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SPACE PHYSICS

Dynamics of Solar Cosmic Ray Energetic Spectra during the Solar Flare on January 20, 2005

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Abstract—Propagation of cosmic rays in the interplanetary space that accelerated during the solar flare on January 20, 2005 is investigated based on the kinetic equation. The cases of instantaneous and continuous particle injection into the interplanetary medium are considered. Based on the analytic solution of the kinetic equation, it is shown that there is a sharp increase in the cosmic ray intensity in the Earth's orbit in the case of the instantaneous particle injection, while their concentration increases gradually in the case of the continuous particle injection. The dynamics of the energy distribution of solar cosmic rays is analyzed.

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INTRODUCTION

Cosmic rays (CR) are one of the key factors influencing the evolution of the biosphere. Galactic cosmic rays, whose origin and energy characteristics are associated with the processes taking place in the Galaxy and Metagalaxy, affect the biosphere for a long period of time. In addition to a relatively slow change in the background of galactic cosmic rays because of the solar activity, there are extreme bursts of CR intensity (e.g., from close supernovas). On the other hand, solar cosmic rays (SCR) generated by the Sun during chromospheric flares impact the biosphere for a short period of time, and their effect is directly associated with the flare's power. In particular, the SCR generation causes a significant increase in the background radiation in the near-Earth space, upper layers of the atmosphere, which is a danger to astronauts and aircraft crews, especially in polar regions, and affects the onboard electronic devices [9, 11, 20, 21]. Thus, along with electromagnetic fields and fluxes of plasma particles of the solar wind, SCRs are one of the components that form the space weather. High-energy charged particles are sometimes generated during solar flares, which are recorded by the world network of neutron monitors and muon telescopes in the form of increase in the intensity of cosmic rays [9, 11, 21]. These relativistic particles move at velocities close to the speed of light and reach the Earth's orbit before the main SCR flux and well before the interplanetary shock wave accompanying the solar flare. Therefore, SCRs are often harbingers of powerful magnetic storms and are an important means of forecasting magnetospheric events [9, 20].

A solar flare on January 20, 2005, (further referred to as event) is the most powerful SCR event in the past half-century [7, 8, 15, 19, 24, 26]. This CR intensity increase was observed by neutron monitors around the globe. The largest increase was recorded at Antarctic CR stations [7–9, 11, 21, 26]. Thus, the CR intensity recorded in some southern polar monitors during the event was several tens of times higher than the background level of galactic CRs [8, 9, 11, 21]. This huge increase in the CR flux rate was caused by a solar flare that started at 6 h 39 min UT at 14″N 61″W. The flare was observed in various ranges of electromagnetic waves (including X-rays and gamma rays) and was accompanied by a rapid coronal mass ejection [9, 19, 21, 25].

An important feature of the event was a pronounced anisotropy of the flux of relativistic particles in the initial phase of the SCR flare [9, 11, 21, 23]. If the CR intensity on the Antarctic neutron monitors rapidly increased in the first few minutes of the flash while the time from the arrival of the first particles to the time of maximum CR intensity was 5–6 minutes, northern monitors at this time did not record increase in the CR intensity [8, 9, 11, 21, 26]. Such differences in CR intensity time profiles show an extremely high anisotropy of the angular distribution of SCR at the beginning of the event [8, 9, 11, 21]. The rapid increase in the CR intensity and a rapid decrease in the flux of relativistic particles in the first 10 minutes after the maximum CR intensity were caused by weak CR scattering by irregularities of the interplanetary magnetic field [9, 23]. The relativistic particle transport range during this flare turned out to be significant, comparable to the distance between the Sun and Earth's orbit.

Relativistic solar cosmic rays contain important information regarding the processes of particle acceleration in the area of the solar flare, as well as their escape from the solar atmosphere and distribution in the interplanetary space [1, 2, 20]. The analysis of observation data recorded during the event by the network of neutron monitors and by spacecraft made it possible to obtain information on the characteristic time of the SCR injection into the interplanetary medium, transport range, anisotropy, and the energy spectrum of CRs [9, 11, 21, 23, 25, 26]. The results obtained based on observational data make it possible to carry out a theoretical study of the propagation of solar cosmic rays from the source to the Earth during the event.

The propagation of particles accelerated in solar flares from the source to the observation point is controlled by heliospheric magnetic fields. The CR propagation depends on the level of disturbance of the interplanetary magnetic field. If the level of turbulence in the interplanetary space is sufficiently high, the particles disperse rapidly, the CR distribution function rapidly becomes isotropic, and the CR distribution is described by diffusion equations [1, 2, 20]. Observation data show that, during some solar flares (including the event), the level of turbulence in the interplanetary medium is low, the particle scattering is weak, and the transport mean free path is comparable with the astronomical unit. In this case, the diffusion approximation cannot be used, and it is necessary to describe the CR propagation based on the kinetic equation [1, 2, 20]. Note that the kinetic description of the SCR propagation in the interplanetary magnetic field was used to analyze a number of solar proton events [4, 10, 14, 17, 22].

