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MECHANICS OF MACHINES

Free Oscillations of a Composite Structurally Orthotropic Cylindrical Shell Stiffened by Discretely Positioned Ring Ribs

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Abstract—A method for calculating the natural oscillations of a cylindrical shell of an orthotropic material is proposed. The shell is stiffened by a set of sufficiently densely positioned transverse-longitudinal ribs, the arrangement of which allows "smearing" and contains sparsely positioned discrete stiffening rings. The shell may have a closed or open cross section; it is considered to be loaded by external pressure and axial forces. The problem is reduced to a set of homogeneous algebraic equations the number of which is equal to twice the number of discrete ribs; the equation was obtained in explicit form. The comparison of the calculated and experimental data is provided.

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Let us consider a reinforced shell (panel) that represents a cylindrical casing stiffened by discretely positioned ribs. We represent the casing in the form of an orthotropic (manufactured of a composite material) shell reinforced by a set of transverse-longitudinal ribs positioned sufficiently densely. The reinforcement system is considered to be positioned asymmetrically with respect to the middle surface of the casing. The influence of the stiffening ribs on the shear and twist of the middle surface of the casing is not taken into consideration. It is assumed that the deformation of the reinforcement is described by the relations of the linear stressed state without considering their interaction.

We take the middle surface of the shell as the coordinate surface and consider that for densely positioned ribs the "smearing" hypothesis is true. Then, the densely positioned reinforcing elements can be represented in the form of an orthotropic layer characterized by some finite tension–compression moduli, Poisson's zero ratios, and the zero shear modulus.

We consider the discretely positioned ribs in the contact definition. When setting up the stability and free-oscillation equations, a shell stiffened by discretely positioned ribs is divided into its constituent members. The shell and the discrete ribs are considered separately. The shell and the ribs are loaded by the distributed contact loads that act along the shell–rib contact band. The distributed contact load is replaced by a linear load reduced to the centerline of the contact band. The interaction between the shell and ribs manifests itself through the conditions of the displacement compatibility and stress equilibrium at the shell–rib contact points.

Let us assume that, between the shell and the ribs, force factors of the contact interaction occur that lie on the ribs' planes (the resistance of the ribs upon deformation out of their plane are not taken into account); this assumption is made when considering problems of this type. When solving the problem, the boundary conditions of the hinge support are used.

Given the above assumptions, the equations of free oscillations of a shell that represents a casing fabricated of an orthotropic material, stiffened by a set of densely positioned transverse-longitudinal ribs and discretely positioned ribs, and loaded by external pressure and axial forces can be presented in the form [1]

$$L_{k1}u + L_{k2}v + L_{k3}w = -B(-1)^k(1-\delta_{k1})\sum_{q=1}^{m}(l_{k2}^q v_q + l_{k3}^q w_q)\delta(x - x_q), \quad (1)$$
$$B = (1-\nu_1\nu_2)R(E_1h)^{-1}; \quad k = 1,2,3,$$

where L_{kr} are the differential operators presented below; u, v, and w are the components of the displacement of the middle surface points; $\delta(x)$ is the delta function; Rx and Ry are the coordinates in the axial and circumferential directions; m is the number of discrete ring ribs; δ_{kr} is the Kronecker symbol; $l_{kr}^{(q)}$ are ordinary differential operators that contain the parameters of the discrete ring ribs and terms with the initial force T_q in the rib; R and h are the radius and thickness of the shell; $x = x_q$ is the contact line between the shell and the qth discrete ring rib; E_1 is the elastic modulus of the shell in the axial direction; ν_1 and ν_2 are Poisson's factors in the axial and circumferential directions; $v_q = v(x_q, y)$; and $w_q = w(x_q, y)$.

We take into consideration that for an orthotropic shell there exists dependence [2] $E_1\nu_2 = E_2\nu_1$, where E_2 is the elastic modulus of the shell in the circumferential direction.

According to the theory of shallow orthotropic cylindrical shells, if the smearing hypothesis holds true for a set of densely positioned transverse-longitudinal ribs [3], the operators L_{kr} that contain the terms with the values of the external pressure p and the axial compressive stress q can be written in the form

$$L_{11} = b_1\frac{\partial^2}{\partial x^2} + K\frac{\partial^2}{\partial y^2} - Bh\rho_0\frac{\partial^2}{\partial \tau^2},\quad L_{12} = L_{21} = (K + \nu_2)\frac{\partial^2}{\partial x\partial y},\quad L_{13} = L_{31} = \nu_2\frac{\partial}{\partial x} + Ba_1R^{-2}\frac{\partial^3}{\partial x^3};$$

