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MECHANICS
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Kinematics of Motions in Rotor System during Contact between Rotor and Floating Elements

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Abstract—The contact regimes in the rotor system when the contact of the rotor with light movable elements of the system have been considered The vibrations of an imbalanced rotor–floating ring system when rotating the rotor inside the floating ring when permanent contact with them have been investigated. Furthermore, the ring vibrations occur, during of which it runs the rotor when permanent contact with it in the direct precession regime (the precession direction coincides with the rotation direction) (hula hoop-type vibration). This article considers the kinematics of ring motion. The trajectories of ring motion in both absolute and relative motion at different ratios between the radii of the ring and rotor have been plotted. It has been revealed that the ring vibrations are a direct asynchronous precession; furthermore, the ring points circumscribe epitrochoids and the trajectory shape depends significantly on the ratio between the radii of the rotor and the ring.

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STATEMENT OF THE PROBLEM

Contact between rotating rotors and stator elements often occurs in rotor systems, as well as dangerous vibrations due to, e.g., the rolling of the rotor along a static stator in the retrograde precession regime, as well as vibroimpact regimes and different combinations of these regimes. These emergency regimes often lead to serious accidents. The works of many authors are devoted to the theoretical and experimentally calculated investigation of these regimes [1–7]. The contact regimes when the rotation of the rotor inside light movable elements of the system can be equally dangerous. These vibrations can occur, e.g., during the rotation of the rotor in bearings with floating sleeves and during the rotation of the rotor inside floating seals [8–10]. The floating seals are currently successfully used in the rotor systems. Unlike the traditional labyrinth and slot seals, they make it possible to respond to a relative displacement between the rotor and the seal. Furthermore, the operational clearance is kept at a minimum, which provides a small gas leak and good sealing closure. The emergence of contact regimes can damage and even destroy these elements and, as a consequence, they disrupt the operation of the entire rotor system.

The similar contact regimes in the contact of rotating rotor with the circular ring is specially kept in vibratory machines when processing and grinding materials in vibroinertial crushers. I. I. Blekhman studied this case of vibration maintenance of stationary planetary motion in detail [2]. Furthermore, ring vibrations occur during which it runs the rotor that is in permanent contact with it in the direct precession regime (the direction of the precession coincides with the rotation direction) (hula hoop-type vibrations) [3, 11]. Further, unlike traditional quite well-studied rolling with retrograde precession, we will call this refer to this regime as running the ring around the rotor. This article investigates the kinematics of motions in the rotor–floating ring system when the rotor is in permanent contact with the ring (Fig. 1).

MATHEMATICAL MODEL OF KINEMATICS OF MOTIONS OF THE ROTOR–FLOATING RING SYSTEM IN THE RUNNING REGIME

In the investigation of kinematics, we assume that there are not the dynamic loads in the system.

The natural assumption should be made that the vibrations of the light ring have little influence on the rotor vibrations. Then, the geometric center O_1 of the rotor under the influence of the imbalance force executes circular vibrations (direct precession) with amplitude a around the rotation axis O (Fig. 1b, circle *3*).

