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Simplified Calculation of the Bending Torques of Steel Sheet and the Roller Reaction in a Straightening Machine

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Abstract—In the straightening of steel sheet, it is necessary to calculate the optimal reduction of the steel blank by the working rollers of the straightening machine so that the sheet produced has the minimum residual stress and curvature. In the simulation of sheet straightening in multiroller machines, the curvature and bending torques of the steel sheet at contact points with the working rollers are first calculated and then the straightening forces are determined. In straightening steel sheet, it is important to calculate the forces in the multiroller straightening machine. Such calculations are based on determination of the reaction of the roller bearings and the forces at the upper and lower working-roller cassettes in straightening. With insufficient bending torque, it is impossible to eliminate harmful residual stress and surface defects in the sheet. Extreme roller torques and forces at the roller cassettes often lead to defects of the sheet, fracture of the working and supporting rollers, and failure of the straightening machine. In the present work, an approximate method is proposed for calculation of the optimal cold-straightening parameters of the steel sheet in a multiroller machine. The calculations permit determination of the curvature of the neutral plane in the sheet on straightening, the residual curvature of the sheet after straightening, the bending torque and the reaction of the working-roller bearings, the residual stress in the sheet, the penetration of the plastic deformation into the depth of the steel sheet, and the relative deformation of the longitudinal surface fibers of the sheet on straightening as a function of the radius of the working rollers, the distance between the rollers of the straightening machine, the reduction of the sheet by the upper rollers, the sheet thickness, and its properties (Young's modulus, yield point, and strengthening modulus). The results may be widely used at manufacturing and metallurgical plants.

Keywords: steel sheet, multiroller straightening machine, working rollers, supporting rollers, sheet curvature, alternating flexure, bending torques, elastoplastic media, linear strengthening

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Rolling mills and multiroller straightening machines are widely used in the production of steel sheet [1–40]. For example, five- and nine-roller SMS Siemag straightening machines are used at 5000 mills, and five-, six-, eleven-, and fifteen-roller machines are used in the Fagor Arrasate transverse-cutting line.

After hot rolling, steel sheet is deformed on cooling because of the residual stress and is often characterized by surface defects when cold (such as buckling, undulation, and tapering). Therefore, steel sheet is straightened in multiroller machines. Straightening in such machines is essential in metallurgical production. It is widely used in Russia (at the Vyksa, Chelyabinsk, Magnitogorsk, and Izhor steel plants) and also in the United States, Germany, Spain, China, India, and elsewhere. In Fig. 1, we show a fifteen-roller machine produced by Fagor Arrasate (Spain).

A precise mathematical model for calculating the parameters of a fifteen-roller machine was developed in [21–24]. The calculation is based on cubic approximation of the sheet's longitudinal line between adjacent straightening rollers. However, this entails numerical solution of a transcendental system of nonlinear equations (which cannot be analytically solved), with a large number of unknowns (equal to the number of rollers). The model must be simplified so that calculation of the straightening machine's parameters does not require numerical solution of a complex nonlinear system of equations but may be confined to simple arithmetical operations: summation, subtraction, multiplication, division, raising to specific powers, and extraction of a root. The precision of such a simplified model must be sufficient for practical purposes at steel plants and must approach that of the precise model. We address this task in the present work.

BENDING TORQUE AND SPRINGBACK COEFFICIENT OF SHEET

Suppose that h and b are the thickness and width of the steel sheet; σ_y, E, are the yield point and Young's

