ISSN 0735-2727, Radioelectronics and Communications Systems, 2019, Vol. 62, No. 11, pp. 577–584. © Allerton Press, Inc., 2019.
Russian Text © The Author(s), 2019, published in Izvestiya Vysshikh Uchebnykh Zavedenii, Radioelektronika, 2019, Vol. 62, No. 11, pp. 678–687.

Pergroup and Joint Optimization of Max-Dmin Precoder for MIMO with LDPC Coding Using QAM Modulation

Sarra Mahi[1*] and Abdelhafid Bouacha[1]**

[1]*Abou Beker Belkaid University, Tlemcen, Algeria*

**ORCID: 0000-0001-6405-9296, e-mail: msarah21@hotmail.fr*

***ORCID: 0000-0003-1827-7975, e-mail: abdelhafid.bouacha@mail.univ-tlemcen.dz*

Received August 27, 2018
Revised August 14, 2019
Accepted November 2, 2019

Abstract—MIMO technology not only offers diversity and capacity gains, but also provides higher spectral efficiency and significant link reliability over SISO systems. Many methods are developed to exploit the diversity offered by multi-antenna systems such as Alamouti code and spatial multiplexing that do not require transmitter-side channel status information (Tx-CSI). Other power allocation optimization techniques, also known as precoding, require a full or partial Tx-CSI. These techniques aim to transform the signal before transmission according to different criteria, among which the minimal Euclidean distance seems to be very effective and continues to interest the researchers. Given perfect channel state information at both sides of the communication, we propose in this paper a novel design of wireless transmission schemes that joint the minimal Euclidean distance precoder and error correction coding based on the non-binary low-density parity-check code (NB-LDPC), to finally determine a power allocation optimization solution that adapts a linear precoding block to an NB-LDPC encoded MIMO transmission. In this paper we use a quadrature amplitude modulation (QAM), over a Rayleigh fading channel with a maximum likelihood detection. Simulations results in term of bit error rate confirmed that NB-LDPC codes are particularly well suited to be jointly used with precoding schemes based on the maximization of the minimum Euclidean distance criterion.

DOI: 10.3103/S0735272719110037

1. INTRODUCTION

The continued development of mobile systems and social networks is driving an exponential increase in data traffic, pushing cellular networks to their capacity limits.

Modern wireless communications, such as the LTE and LTE-A standards [1] and particularly fifth-generation (5G) [2, 3] cellular networks respond to this need, and replacing the current generation in some years. These modern cellular networks require a high data rate with low transmission latency. Especially, high-rate coding, high-order modulation and Multiple-Input Multiple Output (MIMO) technology are essential tools for achieving high data rates.

The advantages of MIMO technology are generally provided by open loop and closed loop techniques [4, 5]. Open loop techniques, such as spatial coding (STC) and spatial multiplexing (SM) [6], are used without the need for Channel State Information (CSI) at the transmitter. To overcome the multipath effect and to improve the robustness of SM systems, closed loop linear pre-coding techniques may be used at the transmitter.

The precoding techniques principle [7] is following: when the channel knowledge is available to the transmitter, a transmit signal is pre-multiplied by a precoding matrix so that the inter-symbol interference (ISI) in the receiver is greatly reduced. This knowledge of the channel characteristics makes possible the anticipation of any damage caused by the propagation to obtain a favorable "global" transmission channel.

CSI at the transmitter (CSIT) can be obtained via a feedback link, but it is difficult to achieve a perfect CSIT in a MIMO system with a rapidly evolving channel. As a result, transmitters in many MIMO systems have no knowledge of the current channel. This motivates the use of limited feedback link precoding methods.

Considering the CSI at the receiver, the antenna power allocation strategies can be realized by the joint optimization of the linear precoder (at the transmitter) and the decoder (at the receiver). This optimization is carried out according to various criteria such as the output capacity maximization [8], signal-to-noise ratio (SNR) [9], mean squared error (MSE) minimization [10], the bit error rate (BER) minimization [11] or the maximization of the minimum singular value of the channel matrix [12]. These optimized precoding matrices are an important set of linear precoding techniques called diagonal precoders.