In this paper, we consider the spatial and temporal distribution and dynamics of the energy spectrum of solar cosmic rays in the event based on the kinetic equation describing the CR propagation in the interplanetary space. An analytic solution of the kinetic equation corresponding to the instantaneous emission of accelerated particles in the interplanetary medium is given below. The continuous SCR injection is also considered. The temporal profile of the SCR intensity in the Earth's orbit and the evolution of the energy distribution of the particles are analyzed.

KINETIC EQUATION

A consistent description of CR propagation is based on the kinetic equation, which describes the scattering of high-energy charged particles in magnetic inhomogeneities and their focusing using a regular interplanetary magnetic field [13, 18]. The kinetic approach makes it possible to calculate the spatial and temporal distribution of the SCR intensity, the anisotropy of the angular distribution of particles, and the evolution of their energy spectrum [3, 4, 6, 13]. Let us write the kinetic equation in the following form

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} = stf + Q, \tag{1}$$

where $f(\mathbf{r}, \mathbf{p}, t)$ is the CR distribution function, $\mathbf{v}$ is the particle velocity, and Q is the particle source. The collision integral stf that describes the particle scattering by irregularities of the magnetic field is determined by the following expression [2, 13, 16, 18]

$$stf = \frac{v}{4\pi\lambda}\int d\Omega f - \frac{v}{\lambda}f, \tag{2}$$

where λ is the CR transport mean free path, and the integration is carried out by particle velocity vector angles.

Suppose that an instant isotropic emission of particles by r_0 radius sphere occurs at the initial time. In this case, the particle source Q, which is recorded in the right part of kinetic equation (1), has the form

$$Q(r, p, t) = q(p)\frac{\delta(r - r_0)\delta(t)}{16\pi^2 r^2}, \tag{3}$$

where $\delta(x)$ is the δ-function, and the value $q(p)$ describes the energy distribution of particles injected into the interplanetary space near the Sun.

Thus, kinetic equation (1) takes the form

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \frac{v}{\lambda}f - \frac{v}{2\lambda}\int d\mu f = q(p)\frac{\delta(r - r_0)\delta(t)}{16\pi^2 r^2}, \tag{4}$$

where $\mu = \cos\theta$ is the cosine of the pitch angle of the particle. The solution of kinetic equation (4) can be obtained using the method of integral transformations. Namely, the Fourier transform was applied to the spatial variable r and the Laplace transform was applied to the time variable t [3, 5, 6, 13].

The concentration of particles with a given momentum value is determined by the relation

$$N(r, p, t) = \int d\Omega f(r, \mathbf{p}, t) = 2\pi \int_{-1}^{1} d\mu f(r, \mathbf{p}, t). \tag{5}$$

It turns out that CR concentration (5) is the sum of concentrations of unscattered (N_0) and scattered (N_s) particles [13]

$$N(r, p, t) = N_0(r, p, t) + N_s(r, p, t). \tag{6}$$

In the case of the instantaneous injection of particles by r_0 radius sphere, when the CR source is defined by formula (3), the concentration of unscattered particles has the form [3]

$$N_0(r, p, t) = \frac{q(p)\exp(-vt/\lambda)}{8\pi r r_0 vt}[\Theta(vt - |r - r_0|) - \Theta(vt - r - r_0)], \tag{7}$$

where $\Theta(x)$ is the Heaviside unit function.

According to relation (7), at the point r, unscattered particles will be observed only in the time interval $(r - r_0)/v < t < (r + r_0)/v$.

The concentration of scattered particles N_s has of the following form [3]

$$N_s(r, p, t) = N_\alpha(r, p, t) + N_\beta(r, p, t), \tag{8}$$

where

$$N_\alpha(r, p, t) = \frac{q(p)}{2\pi^2 \lambda r r_0} \int_0^{\pi/2} d\kappa \frac{\kappa^2 \sin(\kappa r/\lambda)\sin(\kappa r_0/\lambda)}{\sin^2\kappa} \exp\left(\frac{vt}{\lambda}(\kappa\cot\kappa - 1)\right), \tag{9}$$

$$N_\beta(r, p, t) = \frac{q(p)\exp(-vt/\lambda)}{16\pi^2 \lambda r r_0}$$

$$\times\left\{\int_0^{\eta_1} d\eta\left[2\pi\sigma\cos\frac{\pi}{2}\xi_1 + (\sigma^2 - \pi^2)\sin\frac{\pi}{2}\xi_1\right]\exp\left(\frac{\sigma}{2}\xi_1\right)\right. \tag{10}$$

$$\left. - \int_0^{\eta_2} d\eta\left[2\pi\sigma\cos\frac{\pi}{2}\xi_2 + (\sigma^2 - \pi^2)\sin\frac{\pi}{2}\xi_2\right]\exp\left(\frac{\sigma}{2}\xi_2\right)\right\},$$

$$\sigma = \ln\frac{1-\eta}{1+\eta}, \tag{11}$$

$$\xi_1 = \frac{1}{\lambda}(|r - r_0| - vt\eta), \quad \xi_2 = \frac{1}{\lambda}(r + r_0 - vt\eta). \tag{12}$$

Integration limits in (10) are given by relations

$$\eta_1 = \frac{|r - r_0|}{vt}\Theta(vt - |r - r_0|) + \Theta(|r - r_0| - vt), \tag{13}$$

$$\eta_2 = \frac{r + r_0}{vt}[\Theta(vt - r - r_0) + \Theta(r + r_0 - vt)]. \tag{14}$$

Thus, the function η_1 (13) is equal either to the magnitude $|r - r_0|/vt$ or to unity depending on whether the time t exceeds the magnitude $|r - r_0|/v$.