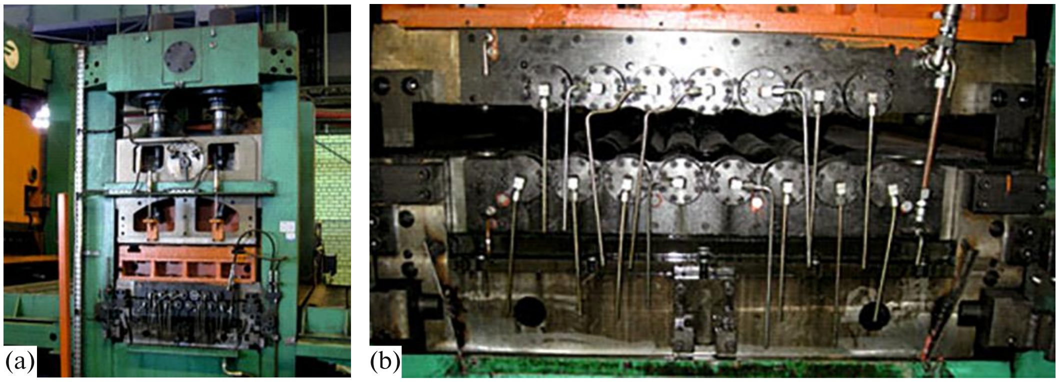

Fig. 1. Fifteen-roller Fagor Arrasate straightening machine: (a) general view; (b) working rollers.

modulus of the steel; Π_{ex}, Π_{co} are the strengthening moduli in extension and compression.

In plastic flexure (when $\rho < \rho_y = \frac{hE}{2\sigma_y}$), the bending torque for the steel sheet is [21–27]

$$M(\rho) = \frac{bh^2\sigma_y}{12}\left[3 - 4\left(\frac{\sigma_y\rho}{Eh}\right)^2\right] + \frac{bh^3(\Pi_{ex} + \Pi_{co})}{24\rho} \times \left(1 - 2\frac{\sigma_y\rho}{Eh}\right)^2 1 + \left(\frac{\sigma_y\rho}{Eh}\right),$$

where ρ is the radius of curvature of the sheet's longitudinal neutral line.

In elastic flexure (when $\rho \geq \rho_y = \frac{hE}{2\sigma_y}$), the bending torque for the steel sheet is [1, 21–27]

$$M(\rho) = \frac{bh^3E}{12\rho}.$$

For high-strength steel, the strengthening moduli in extension and compression are practically equal: $\Pi_{ex} \approx \Pi_{co} = \Pi$.

In plastic flexure (when $\rho < \rho_y = \frac{hE}{2\sigma_y}$), the springback coefficient for the steel sheet is [21–27]

$$\beta(\rho) = \frac{1}{\left(1 - \frac{\Pi_{ex} + \Pi_{co}}{2E}\right)\left(1 - 2\frac{\rho\sigma_y}{hE}\right)^2\left(1 + \frac{\rho\sigma_y}{hE}\right)}.$$

In elastic flexure (when $\rho \geq \frac{hE}{2\sigma_y}$), the springback coefficient $\beta(\rho) = \infty$.

The extent of the plastic deformation over the thickness of the steel sheet (the penetration of the plastic deformation into the depth of the sheet) is

$$\eta = \left\{1 - \frac{2\sigma_y\rho}{Eh},\ \text{if}\ \ \rho \leq \frac{Eh}{2\sigma_y};\ \ 0,\ \text{if}\ \ \rho > \frac{Eh}{2\sigma_y}\right\}.$$

The relative deformation of the longitudinal surface fibers of the sheet is

$$\eta_{lo} = \frac{h}{2|\rho|}.$$

SHEET STRAIGHTENING ON (2*N* + 1)-ROLLER MACHINE

Suppose that steel sheet is straightened by $2N + 1$ driven working rollers: N upper rollers and $N + 1$ lower rollers. The rollers are equipped with individual systems for adjusting their vertical position by means of wedge pairs and hydraulic cylinders [1, 21–24].

We introduce the following notation: t is the distance between the lower straightening rollers; H_i is the reduction of the neutral surface of the steel sheet in roller i; N_i is the reaction of the working rollers at contact with the sheet; R is the radius of the working rollers; $R_0 = R + h/2$; ρ_i are the radii of curvature of the sheet's longitudinal neutral line at contact with the working rollers; $\varepsilon_i = 1/\rho_i$ is its curvature; φ_i ($i = 1, \ldots, 2N + 1$) are the angles of the sheet–roller contact points (Fig. 2).

Note 1. The relation between the actual reduction H_{act} and the reduction of the sheet's longitudinal neutral surface H takes the form

$$H = H_{act} + h.$$

The maximum reduction is

$$H_{max} = 2R_0\left(1 - \sqrt{1 - \left(\frac{t}{4R_0}\right)^2}\right).$$

Without loss of generality, we will assume that the lower (odd) straightening rollers are at the same horizontal level

$$H_{2j+1} = 0,\quad j = 0, \ldots, N,$$

while the upper working rollers are characterized by independent vertical displacement.