Another group of precoding techniques is the non-diagonal linear structure. One of the most efficient non-diagonal precoders is based on maximizing the minimum Euclidean distance (max-dmin) between two received data vectors [13, 14]. The max-dmin precoder offers a significant improvement in BER compared to other precoding strategies [15].

On the other hand, convolution codes (CC) and block codes have been studied for MIMO systems [16]. The CC code attracted more attention because of its decoder process based on the Viterbi decoder, and because of its use in turbo-codes and its capacity performance close to Shannon capacity [17].

The linear block code LDPC (low-density parity-check) is a serious competitor to the turbo codes [18, 19], due to its efficiency it is widely used because of its. LDPC codes have been shown to be good enough to achieve near capacity performance. The potentials of LDPC codes for the MIMO system have been revealed in numerous studies [20–22]. In [23], the simulation results showed that LDPC surpasses the turbo codes in the two correlated and uncorrelated Rayleigh channels. Among the LDPC codes, the non-binary codes (NB-LDPC) outperform the binary LDPC codes, particularly for short-to-moderate length codes, reducing decoding latency.

Thus, the combination of LDPC codes with MIMO systems provides a promising response to reach optimal use of channel capacity with high bit rate.

Although linear precoding techniques and LDPC codes have been developed in literatures [24, 25], it is still difficult to jointly design the linear precoder and the LDPC code for MIMO systems.

In this context, we consider the existence of an upstream external error correction code in the MIMO pre-coding optimization, to finally determine a power allocation solution for a block linear precoding and an LDPC coded MIMO transmission.

In this paper we propose, firstly, to use simple CC to limit the complexity and latency and optimize the error rate performance of MIMO system by defining non-diagonal linear precoder max-dmin. Afterwards, we replace the CC code with the NB-LDPC code in the precoded MIMO system, and we compare the MIMO-LDPC using SM system to the LDPC-MIMO max-dmin system. Finally, we confirm the BER performance, by numerical simulations, for the proposed power allocation optimization scheme.

Rest of this paper is organized as follows: Section 2 presents a description of the proposed system model and the methods used, while Section 3 shows its simulation results. Finally, some conclusions are drawn in Section 4.

2. SYSTEM MODEL

2.1. Channel Model

We consider a MIMO system with n_R receiving antenna and n_T transmitting antenna, and a data flow $b =$ 2. We assume a complete CSI on transmitter and receiver. Therefore the received signal is following:

$$\mathbf{y} = \mathbf{GHFs} + \mathbf{Gn}, \tag{1}$$

where $\mathbf{G}$ is the $(b \times n_R)$ linear decoder matrix, $\mathbf{H}$ is the $(n_R \times n_T)$ channel matrix, $\mathbf{F}$ is the $(n_T \times b)$ linear precoder matrix, $\mathbf{s}$ the $(b \times 1)$ transmitted symbol vector, $\mathbf{n}$ is the $(n_R \times 1)$ additive Gaussian noise vector and $\mathbf{y}$ is the $(b \times 1)$ received symbol vector.

We assume

$$b \le r = \operatorname{rank}(\mathbf{H}) \le \min(n_T, n_R),$$

and $\mathbf{I}_b$ denotes the $(b \times b)$ identity matrix. In addition, $E[\mathbf{ss}^*] = \mathbf{I}_b$, $E[\mathbf{sn}^*] = 0$, and $E[\mathbf{nn}^*] = R$, where R is the noise covariance matrix, and superscript "*" stands for conjugate transposition.

All over the paper, the perfect CSI condition at both the transmitter and the receiver is assumed. Under this condition, the channel can be diagonalized by using the virtual transformation. So, the precoding and decoding matrix can be written as

$$\mathbf{F} = \mathbf{F}_v\mathbf{F}_d \quad \text{and} \quad \mathbf{G} = \mathbf{G}_v\mathbf{G}_d. \tag{2}$$

Then, the new decomposition of $\mathbf{F}_v$ and $\mathbf{G}_v$ matrices into the product of three matrices are considering:

$$\mathbf{F}_v = \mathbf{F}_1\mathbf{F}_2\mathbf{F}_3,$$

$$\mathbf{G}_v = \mathbf{G}_1\mathbf{G}_2\mathbf{G}_3, \tag{3}$$

where ($\mathbf{F}_i$, $\mathbf{G}_i$) perform particular operations: noise whitening, channel diagonalization and dimensionality reduction.