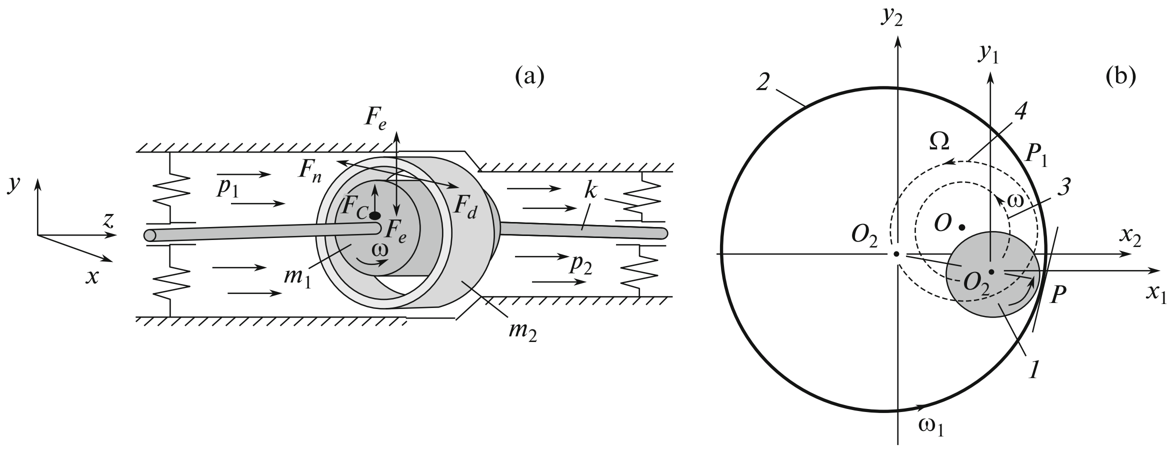

Fig. 1. Rotation of the rotor inside the floating sealing ring: (a) design model; (b) definition of the rotational velocity and the precession velocity of the ring: O is the center of rotation of the system (*1*) rotor, (*2*) floating ring, (*3*) trajectory of rotor center in the direct synchronous precession regime, (*4*) trajectory of the ring center; P is the point of contact of the ring and the rotor in the point of time t_0, P_1 is the position of point P of the ring after the turn at an angle of 2π.

The coordinate system $O_1x_1y_1$ in Fig. 1b is related to the rotor center of $O_1x_1y_1$, and the coordinate system $O_2x_2y_2$ is related to the center of the ring. The precession amplitude a is determined by the rotating velocity of the rotor ω and the magnitude of the imbalance. As a result of the superposition of vibrations with the rotation frequency and the direct precession of the rotor center at the same frequency, the rotor executes a lunar motion, in which each point of the rotor keeps its position relative to the rotation axis [12]. This determines the specificity of the rotor systems. Logically, it is sufficient to suppose that, in this case, the contact of the ring with the rotor takes place at point P of the rotor orbit, which is the most remote from the center of rotation (Fig. 1b). Thus, in the case of the direct synchronous precession, the point of contact P on the rotor always remains the same. When the rotor that rotates this point circumscribes a circle of radius $r + a$, the center of which lies on the rotation axis O (circle *3* in Fig. 2, r is the rotor radius). When analyzing the kinematics, it is assumed that the ring rolls the rotor without breaking or slippage. In this case, the geometric center O_2 of the ring also circumscribes a circle (circle *4* in Fig. 1b) concentric to circle *3*. The radius of this circle equals to $\delta = R - (r + a)$, where R is the ring radius. Depending on the ratio $\delta > a$ or $\delta < a$, circle *4* can be located inside or outside circle *3*. With an increase in the rotation velocity, the probability of the internal location of circle *4* increases. For the rotor systems, one can suppose that circle *4* is outside of circle *1* (Fig. 1b).

When the rotor is in contact with the ring at point P, the geometric center of rotor O_1 is on circle *3* and the geometric center of ring O_2 lies on circle *4*. Since the rotor executes the direct synchronous precession around the rotation center O with rotation velocity ω, then it imposes the precession rate Ω around the same center O on the ring.

<u>*Note.*</u> In the case of the classical Hula Hoop exercise [10], an athlete's body only executes translational motion without rotation, i.e., lunar motion does not occur and, therefore, contact occurs between the body and the ring each time at different points.

DEFINITION OF THE ROTATION VELOCITY AND THE VELOCITY OF THE DIRECT PRECESSION OF THE RING

At point of time of the contact at point P, the velocities of the rotor and the ring are equal to each other as follows: $V_{rot} = \omega(r + a) = V_{ring} = (R + \delta)\omega_1 - \Omega\delta$, ("–" means that the velocity of the ring center O_2 is opposite to the direction of ring velocity at point of contact P). Consequently, the angular velocity of ring rotation ω_1 is determined by the relation

$$\omega r' = (R + \delta)\omega_1 - \Omega\delta, \quad r' = r + a, \quad \delta = R - r' \tag{1}$$

and its direction coincides with the direction of the rotational velocity of the rotor.