If scattering by irregularities of the interplanetary magnetic field is sufficiently weak, so that the CR transport mean free path λ far exceeds the value of vt, the exponent in formula (7) can be set to one. In this case, from relation (7), we obtain the spatial and temporal distribution of the concentration of unscattered particles, which corresponds to the solution of kinetic equation (4) with an infinite increase of λ.

In the opposite limiting case, when the time elapsed after the injection of particles is much greater than the time between collisions ($vt \gg \lambda$), unscattered particles N_0 can be ignored and the value of N_β can be neglected, because of the presence of the exponentially small factor in relations (7) and (10).

The integrand in relation (9) has a sharp maximum at the point $\kappa = 0$ (assuming $vt \gg \lambda$), therefore, by extending κ integration in formula (9) to infinity, we obtain the relation

$$N(r, p, t) = \frac{q(p)}{8\pi^{3/2} r r_0 \sqrt{\chi t}}\left[\exp\left(-\frac{(r - r_0)^2}{4\chi t}\right) - \exp\left(-\frac{(r + r_0)^2}{4\chi t}\right)\right], \tag{15}$$

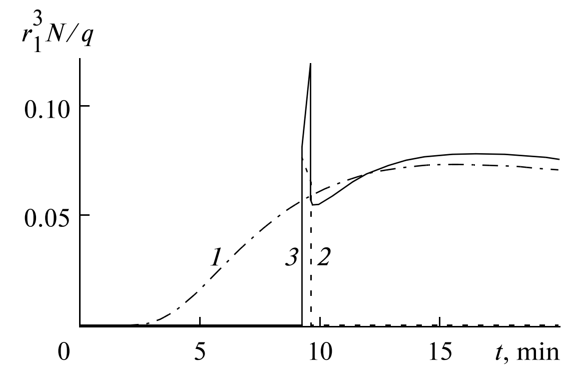

Fig. 1. Dependence of the cosmic ray density on time: (*1*) curves of diffusion approximation, (*2*) unscattered particles, and (*3*) all particles.

which describes the diffusion of particles after their instant emission by the r_0 radius sphere [1]. The value of χ is the CR diffusion coefficient that is proportional to the CR transport mean free path and particle velocity

$$\chi = \frac{v\lambda}{3}. \tag{16}$$

The CR transport mean free path increases with increasing particle energy and can be approximated by a power function of rigidity [2, 12]. Assume that the CR transport mean free path is proportional to the particle momentum to some extent μ

$$\lambda(p) = \lambda_0\left(\frac{p}{p_0}\right)^{\mu}, \tag{17}$$

where the value of p_0 is the proton momentum with a kinetic energy of 1 GeV. The momentum of the particle p can be calculated according to the formula

$$p = \frac{1}{c}\sqrt{\varepsilon_k(\varepsilon_k + 2mc^2)}, \tag{18}$$

where ε_k and mc^2 are the kinetic energy and the rest energy of the particle. The value of λ_0 in formula (17) is the proton transport mean free path with the kinetic energy of 1 GeV.

Figure 1 shows the dependence of the CR concentration on the time at the point $r_1 = 1$ AU.

The emission of particles by the sphere with a radius of $r_0 = 0.02$ AU is expected to be instant and isotropic. Note that the value of r_0 corresponds to the heliocentric coordinate of the flare. The ordinate shows the values of the dimensionless quantity r_1^3N/q, which is proportional to the concentration of particles. The CR transport mean free path is selected to be 0.3 AU. The particle kinetic energy is 1 GeV. The curve *1* corresponds to particle diffusion propagation (15), and the curve *2* refers to unscattered particles (7). The solid curve *3* illustrates the temporal profile of the concentration of all particles (both scattered and unscattered). It can be seen that in the kinetic description the concentration increase occurs abruptly at the time corresponding to the arrival of the first particles (curve *3*) in contrast to the CR diffusion propagation mode (curve *1*). Note that the proton with a kinetic energy of 1 GeV covers the distance to the Earth's orbit for 9.3 min. The pulse CR concentration increase is caused by unscattered particles, and its duration is $2r_0/v$ (Fig. 1). After the impulsive burst, the concentration of unscattered particles (7) becomes zero, so that only scattered particles will be observed at a given point of space. Over time the CR concentration gradually approaches the value corresponding to the diffusion propagation of particles.

A rapid increase in the CR intensity was observed in a number of neutron monitors during the event [11, 21, 25]. However, there was no impulsive increase in the CR intensity corresponding to the arrival of the first particles, and the CR concentration increased gradually during the first 5–6 minutes of this event [9, 11, 21, 25]. Apparently, the used instantaneous injection approximation does not match the dynamics of the emission of accelerated particles in the interplanetary space. The pulse duration at the beginning of the flare depends on the geometry of the interplanetary magnetic field, which requires separate consideration.