Therefore, the input–output relation (1) is:

$$\mathbf{y} = \mathbf{G}_d\mathbf{H}_v\mathbf{F}_d\mathbf{s} + \mathbf{G}_d\mathbf{n}_v, \tag{4}$$

where $\mathbf{H}_v$ is the $(b \times b)$ virtual channel matrix, written as:

$$\mathbf{H}_v = \text{diag}(\sigma_1, \ldots, \sigma_b), \tag{5}$$

where σ_i are entries with: $\sigma_1 > \sigma_2 > \ldots > \sigma_i$, $i = 1, \ldots, b$, and $\mathbf{n}_v = \mathbf{G}_v\mathbf{n}$ is the $(b \times 1)$ transformed additive Gaussian noise vector. Now we obtain the virtual system model as:

$$\mathbf{y} = \mathbf{H}_v\mathbf{F}_d\mathbf{s} + \mathbf{n}_v. \tag{6}$$

The simulator developed during this work is based on a modular approach. This approach allows us to perform the simulations in the same way that communication systems works. The principle of this procedure is to define the initialization parameters and the input data to obtain the output results.

2.2. Spatial Multiplexing

In this work, we assume that the fading of the different channels is independent that leads to a full-rank channel matrix with high probability.

Therefore, it is possible to consider the MIMO channel like several SISO channels in parallel. In each of these channels we transmit information flows, with this technique, we increase the information rate and we obtain a gain called multiplexing gain. This gain is limited by $\min(n_\text{T}, n_\text{R})$.

We consider for our transmission channel a SM where each sublayer is associated with a single transmitting antenna. From there we can state that SM consists of sub-flows obtained by dividing the data stream.

We consider for example $n_\text{T} = n_\text{R} = 2$, so the code matrix can be written as follows:

$$C_\text{SM} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}. \tag{7}$$

The signal received is following:

$$\begin{bmatrix} y_{11} \\ y_{21} \end{bmatrix} = \mathbf{H}\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{21} \end{bmatrix}. \tag{8}$$

2.3. Max-dmin Precoder

The max-dmin is non-diagonal precoder. This property gives additional degree of freedom to the precoder, which can now modify the geometries of the receiving constellations [11]. The principle of this precoder is to maximize the minimal distance between received constellations [11], this value is denoted d_min and given by:

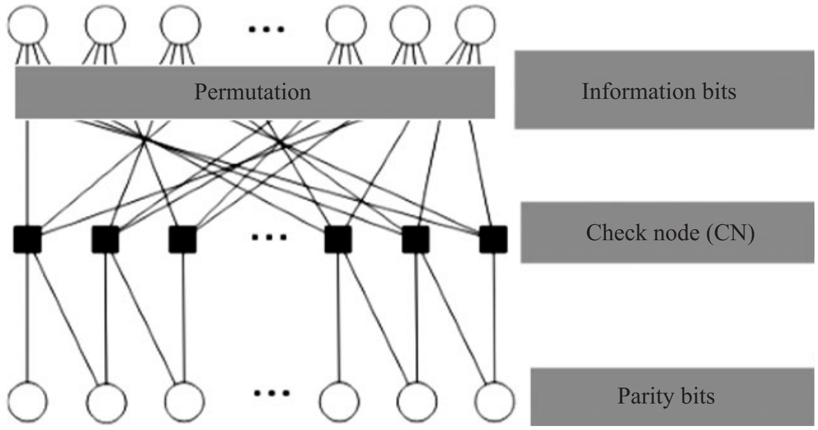

Fig. 1. Tanner graph for LDPC code.

$$d_{\min}(\mathbf{F}_d) = \min_{\substack{(s_k, s_l) \in C^b \\ s_k \neq s_l}} ||\mathbf{H}_v \mathbf{F}_d (s_k - s_l)||, \tag{9}$$

where C represents the set of complex symbols of the constellation, s_k and s_l are two transmit signals, and $\mathbf{S}$ is the set of all these possible transmit vectors.