$$L_{22} = K\frac{\partial^2}{\partial x^2} + E_1E_2^{-1}b_2\frac{\partial^2}{\partial y^2} - Bh\rho_0\frac{\partial^2}{\partial \tau^2},\quad L_{23} = L_{32} = E_1E_2^{-1}b_2\frac{\partial}{\partial y} + Ba_2R^{-2}\frac{\partial^3}{\partial y^3};$$

$$L_{33} = c^2\left[d_1\frac{\partial^4}{\partial x^4} + 2(2K + \nu_2)\frac{\partial^4}{\partial x^2\partial y^2} + d_2\frac{\partial^4}{\partial y^4}\right] + E_1E_2^{-1}b_2 + 2Ba_2R^{-2}\frac{\partial^2}{\partial y^2}$$

$$+ pB\frac{\partial^2}{\partial x^2} + qBhR^{-1}\frac{\partial^2}{\partial x^2} + Bh\rho_0\frac{\partial^2}{\partial \tau^2},$$

here

$$\rho_0 = \rho h + (AR)^{-1}\sum_{k=1}^{m_1}\rho_{1k}F_{1k} + (AR)^{-1}\sum_{k=1}^{m_2}\rho_{2k}F_{2k},$$

$$K = GBhR^{-1},\quad c^2 = h^2(12R^2)^{-1},\quad a_t = -l_t^{-1}\sum_{k=1}^{m_t}\varepsilon_{tk}E_{tk}F_{tk},\quad b_t = \zeta_t + B(R^2l_t)^{-1}\sum_{k=1}^{m_t}E_{tk}F_{tk},$$

$$d_t = \zeta_t + (DRl_t)^{-1}\sum_{k=1}^{m_t}(I_{tk} + \varepsilon_{tk}^2F_{tk}R^2),\quad \zeta_1 = 1,\quad \zeta_2 = E_1E_2^{-1},$$

$$D = E_1h^3\left[12(1 - \nu_1\nu_2)\right]^{-1},\quad t = 1,2,\quad k = 1,2,\ldots,m_t.$$

The subscript $t = 1$ refers to densely positioned longitudinal ribs; the subscript $t = 2$ refers to the transverse ribs; ε_t is the eccentricity of the rib, the distance from the center of mass of the rib section to the shell's middle surface relative to the shell radius, which, in the case of inner ribs, it is a positive quantity; m_t is the number of ribs; $l_1 = 2\pi$; $l_2 = l$; l is the dimensionless shell length; Rl is the shell length; E_t, F_t, and I_t are the elastic modulus, the cross-sectional area, and the intrinsic moment of inertia of the rib section; ρ and ρ_{ik} are the densities of the shell and rib; τ is time; and AR is the size of the panel over the arch (for a closed cylindrical shell, $A = 2\pi$).

If the cross section of the discrete ring rib q has a symmetry axis that passes normally to the shell through the reduced contact point, the operators $l_{kr}^{(q)}$ have the following form:

$$l_{22}^{(q)} = E_q^* F_q^* R^{-2}\frac{\partial^2}{\partial y^2} - \rho_q^* F_q^* \frac{\partial^2}{\partial \tau^2}, \quad l_{23}^{(q)} = E_q^* F_q^* R^{-2}\left(\frac{\partial}{\partial y} + \varepsilon_q^* \frac{\partial^3}{\partial y^3}\right) - \varepsilon_q^* \rho_q^* \frac{\partial^3}{\partial y \partial \tau^2},$$

$$l_{23}^{(q)} = -l_{32}^{(q)}, \quad l_{33}^{(q)} = -E_q^* F_q^* R^{-2}\left(1 + \varepsilon_q^* \frac{\partial^2}{\partial y^2}\right)^2 - E_q^* I_q^* R^{-4}\frac{\partial^4}{\partial y^4} - T_q R^{-2}\frac{\partial^2}{\partial y^2} - \rho_q^* F_q^* \frac{\partial^2}{\partial \tau^2},$$

where F_q^* and I_q^* is the area and the central moment of inertia of the rib section; E_q^* is the elastic modulus of the rib; ε_q^* is the dimensionless eccentricity of the rib similar to ε_t; and T_q is the initial axial force in the discrete ring rib.

When the shell (panel) is between parallel rigid plates, the subcritical momentless stresses in the shell under an axial compressive load are determined from the relation $q = QE_1\left(ARhE_1 + \sum_{k=1}^{m_1} E_{1k}F_{1k}\right)^{-1}$, where Q is an external axial compressive force.

The solution for a hinge-supported shell (a panel the longitudinal edges of which are freely supported) is found in the form

$$\{u, v, w\} = \{f_1(x)\cos\gamma y \sin\varpi\tau, f_2(x)\sin\gamma y\sin\varpi\tau, f_3(x)\cos\gamma y\sin\varpi\tau\}, \quad \gamma = 2\pi n/A,$$

where ϖ is the ring's natural-oscillation frequency and n is the parameter of the wave formation in the circumferential direction.

