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On Bending and Stability of Beams and Plates Laying on a Continuous Nonlocally Elastic Foundation

V. D. Potapov

Moscow State University of Railway Engineering, Moscow, 127994 Russia

e-mail: potapov.vd@mail.ru

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Abstract—When designing beams and plates laying on a continuous elastic foundation, the simplest foundation model proposed by Winkler is normally used. This hypothesis was frequently criticized with good reason, for it does not consider involvement in the work of those areas of the foundation in the vicinity of the concentrated reaction point. In order to refine Winkler's hypothesis, numerous authors have proposed other models that enable the drawbacks of Winkler's model to be smoothed out to different degrees. In recent years, a different approach to solving the same problems is considered when the foundation is regarded as nonlocally elastic. Here, the effect of nonlocality of the foundation on the deformed state and the stability of beams and plates laying on a continuous elastic foundation is analyzed.

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INTRODUCTION

The design of beams and plates laying on a continuous elastic foundation is normally based on the simplest foundation model proposed by E. Winkler [1], according to which the reaction of the foundation at the point in question is considered proportional to the displacement of the edge of the foundation at the same point. In other words, the foundation is represented as linearly elastic springs unconnected with each other; therefore, during a concentrated reaction, only the spring in which this reaction acts becomes strained. Winkler's hypothesis was frequently criticized with good reason, since it does not consider involvement in the work of those areas of the foundation located in the vicinity of the concentrated reaction point. The model proposed by M.M. Filonenko-Borodich [2] can be regarded as a certain refinement of Winkler's model; Filonenko-Borodich considered the model of a soil foundation in the form of springs connected at the upper ends by a unstretchable horizontal cord rigidly fixed outside the foundation.

P.L. Pasternak developed an elastic foundation model remarkable for two soil reaction coefficients [3]. In this model, the reaction of the foundation depends not only on the magnitude of the beam deflection at the point under consideration, but also on the beam's deflection curvature at the same point. In [4], a more advanced three-parameter model of the elastic foundation was proposed. However, the models of Filonenko-Borodich and Pasternak and Kerr [4], like Winkler's model, are local, since they do not consider the effect (reaction) of the elastic foundation at points distant from the point in question. In addition, other models were proposed that are based on solutions to elasticity theory problems for an elastic half-plane, elastic half-space [5, 6], and an elastic layer [7].

A sufficiently thorough review of models and works as a whole on problems related to the design of beams and plates on elastic foundations is presented in [6]. A drawback of the models based on solutions to elasticity theory problems is that they do not sufficiently correctly describe the stress distribution in the soil body under the foundation. This is accounted for, on the one hand, by peculiarities of the solutions to contact problems of elasticity theory, the occurrence of infinitely large stresses in the half-space or the half-plane under the beam ends or plate edges. On the other hand, it is obvious that the soil body must not be equated with an elastic isotropic material. This especially concerns weakly cohesive or completely noncohesive soils. For example, with such soils, the stress distribution under rigid bars (stamps) is not consistent with the solutions to elasticity theory problems [3, 5].

In recent years, in connection with the creation of new composites and nanomaterials, much attention in mechanics is devoted to analyzing the stress–strain state of structures taking into account nonlocal effects [8–10]. Apparently, for the first time, such an approach to a continuous elastic foundation was proposed in [11], where the deflection of the beam at the point in question $w(x)$ is defined by the integral of the reaction intensity $r(x)$ of the elastic foundation as

$$w(x) = (1/c)\int_{-a}^{a} K(|x - x^*|)r(x^*)dx^*, \tag{1}$$

where $2a$ is the length of the beam, c is a constant analogous to the coefficient of the elastic foundation reaction, and $K(|x - x^*|)$ is the influence function taken as an exponent

$$K(|x - x^*|) = (\mu/2)e^{-\mu|x - x^*|},$$

where μ is constant.

Integral relation (1) is reduced to the differential equation

$$\frac{d^2w(x)}{dx^2} - \mu^2 w = -(1/c)\mu^2 r, \tag{2}$$

the solution to which has to meet two boundary conditions.

As a result, the solution to the problem of the beam's deflection reduces to the solution to the differential equation

$$EI\frac{d^4w(x)}{dx^4} - (c/\mu^2)\frac{d^2w(x)}{dx^2} + cw(x) = q(x), \tag{3}$$

where EI is the bending stiffness of the beam and $q(x)$ is the intensity of the distributed load.