Let $\mathbf{x}$ be a difference vector like $\breve{x} = s_k - s_l$ with $s_k \neq s_l$. The max-dmin solution consists of finding the coefficients of the matrix $\mathbf{F}_d$, which maximize the minimum distance of the received constellation:

$$\mathbf{F}_d = \arg \max_{\mathbf{F}_d} \{d_{\min}(\mathbf{F}_d)\} \tag{10}$$

with

$$\text{trace}\,(\mathbf{F}_d \mathbf{F}_d^*) = P_0. \tag{11}$$

Max-dmin solutions for 4QAM modulation and 16QAM modulation are described in [26].

2.4. LDPC Code

In this work we propose the use of the LDPC error correction coding provided by the DVB-S2 standard [27].

In DVB-S2 standard the LDPC code has normal frames with the 64800-bits code word length and short frames with the 16200-bits code word length. Normal frames have 11 different coding rates ranging from 1/4 to 9/10 and short frames have 10 different coding rates ranging from 1/4 to 8/9. The coding procedure comes from the DVB-S2 standard. Figure 1 shows the Tanner graph for the irregular LDPC code in DVB-S2, and it shows some regularities that can be used for hardware implementation.

The coding table of the DVB-S2 standard could be considered as another description of the parity check matrix. Each row of the table presents the corresponding addresses of the verification nodes to these 360 variable nodes. The table has $q = (N - K) / 360$ lines and the degree of information bits can be discovered in Table 1.

In the DVB-S2 standard, the information bits consist of two degrees D_j in the front part and D_3 in the left part of the information bits. Each frequency has a different D_j but has the same $D_3 = 3$. The degree D_c for the control node is uniform and can be calculated from known D_c and D_j:

$$D_\text{c} = \Big([D_j \times \text{Range} + D_3 \times (q - \text{Range})] \times 360 + 2(N - K)\Big)(N - K)^{-1}, \tag{12}$$

where the row is the number of lines that has a degree with D_j.

Table 1. Parameters for LDPC code in DVB-S2 of 11 code rates

Rate (R)	D_j	E[D_j]	E[D_3]	D_c	K
1/4	12	5400	10,800	4	16200
1/3	12	7200	14400	5	21600
2/5	12	8640	172800	6	25920
1/2	8	12960	19440	7	32400
3/5	12	12960	25920	11	38880
2/3	13	4320	38880	10	43200
3/4	12	5400	43200	14	48600
4/5	11	6480	45360	18	51840
5/6	13	5400	48600	22	54000
8/9	4	7200	50400	27	57600
9/10	4	6480	51840	30	58320

In our system the LDPC code is decoded by the Minimum-Sum algorithm. It is a simplification of the belief-based propagation (BP) based on the LLR (Log Likelihood Ratio). It is implemented with a layered decoding approach.

The basic idea of this iterative algorithm is the exchange of messages between Variables Nodes (VN), corresponding to the bits/symbols received, and the variable nodes (CNs), corresponding to the parity control equations, to calculate the new probabilities of a given received bit being “0” or “1”. The messages are signed integers corresponding to the quantized LLRs of the received bits, defined as: parameters for the LDPC code in DVB-S2 of 11 code rates

$$\mathrm{LLR}(x_n) = \log \frac{P(x_n - 0 | y_n)}{P(x_n - 1 | y_n)} \in \Re, \tag{13}$$

where y_n is the received symbol and x_n is the hard bit inside the frame.

In layered decoding, a VN is represented by its soft-output (SO) value, which is the LLR frequently updated during iterations. Each iteration is divided into sub-iterations, a sub-iteration for each layer in one or more CNs.

Assuming that l_n is the initial LLR of bit n, Q_n is the LLR (or SO) of bit n, given all the messages exchanged, Q_{nm} is the message sent by VN n to CN m, R_{mn} is the message sent by CN m to VN n and $N(m)\backslash n$ is the set of VNs connected to CN m excluding n. The steps of the decoding algorithm in layers are:

1) initialization: $Q_n = L_n$, $R_{mn} = 0$; $R_{mn}^{\text{old}} =$ the previous value of R_{mn};

2) calculation the message VN: $Q_{mn} = Q_n - R_{mn}^{\text{old}}$;

3) CN update: $R_{mn}^{\text{new}} = f(Q_n, m)$, porn$' \in N(m) / n$;

4) SO update: $R_{mn}^{\text{new}} = Q_{nm} + R_{mn}^{\text{new}}$.