Figure 2a shows the dependence of the concentration of particles of different energies on time at the point $r_1 = 1$ AU. It is assumed that particles of different energies were injected in the interplanetary space

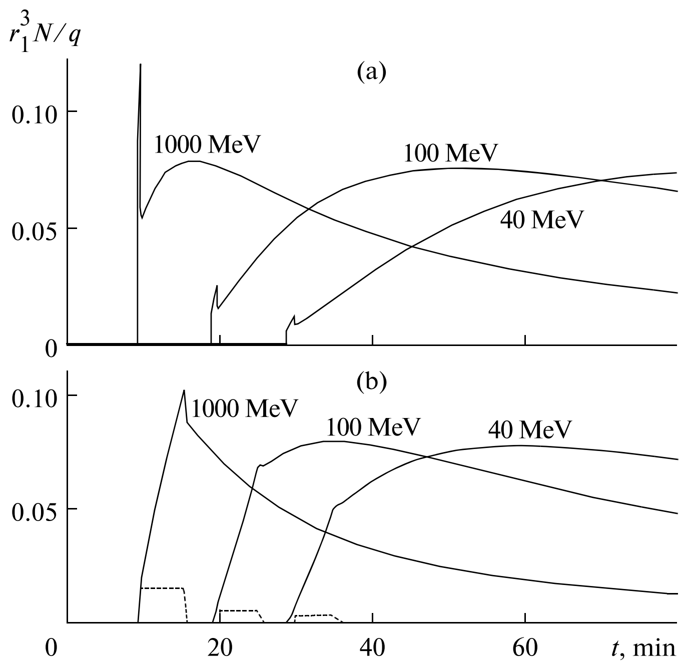

Fig. 2. Dependence of the cosmic ray density on the time elapsed after the start of injection of particles with a kinetic energy of 40, 100, and 1000 MeV. The (a) pulse injection and (b) the continuous injection (dashed curves indicate unscattered particles, and solid curves refer to all particles).

simultaneously. The proton transport mean free path with a kinetic energy of 1 GeV was selected to be $\lambda_0 = 0.3$ AU. The dependence of λ on the particle momentum was assumed to be power dependence (17) with an exponent $\mu = 1/3$. Note that the arrival of the first particles with lower energy was observed later because of the dependence of the velocity on the particle energy, and the impulsive increase of the CR concentration at the beginning of the event was caused by unscattered particles.

SPATIOTEMPORAL DISTRIBUTION OF COSMIC RAYS DURING THE CONTINUOUS PARTICLE INJECTION

Experimental data obtained on the world network of neutron monitors and spacecraft indicate the continuous SCR injection into the interplanetary medium [9, 11, 19, 21, 23–25]. The emission of relativistic protons accelerated during the flare apparently lasted for a few minutes [9, 23, 25]. The anisotropy of the angular particle distribution, which remained significant for a long time, also indicates the continuous nature of the SCR injection [11, 21, 23].

In order to take account of the continuous nature of the SCR injection into the interplanetary medium, it is necessary to determine the particle source Q as follows

$$Q(r, p, t) = q(p)\frac{\delta(r - r_0)\varphi(t)}{16\pi^2 r^2}, \tag{19}$$

where $\varphi(t)$ is the CR injection function, which determines the particle emission duration. In the case of continuous injection, the CR concentration can be computed as the convolution of the particle concentration N corresponding to the instantaneous injection and the function $\varphi(t)$

$$\tilde{N}(r, p, t) = \int_0^t dt_1 N(t - t_1)\varphi(t_1). \tag{20}$$

Suppose that the SCR injection occurs during a finite time t_0 with the constant intensity. Then, the function $\varphi(t)$ has the following form

$$\varphi(t) = \frac{1}{t_0}\Theta(t - t_0). \tag{21}$$

According to formula (21), the particle emission occurs over time t_0 since the time $t = 0$. Let us introduce dimensionless variables

$$\rho = \frac{r}{r_1}, \quad \rho_0 = \frac{r_0}{r_1}, \quad \tau = \frac{ct}{r_1}, \quad \tau_0 = \frac{ct_0}{r_1}, \tag{22}$$

where $r_1 = 1$ AU. Therefore, we will measure the distance in astronomical units, and the dimensionless time unit τ corresponds to the time required for the electromagnetic radiation to reach the Earth's orbit.

Let us integrate CR concentration expression (15) corresponding to the CR diffusion together with SCR injection function (21) according to formula (20). Provided that $t < t_0$, we arrive at the following expression for the particle concentration

