. These trajectories under different ratios between the radii of the ring and the washer within one turn of the washer are presented in Fig. 3. The trajectories are obtained under the same friction coefficients between the ring and the washer and also between them and the substance, and this configuration makes it possible to perform a comparative analysis.

Figures 3a and 3b depict the trajectories of the points of the ring under running around of the washer within one revolution obtained at the experimental test facility. From (2) it follows that the rotation velocity of the ring is lower than the rotation velocity of the washer, and therefore, under the rotations of the washer, the marks of the ring are delayed from the marks of the washer. We see this phenomenon in the experiment. If there is sliding, the rotation velocity of the ring decreases.

Hereby, the center precession of the ring is direct (in contrast to the running round mode, under which the precession is retrograde) and the trajectories of the points of the ring are epitrochoids.

If the radii of ring R and washer r are close (Fig. 3a), the trajectories of the points of the ring are the circumference with radius r^* and it is verified by relationships (1).

The trajectories presented in Fig. 3 are obtained under the same friction coefficients between the ring and the washer and also between them and the material. This makes it possible to compare the velocities of the ring and the washer for different ratios of radii. The following ratios between rotation velocity ω_1 of the ring and its precession velocity Ω are obtained: (a) under $R = 19$ mm: $\omega_1/\Omega = 1.6$ and the trajectory of the ring is epitrochoid with one loop (Fig. 3b); (b) under $R = 30$ mm: $\omega_1/\Omega = 2.7$ and the trajectory of the ring is epitrochoid with two loops (Fig. 3c).

CONCLUSIONS

The trajectories of the point of the ring in rotor systems are epitrochoids, the number of loops in which n is determined by the ratio between the rotation velocity of the ring and the precession velocity of its center.

The center precession of the ring is direct (in contrast to the running around mode, when the precession is retrograde).

The trajectory of the center of the ring is the circumference, and the precession velocity coincides with the rotation velocity of the initiator of oscillations (rotor).

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CONFLICT OF INTEREST

Authors declare that they have no conflict of interest.

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