Decision: if all the VNs satisfy the control equations, a codeword is detected, thus finalizing the algorithm; if the maximum number of iterations was reached, the decoding failed, so finalize the algorithm; otherwise go to step 2 for the next iteration.

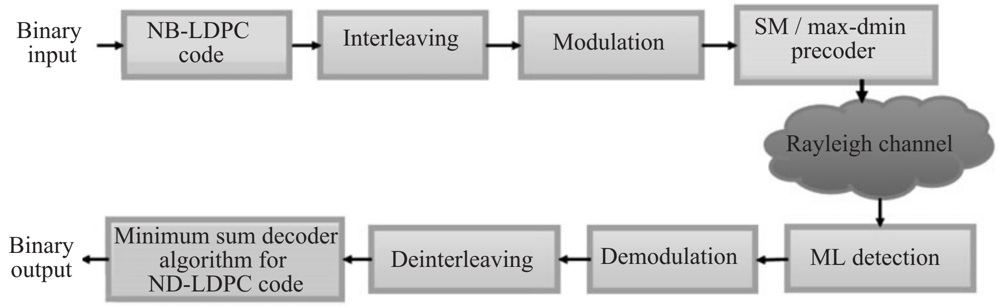

Fig. 2. Transmission chain for max-dmin precoded MIMO system with NB-LDPC coding.

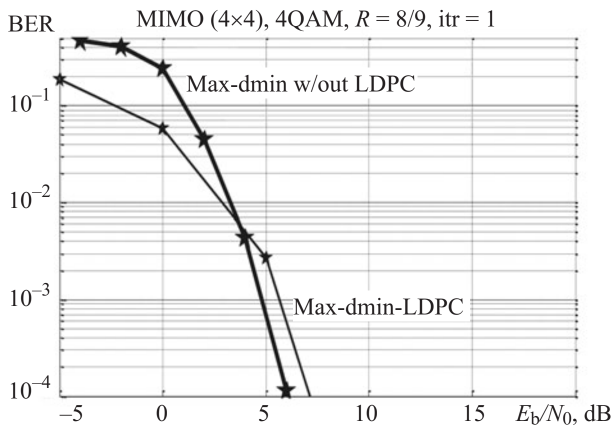

Fig. 3. Evaluation of max-dmin precoder performance with and without LDPC code.

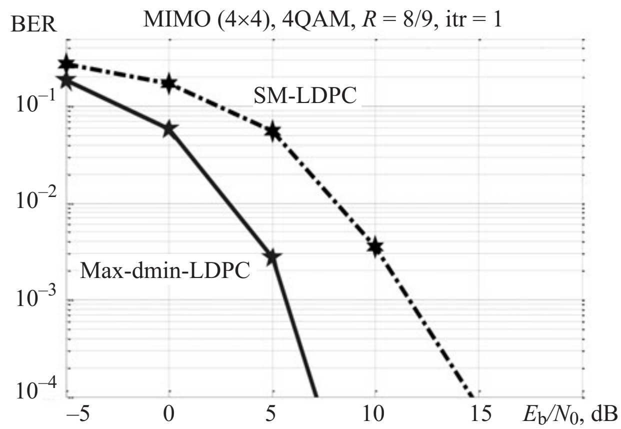

Fig. 4. Comparison between SM-LDPC and max-dmin-LDPC.

3. SIMULATIONS AND RESULTS

To perform the different simulations, we use the transmission chain illustrated in Fig. 2.

The performance of the proposed power allocation system is validated through a series of simulations that evaluate the improvement obtained in terms of BER.

For this simulation, we consider a MIMO system with n_R receive antenna and n_T transmit antenna, and data stream $b = 2$. We assume a complete CSI at the transmitter and at the receiver.

The input element of our chain is a binary train, it is generated in the form of random numbers or a succession of zeros and ones. The length of the information bits depends on the LDPC coding rate. Then the generated data is encoded by the predicted LDPC error correction coding gives by the DBV-S2 standard. The coded word is then randomly interleaved and converted into complex symbols belonging to the constellation alphabet of the modulation. We use 4QAM and 16QAM modulations. The modulated symbol is then either precoded (precoder max-dmin) or multiplexed before being transmitted via the MIMO channel.