$$\begin{aligned}\tilde{N}(\rho, p, \tau) = \frac{q(p)\sqrt{3}}{4\pi^{3/2} r_1^{5/2}\sqrt{\lambda v/c}\,\rho\rho_0\tau_0}\Bigg\{&\sqrt{\tau}\left[\exp\left(-\frac{3}{4}\frac{c}{v}\frac{r_1}{\lambda}\frac{(\rho-\rho_0)^2}{\tau}\right) - \exp\left(-\frac{3}{4}\frac{c}{v}\frac{r_1}{\lambda}\frac{(\rho+\rho_0)^2}{\tau}\right)\right] \\ &- \frac{1}{2}\sqrt{3\pi\frac{c}{v}\frac{r_1}{\lambda}}\left[|\rho-\rho_0|\operatorname{erfc}\left(\frac{|\rho-\rho_0|}{2}\sqrt{3\frac{c}{v}\frac{r_1}{\lambda}\frac{1}{\tau}}\right) - (\rho+\rho_0)\operatorname{erfc}\left(\frac{\rho+\rho_0}{2}\sqrt{3\frac{c}{v}\frac{r_1}{\lambda}\frac{1}{\tau}}\right)\right]\Bigg\},\end{aligned} \tag{23}$$

where erfc(x) is the additional probability integral. If $t > t_0$, the SCR concentration that describes the diffusion of particles during their continuous injection into the interplanetary medium has the form

$$\begin{aligned}\tilde{N}(\rho, p, \tau) = \frac{q(p)\sqrt{3}}{4\pi^{3/2} r_1^{5/2}\sqrt{\lambda v/c}\,\rho\rho_0\tau_0}\Bigg\{&\sqrt{\tau}\left[\exp\left(-\frac{3}{4}\frac{c}{v}\frac{r_1}{\lambda}\frac{(\rho-\rho_0)^2}{\tau}\right) - \exp\left(-\frac{3}{4}\frac{c}{v}\frac{r_1}{\lambda}\frac{(\rho+\rho_0)^2}{\tau}\right)\right] \\ &- \sqrt{\tau-\tau_0}\left[\exp\left(-\frac{3}{4}\frac{c}{v}\frac{r_1}{\lambda}\frac{(\rho-\rho_0)^2}{\tau-\tau_0}\right) - \exp\left(-\frac{3}{4}\frac{c}{v}\frac{r_1}{\lambda}\frac{(\rho+\rho_0)^2}{\tau-\tau_0}\right)\right] + \frac{|\rho-\rho_0|}{2}\sqrt{3\pi\frac{c}{v}\frac{r_1}{\lambda}} \\ &\times\left[\operatorname{erf}\left(\frac{|\rho-\rho_0|}{2}\sqrt{3\frac{c}{v}\frac{r_1}{\lambda}\frac{1}{\tau}}\right) - \operatorname{erf}\left(\frac{|\rho-\rho_0|}{2}\sqrt{3\frac{c}{v}\frac{r_1}{\lambda}\frac{1}{\tau-\tau_0}}\right)\right] - \frac{\rho+\rho_0}{2}\sqrt{3\pi\frac{c}{v}\frac{r_1}{\lambda}} \\ &\times\left[\operatorname{erf}\left(\frac{\rho+\rho_0}{2}\sqrt{3\frac{c}{v}\frac{r_1}{\lambda}\frac{1}{\tau}}\right) - \operatorname{erf}\left(\frac{\rho+\rho_0}{2}\sqrt{3\frac{c}{v}\frac{r_1}{\lambda}\frac{1}{\tau-\tau_0}}\right)\right]\Bigg\},\end{aligned} \tag{24}$$

where erf(x) is the probability integral.

Let us consider the time profile of the CR concentration at a given space point in the kinetic description of the particle propagation in the case of an extended emission in the interplanetary medium. In the case of the instantaneous injection, unscattered particles are observed at the point r for the time $2r_0/v$. If the SCR injection occurs over the time t_0, then the residence time of unscattered particles at a given point is $2r_0/v + t_0$.

Thus, when $t < |r - r_0|/v$ and $t > (r + r_0)/v + t_0$, there will be no unscattered particles at the point r. Let us consider the case where the injection time exceeds $2r_0/v(t_0 > 2r_0/v)$. Then, based on (7), (20), and (21), we obtain the following expression for the concentration of unscattered particles (it holds true for the time interval $|r - r_0|/v < t < (r + r_0)/v$

$$\tilde{N}_0(\rho, p, \tau) = \frac{q(p)}{8\pi r_1^3\rho\rho_0\tau_0}\left[\operatorname{Ei}\left(-\frac{v r_1}{c\lambda}\tau\right) - \operatorname{Ei}\left(-\frac{r_1}{\lambda}|\rho-\rho_0|\right)\right], \tag{25}$$

Ei(x) is the exponential integral function.

If the time elapsed since the start of the CR injection satisfies $(r + r_0)/v < t < |r - r_0|/v + t_0$, the concentration of unscattered particles does not depend on time

$$\tilde{N}_0(\rho, p) = \frac{q(p)}{8\pi r_1^3\rho\rho_0\tau_0}\left[\operatorname{Ei}\left(-\frac{r_1}{\lambda}(\rho+\rho_0)\right) - \operatorname{Ei}\left(-\frac{r_1}{\lambda}|\rho-\rho_0|\right)\right]. \tag{26}$$

Provided that $|r - r_0|/v + t_0 < t < (r + r_0)/v + t_0$, the concentration of unscattered particles has the form

$$\tilde{N}_0(\rho, p, \tau) = \frac{q(p)}{8\pi r_1^3\rho\rho_0\tau_0}\left[\operatorname{Ei}\left(-\frac{r_1}{\lambda}(\rho+\rho_0)\right) - \operatorname{Ei}\left(-\frac{r_1}{\lambda}\frac{v}{c}(\tau-\tau_0)\right)\right]. \tag{27}$$