Then, for functions f_1, f_2, and f_3, we obtain the equations

$$\sum_{r=1}^{3} l_{kr} f_r + B(-1)^k(1 - \delta_{k1})\sum_{q=1}^{m}\sum_{j=2,3} a_{kj}^{(q)} f_{jq}\delta(x - x_q) = 0, \quad k = 1, 2, 3. \tag{2}$$

Here, l_{kr} are ordinary differential operators over the variable x; $f_{jq} = f_j(y_q)$; and $a_{kj}^{(q)}$ ($k = 1, 2, 3$) are constants that have the forms

$$a_{22}^{(q)} = -E_q^* F_q^* R^{-2}(\gamma^2 - M_q), \quad a_{23}^{(q)} = E_q^* F_q^* R^{-2}\gamma[1 - \varepsilon_q^*(\gamma^2 - M_q)], \quad a_{32}^{(q)} = a_{23}^{(q)},$$

$$a_{33}^{(q)} = -E_q^* F_q^* R^{-1}[(1 - \varepsilon_q^*\gamma^2)^2 - M_q] - E_q^* I_q^* R^{-4}\gamma^4 + T_q R^{-2}\gamma^2,$$

$\Omega = 2\pi f\sqrt{BR\rho_0}$, f is the frequency in hertz; and $M_q = \rho_q^* R\Omega^2(\rho_0 B E_q^*)^{-1}$.

For the case of a hinge-supported shell, the solution of Eqs. (2) obtained by the operator method [4] can be written in the form

$$f_r = B\sum_{q=1}^{m}\sum_{k=2}^{3}\sum_{j=2}^{3}(-1)^k a_{kj}^{(q)} f_{jq} K_{kr}(x, x_q),$$

$$K_{kr}(x, x_q) = 2\sum_{t=1}^{4}(\Delta_{kr}(s_t)\Delta'(s_t)^{-1})\{[\sinh s_t x(\sinh s_t l)^{-1}]\sinh s_t(l - x_q) - \sinh s_t(x - x_q)\sigma_0(x - x_q)\}, \tag{3}$$

$$r = 2, 3,$$

where $\Delta(s)$ is the determinant the elements c_{kr} of which are obtained from the operators l_{kr} by replacing the differentiation operation by the transformation parameter s; Δ_{kr} is an algebraic complement to the element c_{kr}; and s_t are the roots of the equation $\Delta(s) = 0$.

The expressions for c_{kr} have the following form:

$$c_{11} = \Omega^2 + b_1 s^2 - K\gamma^2; \quad c_{12} = -(K + \nu_2)\gamma s; \quad c_{13} = \nu_2 - Ba_1R^{-2}s^3; \quad c_{21} = -c_{12};$$

$$c_{22} = \Omega^2 + Ks^2 - E_1E_2^{-1}b_2\gamma^2; \quad c_{23} = E_1E_2^{-1}b_2\gamma + Ba_2R^{-2}\gamma^3; \quad c_{31} = c_{13}; \quad c_{32} = -c_{23};$$

$$c_{33} = c^2[d_1s^4 - 2(2K + \nu_2)\gamma^2 s^2 + d_2\gamma^4] + E_1E_2^{-1}b_2 + 2Ba_2R^{-2}\gamma^2 - pB\gamma^2 + qBhR^{-1}s^2 - \Omega^2.$$

Table 1. Geometric characteristics of shells and ribs

Shell number	Shell number h, cm	Cross-sectional height of rib, cm	Cross-sectional width of rib, cm
1	0.070	–	–
2	0.095	0.5	0.37
3	0.087	0.5	0.37

Table 2. Comparison of calculated and experimental data

n	Shell no. 1			Shell no. 2			Shell no. 3		
	f^t, Hz	f^o, Hz	$\frac{f^t}{f^o}$	f^t, Hz	f^o, Hz	$\frac{f^t}{f^o}$	f^t, Hz	f^o, Hz	$\frac{f^t}{f^o}$
2	362	–	–	351	324	1.08	349	291	1.20
3	199	245	0.81	251	250	1.00	233	236	0.99
4	144	174	0.83	317	348	0.91	263	267	0.98
5	158	162	0.9	366	–	–	318	–	–

Assuming successively that in Eqs. (3) $x = x_1, x_2, \ldots, x_q$, we obtain a system of $2q$ homogeneous algebraic equations with respect to f_{2q} and f_{3q}. The characteristic equation is obtained from the conditions of the determinant of this system being equal to zero.

For identical equally loaded and uniformly positioned discrete ribs, the quantities a_{kj} do not depend on the number of the discrete rib; therefore, below, the subscript q is omitted and $x_q = ql(m+1)^{-1}$.