It must be emphasized that, according to the accepted hypothesis, the solution to Eq. (3) has to meet six boundary conditions. As a consequence of the inconsistency between the order of differential equation (3) and the number of boundary conditions, the latter circumstance leads to the occurrence of apparent concentrated reactions at the beam ends. The solution to this equation looks significantly different from the solution to the equation derived when using the Winkler hypothesis:

$$EI\frac{d^4w(x)}{dx^4} + cw(x) = q(x). \tag{4}$$

Another approach to solving the same problem from the viewpoint of nonlocality was proposed in [12] to determine the natural frequency of the beams resting on a nonlocal foundation taking into account nonlocal damping. The dependence between the reaction $r(x, t)$ and the deflection $w(x, t)$ is proposed in the form of the relation

$$r(x,t) = \int_{x_1}^{x_2} K(x,\xi)w(\xi,t)d\xi + \int_{x_1}^{x_2}\int_{-\infty}^{t} C(x,\xi,t-\tau)\frac{\partial w(\xi,\tau)}{\partial \tau}d\tau d\xi,$$

where $K(x, \xi)$ is a function that takes into account the nonlocality of the foundation; $C(x, \xi, t - \tau)$ is a function that considers the nonlocality of the foundation damping; and x_1, x_2 are the coordinates of the beginning and the end of the elastic foundation.

The function $K(x, \xi)$ is taken as an exponent

$$K(|x - \xi|) = (\mu/2)e^{-\mu|x-\xi|}, \tag{5}$$

a Gaussian function $K(|x - \xi|) = (\mu/\sqrt{2\pi})\exp[-\mu^2(x - \xi)^2/2]$, or a triangular function

$$K(|x - \xi|) = \begin{cases} \frac{\mu}{2}\left(1 - \frac{\mu}{2}|x - \xi|\right) & \text{for} \quad |x - \xi| \le \frac{2}{\mu}, \\ 0, & \text{in other cases,} \end{cases}$$

It should be noted that c, a coefficient in terms of kN/cm^2, should be included in the above expressions for function $K(x, \xi)$.

To solve the problem, the finite element method is used.

However, in this form, the model of nonlocal elasticity of the foundation does not always yield results that considerably differ positively from those obtained using a Winkler foundation model.

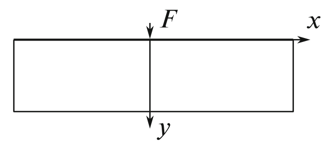

Fig. 1.

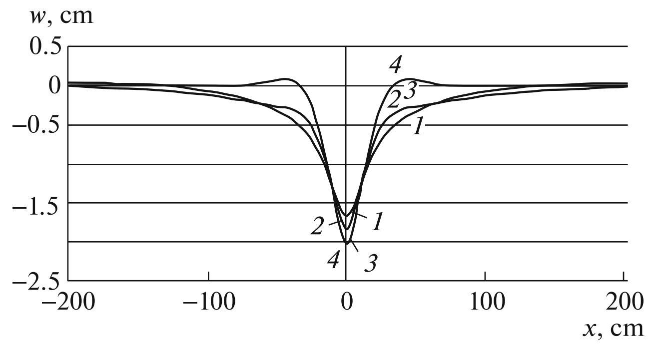

Fig. 2.

In this article, a similar approach, although in a different form, is applied to solve the problem of bending and stability of beams and plates laying on an elastic foundation taking into account the latter's nonlocality.

BENDING OF BEAMS RESTING ON A CONTINUOUS NONLOCALLY ELASTIC FOUNDATION

The deflection of the beam is found in this case from the solution to the integro-differential equation

$$EI\frac{d^4w(x)}{dx^4} + c\int_0^L C(|x-\xi|)w(\xi)d\xi = q(x), \quad (6)$$

where L is the length of the beam, c is a constant, and the function $C(|x-\xi|)$ meets the normalization requirement $\int_{-\infty}^{\infty} C(|x-\xi|)d\xi = 1$.

The solution to Eq. (6) has to meet boundary conditions written in the same way as those for beams that lay on a Winkler foundation.

Then, the expression

$$C(|x-\xi|) = c_0\delta(x-\xi) + c_1(\nu/2)e^{-\nu|x-\xi|}, \quad (7)$$

is taken as the function $C(|x-\xi|)$, where $\delta(x-\xi)$ is the delta function and c_0, c_1, and ν are constants.

The second term in Eq. (7) can be interpreted as a "correction" to Winkler's model.