The precession velocity of the ring can be determined based on the following conditions. The ring radius exceeds the rotor radius; therefore, the complete revolution of the ring around its rotational center

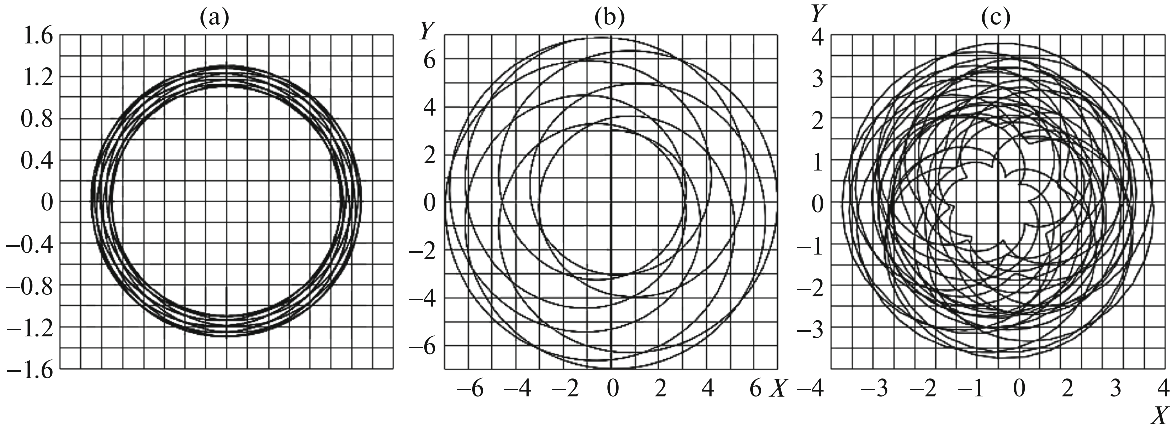

Fig. 2. Ring trajectory with different relations $b = R/r'$ between the radii of rotor R and ring r: (a) $b = 1.1$, (b) $b = 5$; (c) trajectory of relative motion of the ring from the point of view of an observer located on the rotating rotor.

O_2 will be at point P_1 of the rotor so that the distance S along the circular arc PP_1 would turn out to be equal to $S = 2\pi(R - r')$ (Fig. 1b). The rotation of the ring by angle $2\pi + S$ will take place during time $t_1 = 2\pi(R + \delta)/\omega_1$, which determines the velocity of ring precession around point O, the precession radius is equal to δ, and its period equals to $2\pi\delta/\Omega$. Thus, the following equality is fulfilled:

$$\frac{2\pi(R+\delta)}{\omega_1} = \frac{2\pi\delta}{\Omega}, \qquad \Omega = \frac{\delta}{R+\delta}\omega_1. \tag{2}$$

The velocities of ring rotation and ring precession depending on the rotor rotational velocity and the relations between the radii of the ring R and the rotor precession r' are determined from Eqs. (1) and (2) as follows:

$$\omega_1 = \frac{2b-1}{b^2+2b-2}\omega, \qquad \Omega = \frac{b-1}{b^2+2b-2}\omega. \tag{3}$$

Depending on the relation $b = R/r'$, the different relations between the ring rotational velocity ω_1 and the velocity of its precession Ω are possible. Consider these cases.

1. The values of the rotor and ring radii are fairly close to each other, and the rotor imbalance is low. Let $R \approx 1.1r'$. This ratio usually takes place in the case of the sealing ring. Then, as follows from Eqs. (1) and (2), the ring rotational velocity is $\omega_1 = 0.89\omega$, and the precession velocity is smaller by orders of magnitude, i.e., $\Omega \approx 0.1\omega_1$.

2. The radii R and r' are related by the dependence $R = br'$ $(1 < b < 10)$.

When $b = 2$, the ring rotational velocity is $\omega_1 = 0.5\omega$ and the precession velocity is $\Omega \cong 0.165\omega$; when $b = 5$, $\omega_1 = 0.27\omega$ and $\Omega \cong 0.12\omega$.

Thus, unlike traditional rolling with retrograde precession, the ring precession in the running regime is direct and the precession velocity is less than the ring rotational velocity ω_1. With a decrease in the clearance value and imbalance, $R \approx 1.1r'$, these velocities are equalized, and the ring motion tends to direct the synchronous precession.