During the continuous injection, the concentration of scattered particles is described by a relation similar to formula (8)

$$\tilde{N}_s(\rho, p, \tau) = \tilde{N}_\alpha(\rho, p, \tau) + \tilde{N}_\beta(\rho, p, \tau). \tag{28}$$

Taking into account the injection function $\varphi(t)$ (21), from (20), we obtain

$$\tilde{N}_\alpha(\rho, p, \tau) = \int_0^t dt_1 N_\alpha(t-t_1)\varphi(t_1) \equiv \frac{1}{t_0}\left(\int_0^{t_0} dt_1 N_\alpha(t-t_1)\right), \tag{29}$$

where the function N_α is defined by relation (9). The value of $\tilde{N}_\beta$ can be obtained by the temporal integration of the CR concentration N_β (10)

$$\tilde{N}_\beta(\rho, p, \tau) = \frac{1}{t_0}\int_0^{t_0} dt_1 N_\beta(t-t_1). \tag{30}$$

After the integration according to formula (29), we obtain

$$\begin{aligned}\tilde{N}_\alpha(\rho, p, \tau) = {} & \frac{q(p)}{2\pi^2 r_1^3 (v/c)\rho\rho_0\tau_0}\int_0^{\pi/2} dk \frac{k^2}{\sin k(k\cos k - \sin k)}\sin\left(kr_1\frac{\rho}{\lambda}\right)\sin\left(kr_1\frac{\rho_0}{\lambda}\right) \\ & \times\left[\exp\left(\frac{v}{c}\frac{r_1}{\lambda}(k\cot k - 1)\tau\right) - \exp\left(\frac{v}{c}\frac{r_1}{\lambda}(k\cot k - 1)|\rho - \rho_0|\right)\right].\end{aligned} \tag{31}$$

if $|r - r_0|/v < t < |r - r_0|/v + t_0$.

Provided that $t > |r - r_0|/v + t_0$, we obtain the following relationship

$$\begin{aligned}\tilde{N}_\alpha(\rho, p, \tau) = {} & \frac{q(p)}{2\pi^2 r_1^3 (v/c)\rho\rho_0\tau_0}\int_0^{\pi/2} dk \frac{k^2}{\sin k(k\cos k - \sin k)}\sin\left(kr_1\frac{\rho}{\lambda}\right)\sin\left(kr_1\frac{\rho_0}{\lambda}\right) \\ & \times\left[\exp\left(\frac{v}{c}\frac{r_1}{\lambda}(k\cot k - 1)\tau\right) - \exp\left(\frac{v}{c}\frac{r_1}{\lambda}(k\cot k - 1)(\tau - \tau_0)\right)\right].\end{aligned} \tag{32}$$

According to formulae (30), (10), it is possible to change the integration order and integrate N_β with respect to the time t_1. As a result, we obtain the sum of several integrals over the variable η. Because of the fact that expressions for integrands turn out to be very cumbersome, we will not cite the expression for $\tilde{N}_\beta$. Let us only note that, for large values of the time ($t \gg \lambda/v$, $t \gg t_0$), the value of $\tilde{N}_\beta$ becomes negligible compared to $\tilde{N}_\alpha$ (32).

Figure 2b shows the dependence of the CR concentration at a given space point ($r = 1$ AU) on the time in the case of the continuous injection of particles with a kinetic energy of 40, 100 MeV, and 1 GeV. The duration of the SCR emission into the interplanetary medium $t_0 = 6$ min. The dependence of the CR transport mean free path on the particle momentum is assumed to be exponential (17) with an exponent of $\mu = 1/3$. The proton transport mean free path with a kinetic energy of 1 GeV $\lambda_0 = 0.5$ AU. Dashed curves describe the concentration of unscattered particles (25)–(27), and solid curves refer to the concentration of all particles (both scattered and unscattered).

The concentration of particles with the kinetic energy of 1 GeV is characterized by the temporal profile with a sharp maximum, and the time of CR maximum intensity is determined by their injection duration. Note that time profiles of the CR intensity recorded during the event at Antarctic neutron monitors show a sharp increase in the CR intensity in the first 5–6 minutes, and then a rapid decrease in the CR intensity for 10 minutes following the event [7–9, 15, 26]. Note that the shape of the temporal profile of the SCR concentration at the initial phase of the event depends significantly on the particle injection function. The curves corresponding to lower CR energy are characterized by a smoother maximum and a longer period of time from the arrival of the first particles to the maximum intensity (Fig. 2b). As the energy attenuates, the contribution of unscattered particles in the CR concentration decreases.