We use the following generalized two-dimensional problem (Fig. 1) [15] as an example to illustrate the efficiency and accuracy of the proposed foundation model. Let us consider an elastic rectangular plate 1 cm thick, 400 cm long, and 100 cm high. The elasticity modulus of the plate material is 10000 kN/cm^2, and the Poisson ratio is 0.2. Along the side edges, constraints are imposed that exclude any displacement along axes x and z with axis z being perpendicular to the plate plane; along the lower edge, constraints are imposed that exclude any displacement along axes y and z. A steel beam with a square cross section and a side of 1 cm is glued onto the upper edge of the plate. The structure is loaded by a vertical concentrated force of 10000 kN applied to the plate's axis of symmetry and directed from the top downward. We calculate this two-dimensional problem by the finite element method selecting square finite elements with dimensions of 5 × 5 cm. The beam is also divided lengthwise into 5-cm-long finite elements.

The deflection of the steel beam has the shape indicated by line *1* in Fig. 2. The maximum deflection of the beam in the final model is 1.67 cm.

The other two curves were obtained for a beam laying on a solid elastic foundation using the proposed model of a nonlocally elastic foundation and a Winkler foundation. The calculation was performed by the finite difference method with an increment of 5 cm. The error of the obtained approximations was estimated using the mean-square criterion

$$s = \sqrt{\sum_{i=1}^{81} (w[i] - p[i])^2 / 81},$$

where $p[i]$ represents the values of the deflection found by the finite element method.

The following values of the foundation parameters correspond to the local minimum error s for the nonlocal elastic foundation model: $cc_0 = 353.5$ kN/cm^2, $cc_1 = -258.5$ kN/cm^2, $\nu = 0.034$ 1/cm and the error itself $s = 0.009$ cm. These data are represented by curve *2* in Fig. 2 with a maximum of $w_{\max} = 1.83$ cm.

The corresponding data for Winkler's model have the following values: $c = 185$ kN/cm^2, $s = 0.024$ cm, and $w_{\max} = 2$ cm. Curve *3* in Fig. 2 gives an idea of the change in the deflection of the beam in this case. The values of the characteristics for two variants of the elastic foundation model show that the nonlocally elastic foundation model makes it possible to obtain more correct results. The same conclusion can be drawn by comparing curves *2* and *3*.

Let us compare the results with the corresponding results obtained with the nonlocal foundation model described by one exponent (5). In this case, the following values of the characteristics of the foundation and the deflection correspond to the minimum value of error $s = 0.026$ cm: $c = 186$ kN/cm^2, $\mu = 2.48$ 1/cm, and $w_{\max} = 2.02$ cm. The deflection of the rod is represented in this case by curve *4* in Fig. 2, which practically coincides with curve *3* that corresponds to a Winkler foundation. This comparison convincingly proves that the foundation model that uses Eq. (5) does not always appear more efficient compared with Winkler's model.

STABILITY OF THE BEAMS LAUING ON A CONTINUOUS NONLOCALLY ELASTIC FOUNDATION

Let us consider an infinitely long beam compressed by the longitudinal force F; the beam lays on a continuous foundation for which the influence function $C(|x - \xi|)$ has the form of Eq. (7). The equation, analogous to Eq. (6), is written in this case as

$$EI\frac{d^4 w(x)}{dx^4} + F\frac{d^2 w(x)}{dx^2} + c\int_0^L C(|x-\xi|)w(\xi)d\xi = 0. \tag{8}$$

The solution to Eq. (8) is sought in the form

$$w(x) = e^{ikx}\Phi(k). \tag{9}$$

On substituting the expression $e^{ikx}\Phi(k)$ into Eq. (8) and reducing it by e^{ikx}, we obtain

$$\left[k^4 EI - k^2 F + c\int_{-\infty}^{\infty} C(|y|)e^{-iky}dy\right]\Phi(k) = 0, \tag{10}$$

where $y = x - \xi$.

If the function $C(|x - \xi|)$ has the form of (7), Eq. (10) is written as

$$\{k^4 EI - k^2 F + c[(c_0 + c_1\nu^2/(\nu^2 + k^2)]\}\Phi(k) = 0.$$

The following equality corresponds to the critical state of the rod:

$$\{k^4 EI - k^2 F + c[(c_0 + c_1\nu^2/(\nu^2 + k^2)]\} = 0. \tag{11}$$

Hence, we express the force as

$$F = k^2 EI + c\{c_0/k^2 + c_1\nu^2/[k^2(\nu^2 + k^2)]\}. \tag{12}$$

To determine the critical value of the longitudinal force F_{cr}, we differentiate relation (12) with respect to k^2 and set the result equal to zero. From the resulting equation, we find the root and then the value of force F_{cr} from expression (12). If we take the same parameter values as those in the previous section,

$EI = 1750000$ kN/cm^2, $cc_0 = 353.5$ kN/cm^2, $cc_1 = -258.5$ kN/cm^2, and $\nu = 0.034$ 1/cm, we obtain $F_{cr} =$ 48 298.6 kN. The half-wavelength of the mode of the beam's buckling λ appears to be 8.64 cm.

