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Channel Estimation Scheme for MIMO Communication Using Generalized Cholesky Decomposition and Back Substitution Methods

P. Saxena[1*], S. B. Patel[1], and J. K. Bhalani[2,3***]**

[1]*Charotar University of Science and Technology, Changa, India*
[2]*Babaria Institute of Technology, Vadodara, India*
[3]*Gujarat Technological University, Vadodara, India*
**e-mail: payalsaxenakumar@gmail.com*
***ORCID: 0000-0003-4240-6730, e-mail: sagarpatel.phd@gmail.com*
****ORCID: 0000-0001-8471-3208, e-mail: jaymin188@gmail.com*

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Abstract—In this article, a novel semi-blind channel estimation scheme for Multiple-Input Multiple-Output (MIMO) communication system is studied and implemented for quasi-static Rayleigh fading channel, in which channel matrix **H** remains relatively constant throughout the block. Channel matrix **H** can be decomposed as a rotation matrix **Q** and down triangular matrix **R**. The triangular matrix **R** is estimated blindly using the QR-decomposition based Generalized Cholesky Decomposition method (GCD) of output covariance matrix, which exploits Independent Component Analysis (ICA) based blind source separation stochastic method, and **Q** is estimated from the orthogonal pilot symbols using QR-based novel approach to minimize the cost function. In this novel approach, orthogonal pilots can be decomposed as a deterministic Hermitian matrix and the upper triangular matrix using the QR-decomposition, and finally matrix **Q** can be estimated by using the back substitution method, which is presented in this paper. Simulations are demonstrated using the Alamouti space-time coded 2 transmitter antennas and different combinations of 2 and 6 receiver antennas to showcase the performance of the novel scheme as compared to the conventional Least Squares (LS) and Maximum a posterior (MAP) estimation schemes using the BPSK data modulation scheme. The results indicate that the novel scheme outperforms the other schemes and shows a considerably better result in terms of bit error rate (BER). Hence, the novel scheme is quite useful for solving a complex problem of semi-blind MIMO channel estimation by using the QR matrix decomposition technique. Further error analysis is presented in terms of the error covariance matrix by considering the noise for non-zero error case (practical case) as compared to zero-error case.

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1. INTRODUCTION

MIMO communication systems may be considered among the most promising ones providing the benefits of variety of gains with the diversity perspectives and additional advantages of increased data rates using spatial multiplexing provisions. Wireless systems consider the opportunities of employing several antennas at the transmitter and receiver sides to exploit different advantages like array gain, high data rates, high efficiency, etc. [1], [2].

Channel state information (CSI) is one of the most important information for ensuring the performance of the system; hence, there is a need to implement a robust channel estimation technique [3]–[5].

There are three methods for channel estimation, namely, the pilot-based channel estimation, blind channel estimation, and the semi-blind channel estimation. In pilot-based channel estimation, special pilot symbols involve the need to estimate the channel information, which is not bandwidth efficient. In the blind technique, channel is estimated without using pilot symbols and CSI is calculated by using only the received signal statistics.

As is known, blind methods are more accurate but they are complex and feature the problems of poor convergence. Therefore, to exploit the benefits of both pilot-based estimation and blind estimation, we propose to use a semi-blind channel estimation technique in order to find the channel state information by using quasi-static channel (block fading channel).

Here semi-blind channel estimation techniques are presented by decomposing channel matrix **H** as a product, where **R** is the down triangular matrix, which is estimated blindly by using the QR-decomposition based generalized Cholesky decomposition (GCD) method for received data symbols and **Q** is the matrix estimated from pilot symbols by using the QR-decomposition based novel approach, which is presented in Section 3 [6]–[8].

2. SYSTEM MODEL

In this article, we use a quasi-static flat fading MIMO channel matrix $\mathbf{H} \in C^{r \times t}$, where t denotes the number of transmit antennas, r denotes the number of receive antennas in the prescribed system, and each h_{ij} signifies the i-th receiver and j-th transmitter in the flat-fading channel coefficients, which are unknown and need to be estimated. Here the channel matrix is considered for the case of the quasi-static fading, so **H** cannot be ill conditioned due to spatial correlation and also due to the absence of significant effects of channel fluctuations on the matrix close traces.