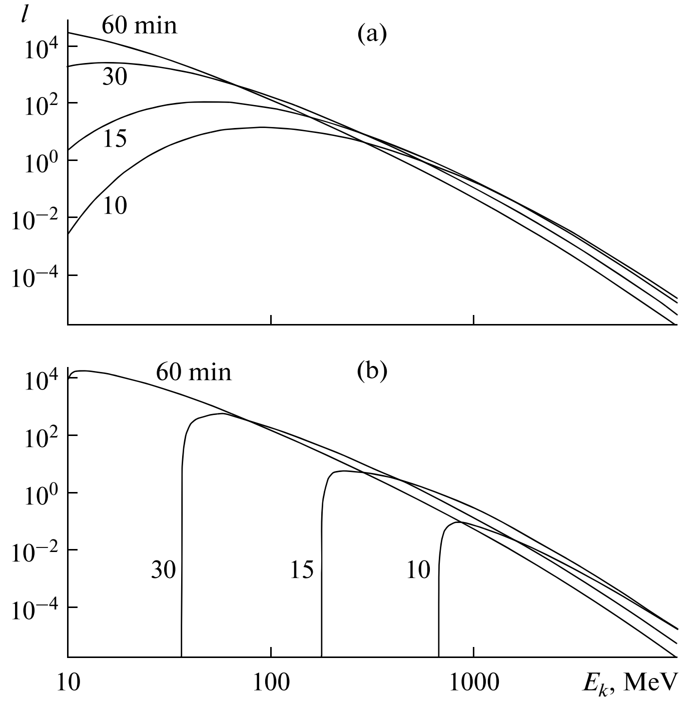

Fig. 3. Dependence of the intensity I of solar cosmic rays on the kinetic energy E_k of particles. Calculations in the (a) diffusion approximation and (b) based on the kinetic equation. The numbers on the curves are the time (in minutes) since the beginning of the injection of particles.

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It is possible to calculate the CR intensity, which is proportional to the particle concentration based on these relations

$$I(r, p, t) = p^2 N(r, p, t). \tag{33}$$

Figure 3 shows the dependence of the SCR intensity (33) on the particle kinetic energy. The numbers on curves correspond to the time (in minutes) elapsed since the emission of particles in the interplanetary medium. The SCR injection duration $t_0 = 6$ min. The transport mean free path of particles with a kinetic energy of 1 GeV is 0.5 AU. The dependence of the mean free path on the momentum is described by formula (17), while $\mu = 1/3$. The CR energy spectrum generated by the source is assumed to be exponential in terms of rigidity with an exponent of -5 [9, 23]. Relations (23), (24) describing the CR concentration in the diffusion particle propagation mode were used for the calculation of SCR energy spectra. It can be seen that the high-energy CR intensity decreases over time, while the low-energy CR intensity increases. The CR spectrum at the beginning of the flare (10 min) is depleted of low-energy particles in comparison with the energy distribution at a later time. Thus, the SCR energy spectrum becomes less rigid over time.

Let us consider the CR energy distribution evolution based on the kinetic equation. The CR intensity (33) at the heliocentric distance of 1 AU will be calculated using the relationships obtained for the CR concentration in the case of the continuous particle injection. The dependence of the SCR intensity on the particle kinetic energy is shown in Fig. 3b. The numbers on the curves are the time elapsed after the start of the SCR injection into the interplanetary medium. Values of the parameters are the following: $t_0 =$ 6 min, $\lambda_0 = 0.5$ AU, and $\mu = 1/3$. The SCR spectrum index at the source is -5. It can be seen that SCR spectra obtained by solving the kinetic equation are abruptly cut off at low energies. However, the low-energy boundary of the CR spectrum and the maximum of the energy particle distribution are shifted over time toward lower energies. This profile of energy spectra is caused by the later arrival of low-energy CRs with the simultaneous emission of particles of the whole energy range. Note the difference of the above-mentioned energy dependences (Fig. 3b) on corresponding curves calculated in the diffusion approximation (Fig. 3a). In particular, in the kinetic description of the CR propagation, there is the SCR spectrum boundary at low energies, which is not found in the diffusion approximation.

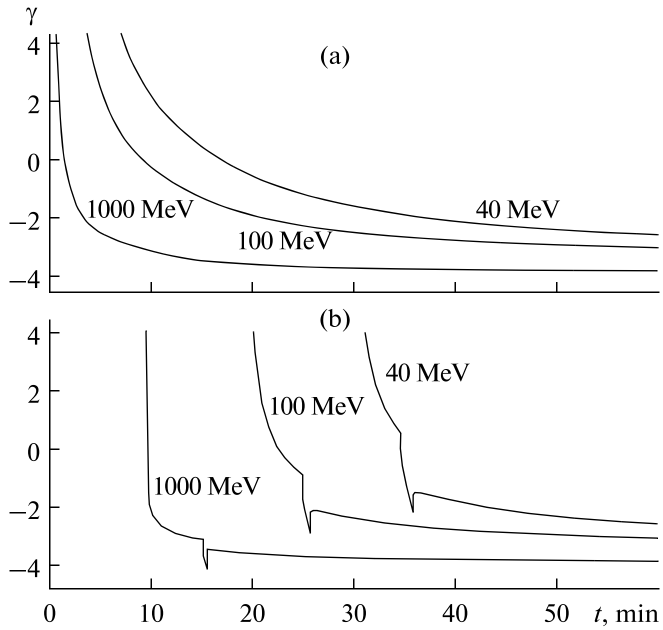

Fig. 4. Dependence of the spectrum of cosmic rays on the time elapsed after the start of the injection of particles with a kinetic energy of 40, 100, and 1000 MeV. The (a) diffusion approximation and (b) the kinetic analysis.