The formatting, either by SM or by the precoder max-dmin, is disturbed by the propagation channel. The Rayleigh channel model is considered in our system.

At the reception, we use an ML (Maximum Likelihood) equalization for the max-dmin precoder and SM. Then after demodulation and de-interleaving we use the minimum-sum algorithm for LDPC decoding.

Finally, the received and transmitted data are compared to determine BER. The target BER = 10^{-4}.

We demonstrate in the first simulation the BER performance of the different precoders and the SM technique for MIMO system with two transmit, two receive antennas, and 4QAM modulation.

In Fig. 3 it is shown the BER performance of the MIMO schema with LDPC coding where optimization of the power allocation is performed by applying the max-dmin precoder. This system is designated as "Max-dmin-LDPC" and its performance is compared to performance of the MIMO system without LDPC.

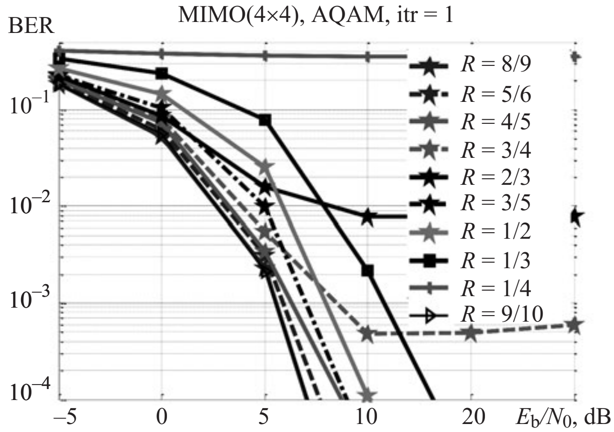

Fig. 5. Influence of coding rate R on max-dmin-LDPC system.

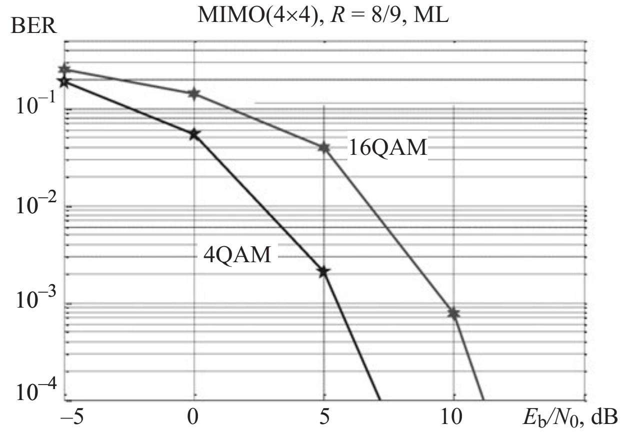

Fig. 6. Influence of modulation order on max-dmin-LDPC system.

The simulation results show that for low SNRs (in the range of [–5, +6]), the "Max-dmin-LDPC" system presents significant improvement compared to the "Max-dmin w/out LDPC" system. This is due to the LDPC decoding algorithm which significantly reduces errors.

As shown from Fig. 4 the SM-LDPC system reaches the BER = 10^{-4} for SNR = 14.5 dB while the max-dmin-LDPC system reaches the same BER value for only 7 dB and that's for an LDPC coding rate R = 8/9. Therefore, the max-dmin-LDPC system has a 7.5 dB gain on the SM-LDPC system for BER = 10^{-4}. This equates to a 67% increase in transmission rate for the same BER performance, through the use of max-dmin precoder with LDPC code. The advantage of the combination of max-dmin precoders and the LDPC code is clearly established.

In Fig. 5 it is represented the simulation results of "Max-dmin-LDPC" in the MIMO 4×4 system with a 4QAM modulation and different LDPC code rate. The gain brought by the max-dmin precoder becomes more important for LDPC codes with higher rates. The best BER is obtained for a coding rate of R = 8/9 (which means an **H** matrix (57600, 64800)) with a gain of 3 dB compared to the coding rate of R = 1/2.