Complex received data denoted by $\mathbf{Y} \in C^{r \times 1}$, hence the equivalent received base-band system can be modeled as follows:

$$\mathbf{Y}(k) = \mathbf{H}\mathbf{X}(k) + \eta(k), \tag{1}$$

where k denotes the time instant, $\mathbf{X} \in C^{t \times 1}$ denotes the complex transmitted symbol vector, η represents the additive white Gaussian noise such that $\mathrm{E}\{\eta(k)\eta(l)\} = \delta(k,l)\sigma_n^2 I$, where $\delta(k,l) = 1$ if $k = 1$, and 0 or else. Furthermore, the sources are assumed to be spatially and temporally independent with identical source power σ_s^2, i.e., $\mathrm{E}\{X(k)X(l)\} = \delta(k,l)\sigma_s^2 I$.

The signal-to-noise ratio (SNR) is given by expression σ_s^2 / σ_n^2. Presuppose that the communication channel has been utilized for a total of N symbol transmissions. Among these N transmissions, the initial L symbols are the known training symbols and the rest of $(N - L)$ symbols are the blind data symbols.

3. CHANNEL ESTIMATION TECHNIQUES

3.1. Pilot-based LS and MAP Channel Estimation Techniques

These conventional techniques solve the channel estimation problem by minimizing Euclidean norms of the following cost function given below, where $\mathbf{X}_p$ are the orthogonal pilot symbols, $\mathbf{Y}_p$ is the output data, and $\hat{\mathbf{H}}$ is the estimated channel matrix [9]–[12], because the task of channel estimation algorithm is to recover matrix **H** using the knowledge of $\mathbf{Y}_p$ and $\mathbf{X}_p$:

$$\min \mathrm{E}\left\{ ||\mathbf{H} - \mathbf{H}_{\mathrm{LS}}||_F^2 \right\} = ||\mathbf{X}_p\hat{\mathbf{H}} - \mathbf{Y}_p||_F^2 = (\mathbf{X}_p\hat{\mathbf{H}} - \mathbf{Y}_p)^{\mathrm{H}}(\mathbf{X}_p\hat{\mathbf{H}} - \mathbf{Y}_p), \tag{2}$$

where $\mathrm{E}\{\bullet\}$ is the statistical expectation, $\| \ \|_F$ is Frobenius norm and $\mathbf{H}_{\mathrm{LS}}$ is a least square solution of estimated channel matrix.

The solution of the following equation can be found in the form:

$$\hat{\mathbf{H}}_{\mathrm{LS}} = (\mathbf{X}_p^{\mathrm{H}}\mathbf{X}_p)^{-1}\mathbf{X}_p^{\mathrm{H}}\mathbf{Y}_p = \mathbf{X}_p^{\dagger}\mathbf{Y}_p, \tag{3}$$

where $(\ )^{\dagger}$ denotes pseudo inverse (Moore–Penrose inverse).

Now, in the case of MAP estimation, we require orthogonal pilot symbols $\mathbf{X}_p$, channel covariance matrix $\mathbf{C}_H$, and noise covariance matrix $\mathbf{C}_n$:

$$\hat{\mathbf{H}}_{\mathrm{MAP}} = (\mathbf{X}_p^{\mathrm{H}} \mathbf{C}_n^{-1} \mathbf{X}_p + \mathbf{C}_H)^{-1} \mathbf{X}_p^{\mathrm{H}} \mathbf{C}_n^{-1} \mathbf{Y}_p. \quad (4)$$

3.2. Proposed Semi-blind Channel Estimation Technique

Here MIMO channel matrix $\mathbf{H}$ can be decomposed as the product

$$\mathbf{H} = \mathbf{R}\hat{\mathbf{Q}}^{\mathrm{H}},$$

where $\mathbf{R}$ denotes the down triangular matrix and $\mathbf{Q}$ denotes the unitary rotation matrix, i.e.,

$$\mathbf{Q}^{\mathrm{H}}\mathbf{Q} = \mathbf{Q}\mathbf{Q}^{\mathrm{H}} = \mathbf{I}.$$

Here $\mathbf{R}$ is the blindly estimated matrix using the covariance matrix of output data, which exploits ICA (Independent Component Analysis) based statistical approach to find unknown channel coefficients.