For comparison, we present the expression and values of the critical force for a beam laying on Winkler's foundation (4) with the characteristic $c = 186$ kN/cm^2:

$$E_{cr} = \sqrt{4EIc} = 36083.2 \text{ kN}.$$

The half-wavelength λ appears to be 30.94 cm in this case.

It can be seen that consideration of nonlocality of the foundation results in a noticeable increase in the critical force and a reduction in the half-wavelength of the mode of the baem's buckling.

STABILITY OF PLATES LAYING ON A CONTINUOUS NONLOCALLY ELASTIC FOUNDATION

Let us consider an infinite elastic plate that layers on a continuous nonlocally elastic foundation and is uniformly compressed in the direction of two coordinate axes x and y. The deflection of the plate in the case of bifurcation of the deformed state is determined by the equation

$$D\nabla^4 w + N_x \partial^2 w/\partial x^2 + N_y \partial^2 w/\partial y^2 + c\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(x-\xi, y-\zeta) w(\xi,\zeta) d\xi d\zeta = 0, \tag{13}$$

where N_x and N_y are the intensities of the uniformly distributed compression loads that act in the direction of axes x and y.

By analogy with the representation of the function $C(|x-\xi|)$, we select the function $C(x-\xi, y-\zeta)$ in the one-dimensional case in the form [14]

$$C(x-\xi, y-\zeta) = [c_0\delta(d) + c_1(\eta^2/2\pi)e^{-\eta d}], \tag{14}$$

where η is a parameter that characterizes the scale of nonlocality; $d = \sqrt{(x-\xi)^2 + (y-\zeta)^2}$ is the distance between two points of the plate with the coordinates x and y and ξ and ζ.

This variant of the function $C(x-\xi, y-\zeta)$ can be interpreted as a variant of an "isotropic" nonlocally elastic foundation.

If the foundation material has different reactive properties in the directions of coordinate axes x and y, a model of an "orthotropic" nonlocally elastic foundation can be proposed. In particular, the exponential form of the function $C(x-\xi, y-\zeta)$ can be applied [14] as

$$C(x-\xi, y-\zeta) = [c_0\delta(d) + c_1(\eta\nu/4)e^{(-\eta|x-\xi|-\nu|y-\zeta|)}], \tag{15}$$

where η and ν are parameters that characterize the scales of nonlocality in the direction of axes x and y, respectively.

We seek function $w(x, y)$ in the form

$$w(x,y) = e^{i(k_x x + k_y y)}\Phi(k_x, k_y). \tag{16}$$

Substituting expression (16) into Eq. (15), we obtain the equation for the values of bifurcation loads N_x and N_y:

$$(k_x^2 + k_y^2)^2 D - (k_x^2 N_x + k_y^2 N_y) + c\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(|z|, |\chi|) e^{-i(k_x z + k_y \chi)} dz d\chi = 0,$$

where $z = x - \xi$ and $\chi = y - \zeta$.

Then, we restrict ourselves to the case when the intensities of loads N_x and N_y are considered equal to $N = N_x = N_y$.

Then, in the case of an isotropic foundation material, we obtain

$$N = (k_x^2 + k_y^2)D + cc_0/(k_x^2 + k_y^2) + cc_1\eta^2/[2\pi(k_x^2 + k_y^2)]\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\eta d} e^{-i(k_x z + k_y \chi)} dz d\chi. \tag{17}$$

k_x, cm^{-1}	k_y, cm^{-1}	N_{cr}, kN/cm^2.
0	0.1153	51716.2
0.02	0.1135	51716.3
0.04	0.1081	51716.3
0.06	0.0982	51716.6
0.08	0.08191	51716.4
0.08151	0.08151	51716.3

Let us consider a 1-cm-thick steel plate ($D = 1\,923\,076.9$ kN/cm^2) that rests on a foundation with the following characteristics: $cc_0 = 353.50$ kN/cm^3, $cc_1 = -258.5$ kN/cm^3, and $\eta = 0.034$ 1/cm. When calculating the integrals in relation (17), an increment of $\Delta z = \Delta\zeta = 0.1$ cm was used and the number of increments in the direction of either axis was 3000. The calculated results are presented in the table; these results indicate that an infinite set of buckling (bifurcation) modes corresponds to the same critical load value $N_{cr} \approx 51716$ kN/cm^2. These modes change from the mode with waves only along axis y with a half-wavelength of 27.24 cm to the mode with square prominences with a half-wavelength in the direction of either of the coordinate axes of 38.3 cm. The shape of the prominence also changes in a similar way in another case when waves are formed only in the direction of axis x, as well as to the same square prominences.