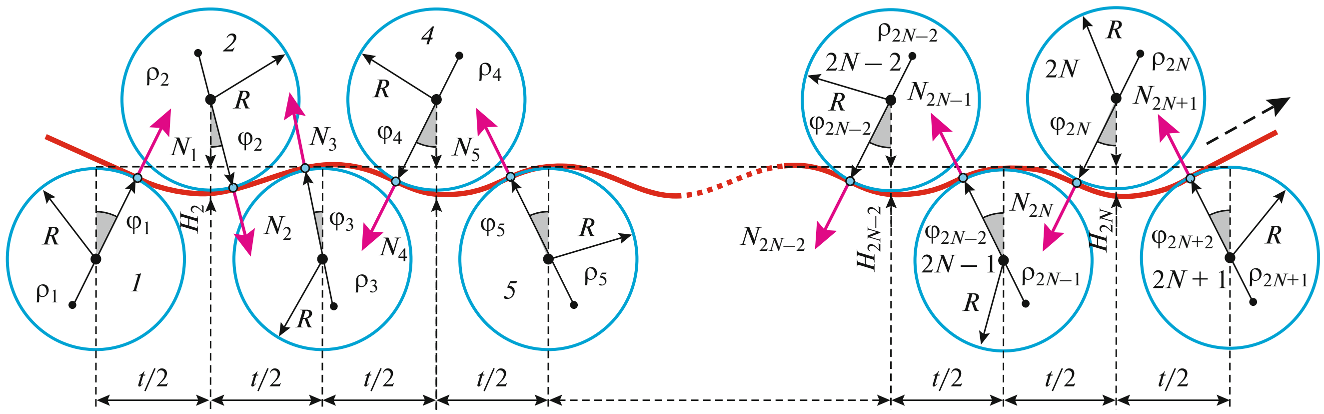

Fig. 2. Straightening of steel sheet by $2N + 1$ working rollers.

In straightening at metallurgical plants, the actual sheet–roller contact angles are small (up to 1°–3°) for the second and subsequent rollers. Therefore, we assume that all the contact angles are zero

$$\varphi_i = 0,\ i = 0, \dots, 2N + 1.$$

The approximate radii of curvature of the sheet's longitudinal neutral line at the contact points with the even and odd working rollers are as follows

$$\rho_{2k} = \frac{t^2}{24H_{2k}},\quad k = 1, \dots, N;$$

$$\rho_{2k+1} = \frac{t^2}{12(H_{2k} + H_{2k+2})},\quad k = 1, \dots, N - 1.$$

The approximate radius of curvature at the last working roller is

$$\rho_{2N+1} = \beta(\rho_{2N})\rho_{2N}.$$

Note 2. If

$$H_{2k} \geq \frac{t^2}{24R_0},$$

the street surface will wrap around the surface of the working roller. Hence, $\rho_{2k} = R_0$. Analogously, if

$$H_{2k} + H_{2k+1} \geq \frac{t^2}{12R_0},$$

then $\rho_{2k+1} = -R_0$.

Note 3. We may assume that the approximate odd radii of curvature are equal

$$\rho_{2k+1} = -\frac{t^2}{48}\left(\frac{1}{H_{2k}} + \frac{1}{H_{2k} + 1}\right),\quad k = 1, \dots, N - 1.$$

However, the accuracy in calculating the odd radii of curvature falls in this case (especially for the last odd rollers, where the radii of sheet curvature are large).

Note 4. Since $2H_{2k}/t \ll 1$, we may write

$$\rho_{2k} = \frac{t^2}{24H_{2k}} \approx \frac{t^2}{24H_{2k}}\left[1 + \left(\frac{2H_{2k}}{t}\right)^2\right] = \frac{\sqrt{\left(\frac{t}{2}\right)^2 + H_{2k}^2}}{6\cos\left[\arctan\left(\frac{t}{2H_{2k}}\right)\right]}.$$