We seek a solution to system (3) in the form $f_{2r} = a\sin(\pi N x_q)l^{-1}$, $f_{3r} = b\sin(\pi N x_q)l^{-1}$, $1 \le N \le m+1$, where a and b are constants and N is an integer that characterizes the oscillation mode; we obtain the characteristic equation as follows:

$$B^2(a_{22}a_{23} - a_{23}a_{32})(\Phi_{32}\Phi_{23} - \Phi_{22}\Phi_{33}) - B(a_{22}\Phi_{22} - a_{23}\Phi_{32} + a_{32}\Phi_{23} - a_{33}\Phi_{33}) + 1 = 0,$$

where

$$\Phi_{kr} = \sum_{t=1}^{4} \Delta_{kr}(s_t)\sinh\alpha_t[\Delta(s_t)(\sinh\alpha_t - \cos\beta)]^{-1},$$

$$\alpha_t = s_t l(m=1)^{-1}, \quad \beta = \pi N(m+1)^{-1}, \quad \Phi_{23} = -\Phi_{32}.$$

The characteristic equation solved with respect to the flexural rigidity of a discrete rib in the dimensionless form has the form

$$E^*I^*(DR)^{-1} = TR^2(D\gamma^2)^{-1} + \Psi_1\Psi_2^{-1}, \tag{4}$$

where

$$\Psi_1 = 1 + z[(\gamma^2 - M)\Phi_{22}\Phi_{23} - A_1\Phi_{23} - A_2\Phi_{33}] + z^2A_3(\Phi_{23}^2 + \Phi_{22}\Phi_{33}),$$

$$\Psi_2 = c^2\gamma^4[\Phi_{33} + z(\gamma^2 - M)(\Phi_{23}^2 + \Phi_{23}\Phi_{33})], \quad A_1 = 2\gamma[1 - \varepsilon^*(\gamma^2 - M)],$$

$$A_2 = (1 - \varepsilon^*\gamma^2)^2 - M(1 - \varepsilon^*\gamma^2), \quad A_3 = (1 - \gamma^2 - M), \quad z = BE^*F^*R^{-2}.$$

Equation (4) is a closed expression and allows determining the free-oscillation frequency of a reinforced shell (panel) loaded by external pressure and axial forces.

Prescribing different integer values on n and N, where $1 \le N \le m+1$, we find the value of the stiffness of a discrete rib that corresponds to a specific oscillation frequency of the shell. And vice versa, this frequency will correspond to the found stiffness of the rib.

To confirm the possibility of using the obtained data to calculate real structures, the calculated and experimental data on the oscillations of model shells were compared.

A shell manufactured of fiberglass had the following mechanical characteristics: $E_1 = 1.76 \times 10^{10}$ N/m^2; $E_2 = 2.74 \times 10^{10}$ N/m^2; $G = 9.46 \times 10^9$ N/m^2; $\nu_1 = 0.12$; $\nu_2 = 0.187$; and $\rho = 1.9$ g/cm^3.

Three shells were tested, one of which was smooth and two shells were reinforced in the midsection by an internal ring rib. The ribs manufactured of the D16-A5 material (ASM AA2024) had the following mechanical characteristics: $E^* = 6.66 \times 10^{10}$ N/m^2 and $\rho^* = 2.78$ g/cm^3.

The geometric characteristics of the shells and ribs are presented in Table 1.

In the calculations according to Eq. (4), the initial force in the ribs T was assumed to be zero. The calculated natural oscillation frequencies f^t and the experimental natural oscillation frequencies f^o for different wave formation parameters n in the circumferential direction for the first oscillation mode ($N = 1$) are presented in Table 2. It can be seen that the maximum divergence between the calculated and experimental data does not exceed 20%. There is satisfactory agreement between the calculated and experimental data; consequently, the proposed method can be used in the calculation in practice.

REFERENCES

1. Solomonov, Yu.S., Georgievskii, V.P., Nedbai, A.Ya., and Andryushin, V.A., *Prikladnye zadachi mekhaniki kompozitnykh tsilindricheskikh obolochek* (Applied Problems of Mechanics of Composite Cylindrical Shells), Moscow: Fizmatlit, 2013.
2. Ambartsumyan, S.A., Teoriya anizotropnykh obolochek (A Refined Theory of Anisotropic Shells), Moscow: Fizmatgiz, 1961.
3. Karmishin, A.V., Lyaskovets, V.A., Myachenkov, V.I., and Frolov, A.N., *Statika i dinamika tonkostennykh obolochechnykh konstruktsii* (Statics and Dynamics of Thin-Walled Shelled Structures), Moscow: Mashinostroenie, 1975.
4. Lur'e, A.I., *Operatsionnoe ischislenie i ego prilozhenie k zadacham mekhaniki* (Operational Calculus and its Application to Problems of Mechanics), Moscow: Gostekhizdat, 1950.

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