As follows from Eqs. (2) and (3), the trajectory of ring points is a superposition of vibrations with two frequencies, i.e., with the rotational frequency ω_1 and the frequency of the direct asynchronous precession Ω. The equation of the ring trajectory is

$$\begin{aligned} x &= (R+\delta)\sin\omega_1 t - \delta\sin\Omega t = (2R - r')\sin\omega_1 t - (R - r')\sin\Omega t; \\ y &= (R+\delta)\cos\omega_1 t - \delta\cos\Omega t = (2R - r')\cos\omega_1 t - (R - r'))\cos\Omega t. \end{aligned} \tag{4}$$

Using relations (4), let us plot the ring trajectories when different ratios of the radii R to r'.

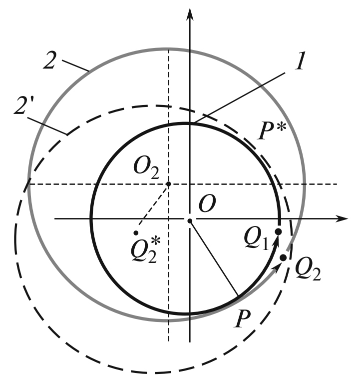

Fig. 3. (*1*) Circular orbit of point P of rotor maximum remote from rotation axis O, (*2*) ring position when contact with rotor at point P at point of time t_0, (*2*') ring position at point of time $t_0 + \Delta t$ with its contact with the rotor at point P^*.

In this case, the ring in accordance with Eq. (5) executes vibrations close to harmonic oscillations because the amplitudes of the slow vibrations with the frequency of direct precession are fairly small (Fig. 2a).

$$R \approx 1.1r':$$
$$x = 1.2\sin\omega_1 t - 0.1\sin(0.1\omega_1 t);$$
$$y = 1.2\cos\omega_1 t - 0.1\cos(0.1\omega_1 t). \tag{5}$$

In this case, the ring trajectory is the superposition of vibrations with two commensurable frequencies (Fig. 2b). The trajectory of the relative motion of the ring is also of interest from the point of view of an observer located on the rotating rotor as shown below (Fig. 2c):

$$R = 5r':$$
$$x = 5\sin\omega_1 t - 2\sin(0.44\omega_1 t);$$
$$y = 5\cos\omega_1 t - 2\cos(0.44\omega_1 t). \tag{6}$$

GEOMETRIC CONSTRUCTION OF CONSECUTIVE POINTS OF CONTACT BETWEEN THE ROTOR AND RING

One can graphically define the consistent points of contact between the rotor and ring, as is shown in Fig. 3. Point P of the rotor, which is maximally remote from the rotational center O circumscribes circle *1* with radius $OP = r + a$. According to the above assumption, the points of contact of the ring with the rotor lie on this circle. For the time interval Δt, the point P on the rotor will move to point Q at distance $\Delta s = \omega(r + a)\Delta t$. For the same time interval, due to its rotation, point P on the ring will move to point Q_1 at a distance of $\Delta s = \omega_1 R\Delta t$; furthermore, due to the precession, the ring will move at the distance $\Delta s_1 = \Omega(R - r')\Delta t$ along the circle of precession; in Fig. 3, this offset is designated as $O_2O_2^*$. As a result, the next point of intersection of the ring with circle *1*, i.e., the point of rotor contact with the ring, will be in the point P^*.

When the graphical definition of the points of contact of the ring with the rotor above, it was assumed that the rotor executes the direct synchronous precession (lunar motion). However, the presented graphical construction is right and, in the general case of rotor motion, when its trajectory will no longer be a circle and the radius of its precession will depend on time $a = a(t)$. In this case, the ring every time can come into contact with the rotor in the points with different distance from the rotation axis and, hence, the ring velocity that the rotor imposes on it, as well as the velocity of its precession, will be a variable. However, in this case, the regime of direct asynchronous precession for the ring will also be kept.

CONCLUSIONS

—The ring rotational velocity is less than the rotational velocity of the rotor.

—The precession of the ring in the running regime is direct, and the precession velocity is less than the ring rotational velocity ω_1, which essentially distinguishes it from the rolling regime.

—If the rotor and the ring radii are close, then the ring executes vibrations close to harmonic oscillations with the rotational frequency of the rotor.

—With an increase in the relation R/r', the motion of the ring is a direct asynchronous precession.

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