At the sheet–roller contact points, the bending torques for the sheet are determined from the formulas

$$M_1 = 0;\quad M_{2k} = M(\rho_{2k}),\quad k = 1, \dots, N;$$

$$M_{2k+1} = -M(|\rho_{2k+1}|),\quad k = 1, \dots, N - 1;\quad M_{2N+1} = 0.$$

The reaction of the working rollers at the contact points takes the form

$$N_1 = \frac{2}{t}M_2;\quad N_2 = \frac{2}{t}(-M_3 + 2M_2),$$

$$N_{2k-1} = \frac{2}{t}(M_{2k} - 2M_{2k-1} + M_{2k} - 2),\quad k = 2, \dots, N;$$

$$N_{2k-2} = \frac{2}{t}(-M_{2k-1} + 2M_{2k-2} - M_{2k-3}),\quad k = 3, \dots, N;$$

$$N_{2N} = \frac{2}{t}(2M_{2N} - M_{2N-1}),\quad N_{2N+1} = \frac{2}{t}M_{2N}.$$

The vertical force exerted by the upper and lower rollers on the sheet is as follows

$$F_{\text{upp}} = \sum_{k=1}^{N} N_{2k} = \frac{4}{t}\sum_{k=2}^{2N}(-1)^k M_k;$$

$$F_{\text{low}} = \sum_{k=1}^{N} N_{2k+1} = \frac{4}{t}\sum_{k=2}^{2N}(-1)^k M_k = F_{\text{upp}}.$$

The total force of all the rollers is

$$F_{\text{tot}} = F_{\text{upp}} + F_{\text{low}} = \sum_{k=1}^{2N+1} N_i = \frac{8}{t}\sum_{k=2}^{2N}(-1)^k M_k.$$

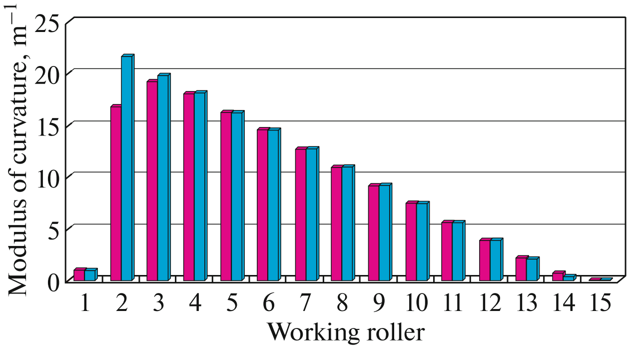

Fig. 3. Modulus of curvature of the sheet in the straightening machine.

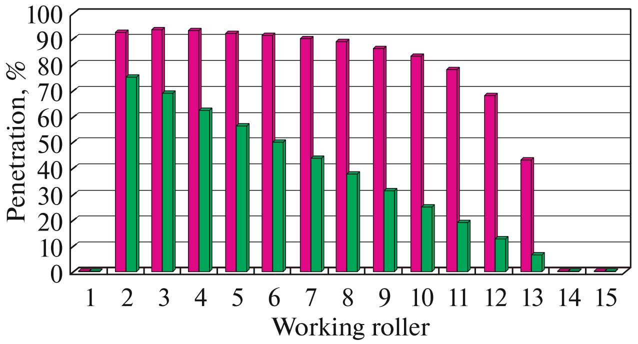

Fig. 4. Penetration of the plastic deformation into the depth of the steel sheet.

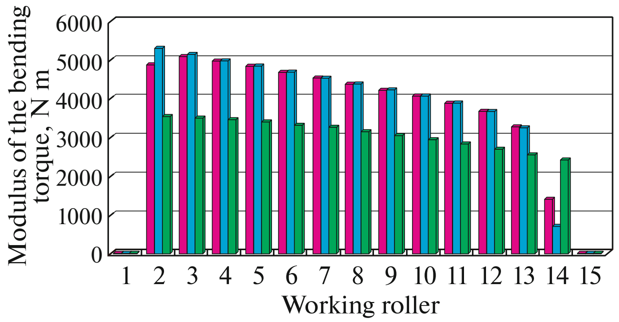

Fig. 5. Modulus of the bending torque in straightening.