Now from equation (1), we can write the output covariance matrix in the form:

$$\mathbf{R}_{YY} = \mathrm{E}[\mathbf{Y}_b \mathbf{Y}_b^{\mathrm{H}}] = \mathbf{H}\mathbf{X}_b (\mathbf{H}\mathbf{X}_b)^{\mathrm{H}} + \sigma_n^2 \mathbf{I}, \quad (5)$$

where $\mathbf{Y}_b$ is the received output data, $\mathbf{X}_b$ is the blind input data, and σ_n^2 is the noise power.

Next, we can write

$$\mathbf{R}_{YY} = \mathbf{H}\mathbf{H}^{\mathrm{H}} + \sigma_n^2 \mathbf{I}, \quad (6)$$

$$\mathbf{H}\mathbf{H}^{\mathrm{H}} = \mathbf{R}_{YY} - \sigma_n^2 \mathbf{I} = \tilde{\mathbf{R}}_{YY}. \quad (7)$$

Using the QR-decomposition, we can write

$$\tilde{\mathbf{R}}_{YY} = (\mathbf{R}_b \mathbf{Q}_b)(\mathbf{R}_b^{\mathrm{H}} \mathbf{Q}_b^{\mathrm{H}}) = \mathbf{R}_b \mathbf{R}_b^{\mathrm{H}}, \quad (8)$$

$$\mathbf{Y}_b \mathbf{Y}_b^{\mathrm{H}} = (\mathbf{R}_b \mathbf{Q}_b)(\mathbf{R}_b^{\mathrm{H}} \mathbf{Q}_b^{\mathrm{H}}) = \mathbf{R}_b \mathbf{R}_b^{\mathrm{H}} = \mathbf{R}\mathbf{R}^{\mathrm{H}}. \quad (9)$$

Thus, matrix $\mathbf{R}$ can be estimated blindly by using the QR-decomposition based on GCD method. Here $\mathbf{R}_b$ and $\mathbf{Q}_b$ can be considered as the down triangular matrix and rotation matrix, which can be calculated using the QR-decomposition of $\mathbf{Y}_b$.

We can now find the unitary rotation matrix $\mathbf{Q}$ using orthogonal pilot symbols $\mathbf{X}_p$ by minimizing the cost function $\arg\min \|\mathbf{Y}_p - \mathbf{H}\mathbf{X}_p\|_F^2$.

Then the following relationship is valid:

$$\|\mathbf{Y}_p - \mathbf{H}\mathbf{X}_p\|_F^2 = \|\mathbf{Y}_p - \mathbf{R}\mathbf{Q}^{\mathrm{H}}\mathbf{X}_p\|_F^2. \quad (10)$$

Hence, it can be shown that error ε can be minimized in the following way:

$$\varepsilon = \mathbf{Y}_p - \mathbf{R}\mathbf{Q}^{\mathrm{H}}\mathbf{X}_p, \quad (11)$$

$$\varepsilon = 0 \Rightarrow \mathbf{Y}_p = \mathbf{R}\mathbf{Q}^{\mathrm{H}}\mathbf{X}_p. \tag{12}$$

Now matrix $\mathbf{X}_p$ can be decomposed into Hermitian matrix $\mathbf{Q}_p$ and upper triangular matrix $\mathbf{R}_p$ by using the QR-decomposition

$$\mathbf{Y}_p = \mathbf{H}\mathbf{X}_p = \mathbf{R}\mathbf{Q}^{\mathrm{H}}\mathbf{X}_p = \mathbf{R}\mathbf{Q}^{\mathrm{H}}\mathbf{Q}_p\begin{bmatrix}\mathbf{R}_p \\ 0\end{bmatrix}. \tag{13}$$