For comparison, let us consider a plate laying on a Winkler foundation. In this case, instead of expression (16), we have $N = (k_x^2 + k_y^2)D + c/(k_x^2 + k_y^2)$.

An undulation in the form of a prominence with parameters that obey the equality below corresponds to the critical compression load value:

$$k_x^2 + k_y^2 = \sqrt{c/D}. \qquad (17)$$

In this case, an infinite set of buckling modes also corresponds to the same critical load value N_{cr} for the shape of the undulation of which parameters k_x and k_y obey equality (17). If the rebound the coefficient of the elastic foundation reaction coefficient c in Winkler's model is 186 kN/cm^3, then $k_x^2 + k_y^2 = 0.00991$ 1/cm^2 and $N_{cr} = 37825.5$ kN/cm^2. With $k_x = 0$ and $k_y = \sqrt{0.00991} = 0.0996$ 1/cm, we have a buckling mode in the shape of waves only in the direction of axis y with a half-wavelength of 31.55 cm, while in the case of square prominences in the buckling mode ($k_x = k_y = 0.070$ 1/cm) the width and length of these prominences is 44.62 cm. Consequently, the behavior of a plate laying on a Winkler foundation is similar to the behavior of a plate laying on a nonlocally elastic foundation in the sense of an infinite set of the buckling modes. The critical load values of the plates of these two variants differ considerably. The geometry characteristics of the plates' buckling modes also differ, although this difference is less than the difference in the critical load values.

Let us consider another variant of nonlocality of an elastic foundation, the orthotropic one. The expression, analogous to expression (17), is written as

$$N = (k_x^2 + k_y^2)D + cc_0/(k_x^2 + k_y^2) + cc_1\eta\nu/[4(k_x^2 + k_y^2)]\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\eta|z| - \nu|\chi|} e^{-i(k_x z + k_y \chi)} dz d\chi. \qquad (18)$$

On calculating the integrals, we obtain

$$N = (k_x^2 + k_y^2)D + cc_0/(k_x^2 + k_y^2) + cc_1/[(k_x^2 + k_y^2)(1 + k_x^2/\eta^2)(1 + k_y^2/\nu^2)].$$

Let us assume that the orthotropic foundation of the plate has the following characteristics: $cc_0 = 353.5$ kN/cm^3, $cc_1 = -258.5$ kN/cm^3, and $\eta = \nu = 0.034$ 1/cm. The buckling mode with undulation only in the direction of either of the coordinate axes corresponds to the critical load value N for the same plate, i.e., $k_x = 0.1127$ 1/cm, $k_y = 0$ or $k_y = 0.1127$ 1/cm, and $k_x = 0$, which corresponds to the half-wavelength $\lambda = 27.88$ cm. The N_{cr} value appears to be 50559.6 kN/cm^2 in both cases. It can be seen that the critical load values are very close in the cases of the isotropic and orthotropic foundations; the buckling modes in the case of the isotropic foundation, however, appear more diverse.

CONCLUSIONS

A new version of the one- and two-dimensional model of a continuous nonlocally elastic foundation is proposed, which is used to solve problems of bending and stability of beams and plates. Two variants of nonlocality of a two-dimensional elastic foundation—when plates lay on an elastic foundation—are considered: isotropic and orthotropic foundations; these models enable a more correct description of the features of a deformed nonlocally elastic foundation. A numerical experiment showed that this model satisfactorily describes the deformation of a beam laying on an elastic foundation when the nonlocality of the foundation is taken into account by the following integro-differential equation kernel:

$$C(|x-\xi|) = c_0\delta(x-\xi) + c_1(\nu_1/2)\exp(-\nu_1|x-\xi|).$$

In a more general case, the expression $C(|x-\xi|)$ can be taken as the sum of exponents as

$$C(|x-\xi|) = c_0\delta(x-\xi) + \sum_{i=1}^{n} c_i(\nu_i/2)\exp(-\nu_i|x-\xi|),$$

where c_i and ν_i are constants.

Analysis of the stability of a beam and plates has shown that taking into account the nonlocality of the foundation results in a considerable increase in the critical load value compared with similar loads in the case of a Winkler foundation.

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