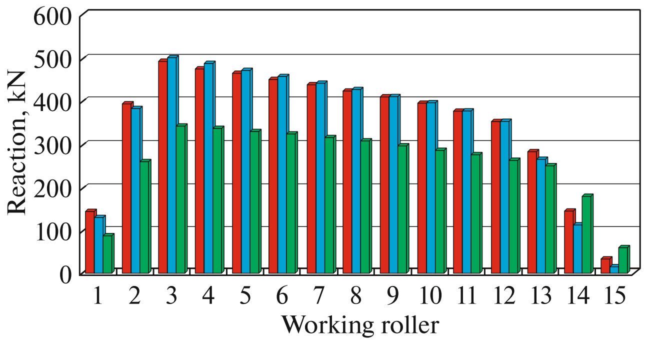

Fig. 6. Normal reaction of the working rollers.

NUMERICAL CALCULATION FOR FIFTEEN-ROLLER MACHINE

For the Fagor Arrasate fifteen-roller machine

$$N = 7(2_{N+1} - 15), \quad t = 0.245/3 = 0.08167 \text{ m},$$
$$R = 0.075 \text{ m}.$$

In Figs. 3–6, we show the calculation results for the straightening of steel sheet when

$$h = 0.004 \text{ m}, \quad b = 1.8 \text{ m}, \quad E = 2 \times 10^{11} \text{ Pa},$$
$$\sigma_y = 500 \times 10^6 \text{ Pa}, \quad \Pi_{ex} = \Pi_{co} = 8.8 \times 10^9 \text{ Pa},$$

when the positions H_i of the upper rollers in the flat upper cassette are as follows

$$H_2 = 0.006 \text{ m}, H_4 = 0.00502 \text{ m}, H_6 = 0.00403 \text{ m},$$
$$H_8 = 0.00305 \text{ m}, \quad H_{10} = 0.00207 \text{ m},$$
$$H_{12} = 0.00108 \text{ m}, \quad H_{14} = 0.0001 \text{ m}.$$

and when $\rho_1 = -1$ m.

The vertical force exerted by the upper and lower working rollers on the steel sheet are $F_{upp} = F_{low} =$ 2634 N, according to the precise solution. According to the simplified method described earlier. $F_{upp} = F_{low} = 2612$ kN; the discrepancy is 22 kN or 0.8%. By contrast, $F_{upp} = F_{low} = 1952$ kN, according to the method in [1]; the discrepancy is 682 kN or 25.9%.

In Fig. 3, the left column corresponds to the precise solution, and the right column to our simplified method. We see that, when the upper rollers are in a flat cassette, the modulus of curvature of the sheet at contact points varies almost linearly from the third to the fourteenth roller. If the upper working rollers may adjust their positions independently, nonlinear variation of the modulus of curvature is possible.

COMPARISON OF THE CALCULATION METHODS

The penetration of the plastic deformation into the depth of the steel sheet is used in calculating the bending torques by the method in [1]. In this method, the bending torques are calculated on the basis of the penetration of the plastic deformation into the depth of the sheet. It is assumed that the plastic deformation of the sheet's surface layer varies linearly from the second to the penultimate roller. (The first and last rollers do not bend the sheet.) This assumption is inaccurate, as we see in Fig. 4, where the left column corresponds to the precise solution, and the right column to the method in [1].

In practice, if the upper rollers are in a flat cassette, the penetration of the plastic deformation into the depth of the steel sheet resembles an inverted parabola between the third and penultimate rollers, with a maximum at the third roller (Fig. 4).

The curvature and radii of curvature of the steel sheet are not calculated in the method of [1]. That prevents accurate assessment of the sheet's bending torques, which depend on its curvature.

For current multiroller straightening machines with independent motion of the working rollers, we cannot use the method of [1]. That method assumes that the upper and lower rollers occupy fixed positions in the upper and lower cassettes.

The strengthening modulus of the sheet in flexure is assumed to be zero (Prandtl diagram) in the method of [1]. That is its major flaw, leading to very significant errors (up to 26%) in calculating the parameters of the straightening machine (Figs. 5 and 6). This flaw may result in failure of the straightening machine.

In Figs. 5 and 6, the left column corresponds to the precise solution, the middle column to our simplified method, and the right column to the method of [1].

CONCLUSIONS

We have proposed an approximate method for calculation of the optimal cold-straightening parameters of the steel sheet in a multiroller machine: the curvature of the neutral plane in the sheet on straightening; and the bending torques and the reactions of the working rollers.

The proposed simplified method is considerably more accurate than the Korolev method [1]. The results may be used in the production of steel sheet at metallurgical plants.

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