As is known that $\mathbf{Q}_p\mathbf{Q}_p^{\mathrm{H}} = \mathbf{I}$, we can write:

$$\mathbf{R}\mathbf{Q}^{\mathrm{H}}\begin{bmatrix}\mathbf{R}_p \\ 0\end{bmatrix} = \mathbf{Y}_p\mathbf{Q}_p^{\mathrm{H}}, \tag{14}$$

and $\mathbf{Q}^{\mathrm{H}}$ can be found by using the back substitution method:

$$\mathbf{Q}^{\mathrm{H}}\begin{bmatrix}\mathbf{R}_p \\ 0\end{bmatrix} = \mathbf{R}^{\dagger}\mathbf{Y}_p\mathbf{Q}_p^{\mathrm{H}}, \tag{15}$$

$$\hat{\mathbf{Q}}^{\mathrm{H}} = \mathbf{R}^{\dagger}\mathbf{Y}_p\mathbf{Q}_p^{\mathrm{H}} \bigg/ \begin{bmatrix}\mathbf{R}_p \\ 0\end{bmatrix}, \tag{16}$$

where $\mathbf{R}^{\dagger} = (\mathbf{R}^{\mathrm{H}}\mathbf{R})^{-1}\mathbf{R}^{\mathrm{H}}$ is the Moore–Penrose inverse.

Hence, we can calculate the final estimate of $\mathbf{H}$ by using equation (8), (9) and (16):

$$\hat{\mathbf{H}} = \mathbf{R}\hat{\mathbf{Q}}^{\mathrm{H}}. \tag{17}$$

Now, if we consider noise at non-zero error in the above case, then equation (13) can be rewritten in the form:

$$\mathbf{Y}_p = \mathbf{H}\mathbf{X}_p + \eta = \mathbf{R}\mathbf{Q}^{\mathrm{H}}\mathbf{X}_p + \eta = \mathbf{R}\mathbf{Q}^{\mathrm{H}}\mathbf{Q}_p\begin{bmatrix}\mathbf{R}_p \\ 0\end{bmatrix} + \eta, \tag{18}$$

$$\mathbf{R}\mathbf{Q}^{\mathrm{H}}\mathbf{Q}_p\begin{bmatrix}\mathbf{R}_p \\ 0\end{bmatrix} = \mathbf{Y}_p - \eta, \tag{19}$$

$$\mathbf{R}\mathbf{Q}^{\mathrm{H}}\begin{bmatrix}\mathbf{R}_p \\ 0\end{bmatrix} = \mathbf{Q}_p^{\mathrm{H}}(\mathbf{Y}_p - \eta), \tag{20}$$

$$\mathbf{Q}^{\mathrm{H}}\begin{bmatrix}\mathbf{R}_p \\ 0\end{bmatrix} = \mathbf{R}^{\dagger}\mathbf{Q}_p^{\mathrm{H}}(\mathbf{Y}_p - \eta) = \mathbf{R}^{\dagger}\mathbf{Y}_p\mathbf{Q}_p^{\mathrm{H}} - \mathbf{R}^{\dagger}\eta\mathbf{Q}_p^{\mathrm{H}}, \tag{21}$$

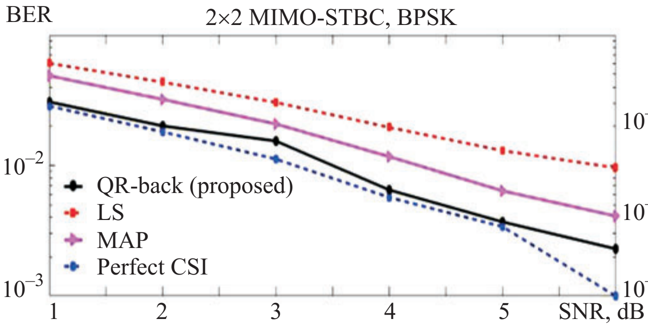

Fig. 1. Performance comparison in terms of BER vs SNR for 2 transmitter and 2 receiver antennas.

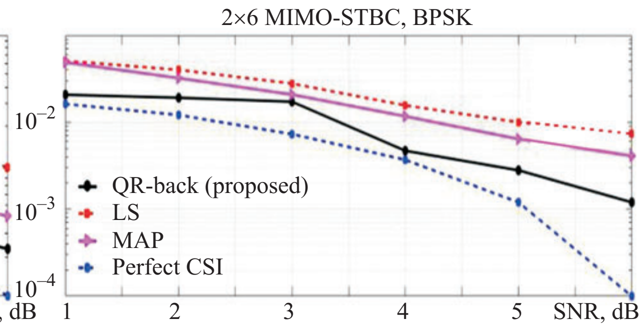

Fig. 2. Performance comparison in terms of BER vs SNR for 2 transmitter and 6 receiver antennas.