Let us consider how the CR energy spectrum index changes over time

$$\gamma = \frac{\varepsilon_k}{I(r, \varepsilon_k, t)} \frac{\partial I(r, \varepsilon_k, t)}{\partial \varepsilon_k}. \tag{34}$$

If the source of the particles is instantaneous and the diffusion approximation is applicable, the CR concentration is described by relation (15), and the spectrum index has the form

$$\gamma = \gamma_0 + \frac{r^2 + r_0^2 - 2\chi t - 2rr_0 \coth[rr_0/(2\chi t)]}{4\chi t} \frac{\varepsilon_k}{\chi} \frac{\partial \chi}{\partial \varepsilon_k}, \tag{35}$$

where γ_0 is the injected particle spectrum index.

At short times after the injection of particles ($t \ll rr_0/\chi$) from (35), it follows that

$$\gamma = \gamma_0 + \left[\frac{(r - r_0)^2}{4\chi t} - \frac{1}{2}\right] \frac{\varepsilon_k}{\chi} \frac{\partial \chi}{\partial \varepsilon_k}. \tag{36}$$

The CR diffusion coefficient χ increases with the growing particle energy; thus, it follows from (36) that the spectral index γ (36) is positive for short times. Thus, the SCR intensity increases with the growing particle energy. At large times, after the injection of particles ($t \gg rr_0/\chi$) from (35), we obtain

$$\gamma = \gamma_0 + \left[\frac{r^2 + r_0^2}{4\chi t} - \frac{r^2 r_0^2}{12\chi^2 t^2} - \frac{3}{2}\right] \frac{\varepsilon_k}{\chi} \frac{\partial \chi}{\partial \varepsilon_k}. \tag{37}$$

From these relations, it is clear that the SCR spectral index tends to the limit over time

$$\gamma = \gamma_0 - \frac{3}{2} \frac{\varepsilon_k}{\chi} \frac{\partial \chi}{\partial \varepsilon_k}. \tag{38}$$

If the CR transport range λ is a power function of particle momentum (17), then

$$\frac{\varepsilon_k}{\chi} \frac{\partial \chi}{\partial \varepsilon_k} = \frac{m^2 c^4}{(\varepsilon_k + mc^2)(\varepsilon_k + 2mc^2)} + \mu \frac{\varepsilon_k + mc^2}{\varepsilon_k + 2mc^2}. \tag{39}$$

Note that SCR spectral index (38) turns out to be greater in magnitude than the index of the spectrum of emitted particles γ_0. Thus, the spectrum of particles at large times is softer than the SCR spectrum in the source.

The dependence of the spectrum index on the time is shown in Fig. 4a. The calculation is performed in the diffusion approximation when the SCR concentration is represented by relations (23) and (24). The kinetic energy magnitude is shown by the corresponding curve. The transport mean free path of the particle with a kinetic energy of 1 GeV $\lambda_0 = 0.5$ AU, and the dependence of the transport mean free path on the particle momentum is exponential (17) with an exponent of $\mu = 1/3$. The particle injection time is 6 min. It can be seen that the energy spectrum index γ decreases monotonically over time in the diffusion approximation. At short times after the start of the particle emission, the spectral index is positive, while it is close to the value of (38) at large times γ, which depends on the particle energy.

Figure 4b shows the dependence of the SCR spectral index γ (34) on time in the case of the kinetic particle propagation mode. Values of the kinetic energy of particles are given on the curves. The following parameters were selected: $\lambda_0 = 0.5$ AU, $\mu = 1/3$, and $t_0 = 6$ min. It can be seen that particles that arrived at a given point first have a positive spectrum index, which rapidly decreases over time. The greater the particle energy, the faster the sign of the SCR spectral index changes. With further increase in time, the value of $\gamma(t)$ asymptotically approaches the value of (38).

Note the characteristic feature of the dependence $\gamma(t)$ which occurs at the end of the particle injection. Namely, there is a sharp abrupt decrease in the spectrum index to a local minimum, and then a rapid increase to the local maximum value of γ. This type of the dependence of $\gamma(t)$ at the end particle emission is caused by the selection of the SCR injection function in the form (21). Note that, in the case of the smoother temporal injection function profile, this feature of the time dependence of the SCR spectrum index is not observed [4].

CONCLUSIONS

The propagation of solar cosmic rays in the interplanetary medium was considered based on the kinetic equation. It was shown that, in the case of the instantaneous injection of particles accelerated during the solar flare, there is a sharp impulsive increase in the CR intensity at the beginning of the event because of unscattered particles. In the case of the continuous injection, the CR concentration increases gradually, and the particle emission duration into the interplanetary environment significantly affects the shape of the temporal profile of the SCR intensity at the initial phase of the flare. It was shown that, in the case of the simultaneous injection of particles of different energies, the SCR spectrum is cut off abruptly at low energies. The maximum of the SCR spectrum and its low-energy boundary shift to low-energies over time, and the energy distribution of particles gradually becomes less rigid.

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