$$\hat{\mathbf{Q}}^{\mathrm{H}} = (\mathbf{R}^{\dagger}\mathbf{Y}_p\mathbf{Q}_p^{\mathrm{H}} - \mathbf{R}^{\dagger}\eta\mathbf{Q}_p^{\mathrm{H}}) \Big/ \begin{bmatrix}\mathbf{R}_p\\0\end{bmatrix}$$

$$= \underbrace{\left(\mathbf{R}^{\dagger}\mathbf{Y}_p\mathbf{Q}_p^{\mathrm{H}} \Big/ \begin{bmatrix}\mathbf{R}_p\\0\end{bmatrix}\right)}_{\text{signal}} - \underbrace{\left(\mathbf{R}^{\dagger}\eta\mathbf{Q}_p^{\mathrm{H}} \Big/ \begin{bmatrix}\mathbf{R}_p\\0\end{bmatrix}\right)}_{\text{error}}. \tag{22}$$

Here error e can be considered as

$$e = \left(\mathbf{R}^{\dagger}\eta\mathbf{Q}_p^{\mathrm{H}} \Big/ \begin{bmatrix}\mathbf{R}_p\\0\end{bmatrix}\right),$$

hence we can find the covariance matrix of error as follows:

$$R_e = \mathrm{E}\{ee^{\mathrm{H}}\} = \mathrm{E}\left\{\left(\mathbf{R}^{\dagger}\eta\mathbf{Q}_p^{\mathrm{H}} \Big/ \begin{bmatrix}\mathbf{R}_p\\0\end{bmatrix}\right)\left(\mathbf{R}^{\dagger}\eta\mathbf{Q}_p^{\mathrm{H}} \Big/ \begin{bmatrix}\mathbf{R}_p\\0\end{bmatrix}\right)^{\mathrm{H}}\right\}$$

$$= \mathrm{E}\left\{\left(\mathbf{R}^{\dagger}(\mathbf{R}^{\dagger})^{\mathrm{H}}\eta\eta^{\mathrm{H}}\mathbf{Q}_p^{\mathrm{H}}\mathbf{Q}_p \Big/ \begin{bmatrix}\mathbf{R}_p\\0\end{bmatrix}\begin{bmatrix}\mathbf{R}_p\\0\end{bmatrix}^{\mathrm{H}}\right)\right\}$$

$$= \sigma_n^2\mathrm{E}\left\{\left((\mathbf{R}\mathbf{R}^{\mathrm{H}})^{\dagger} \Big/ \begin{bmatrix}\mathbf{R}_p\\0\end{bmatrix}\begin{bmatrix}\mathbf{R}_p\\0\end{bmatrix}^{\mathrm{H}}\right)\right\} = \sigma_n^2\mathrm{E}\left\{\left((\mathbf{R}\mathbf{R}^{\mathrm{H}})^{\dagger} \Big/ \mathbf{R}_p\mathbf{R}_p^{\mathrm{H}}\right)\right\}. \tag{23}$$

4. SIMULATION RESULTS

In this simulation setup, the digital data modulation scheme is BPSK and the quasi-static Rayleigh fading channel is generated. We have taken setup for 2 transmitter antennas and 2 and 6 receiver antennas with 4 pilot symbols and 100 blind symbols, which are drawn from BPSK modulation constellations.

Figure 1 presents the simulation results of the proposed semi-blind channel estimation technique (QR-back) in comparison with well-known conventional pilot based LS and MAP channel estimation techniques [9]–[11] for 2 transmitter and 2 receiver antennas. The BER characteristic obtained reveals that the proposed technique outperforms the other two techniques and shows the near perfect CSI (for known channel).

Figure 2 shows the results for 2 transmitter and 6 receiver antennas. It can be seen that due to diversity when the number of receiver antennas is increased, the BER characteristics improve considerably by showing a 2–3 dB improvements.

5. CONCLUSIONS

This study involved the investigation of the QR-decomposition based novel semi-blind channel estimation technique for MIMO systems employing the Rayleigh fading quasi-static channel. After the study and investigation of the proposed technique, it can be proved that the proposed technique QR-back outperforms the pilot-based two conventional techniques in terms of BER performance by 2–3 dB. In addition, with the rise of the number of receiver antennas, the system performance improves significantly.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

ADDITIONAL INFORMATION

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