In Fig. 6 it is represented a performances comparison of the max-dmin-LDPC MIMO system with 4×4 configuration and coding rate R = 8/9, for two different modulations 4QAM and 16QAM.

From the results we note that 4QAM modulation has better BER performance compared to 16QAM with 4 dB gain. This is mainly due to the reduction points number of the QAM constellation.

4. CONCLUSIONS

The purpose of this paper is to determine a power allocation optimization solution that matches a linear precoding block to an LDPC encoded MIMO transmission.

To do this we proposed to study the combination of a non-binary LDPC code due to its efficiency with spatial multiplexing (SM) in a MIMO system.

To further improve the quality of the transmission we proposed the association of NB-LDPC codes with a non-diagonal precoder which optimizes the criterion of the minimal Euclidean distance. The results of the simulations show the interest to associate the precoder to the NB-LDPC codes in the MIMO systems, especially for the low SNRs, which is particularly well suited to high-speed transmissions.

In addition, in many highlighted results it is shown that the performance in terms of BER is significantly influenced by the modulation techniques and the number of antennas in transmission and reception. We clearly see that MIMO 4×4 configuration and 4QAM modulation for max-dmin-LDPC have the best BER performance for the proposed system.

Throughout this work, we assume a perfect CSI at the transmitter. In practical applications, the imperfect CSI at the transmitter can lead to a loss of performance. Therefore, extending the studies of this work to the imperfect CSI system could also be of interest for future works.

REFERENCES

1. I. V. Strelkovskaya, T. I. Grygoryeva, I. N. Solovskaya, "Self-similar traffic in G/M/1 queue defined by the Weibull distribution," *Radioelectron. Commun. Syst.* **61**, No. 3, 128 (2018). DOI: 10.3103/S0735272718030056.

2. D. A. Pokamestov, Y. V. Kryukov, E. V. Rogozhnikov, R. R. Abenov, A. Y. Demidov, "Concepts of the physical level of the fifth generation communications systems," *Radioelectron. Commun. Syst.* **60**, No. 7, 285 (2017). DOI: 10.3103/S0735272717070019.
3. A. Morgado, K. M. S. Huq, S. Mumtaz, J. Rodriguez, "A survey of 5G technologies: regulatory, standardization and industrial perspectives," *Digit. Commun. Netw.* **4**, No. 2, 87 (2017). DOI: 10.1016/j.dcan.2017.09.010.
4. V. Tarokh, H. Jafarkhani, A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory* **45**, No. 5, 1456 (1999). DOI: 10.1109/18.771146.
5. V. B. Kreyndelin, T. B. K. Ben Rejeb, "Nonlinear iterative precoding algorithm for MIMO multiuser systems," *Radioelectron. Commun. Syst.* **60**, No. 10, 449 (2017). DOI: 10.3103/S073527271710003X.
6. Q. Jing, Jiajia Wu, "Performance comparison of space-time block and trellis codes in the MIMO land mobile satellite channels," *Radioelectron. Commun. Syst.* **60**, No. 1, 1 (2017). DOI: 10.3103/S0735272717010010.
7. Nhat-Quang Nhan, Philippe Rostaing, Karine Amis, Ludovic Collin, Emanuel Radoi, "Complexity reduction for the optimization of linear precoders over random MIMO channels," *IEEE Trans. Commun.* **65**, No. 10, 4205 (2017). DOI: 10.1109/TCOMM.2017.2716375.
8. E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecommun.* **10**, No. 6, 585 (Nov./Dec. 1999). DOI: 10.1002/ett.4460100604.
9. P. Stoica, G. Ganesan, "Maximum-SNR spatial-temporal formatting designs for MIMO channels," *IEEE Trans. Signal Process.* **50**, No. 12, 3036 (2002). DOI: 10.1109/TSP.2002.805266.
10. H. Sampath, P. Stoica, A. Paulraj, "Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion," *IEEE Trans. Commun.* **49**, No. 12, 2198 (2001). DOI: 10.1109/26.974266.
11. P. Rostaing, O. Berder, G. Burel, L. Collin, "Minimum BER diagonal precoder for MIMO digital transmissions," *Signal Process.* **82**, No. 10, 1477 (2002). DOI: 10.1016/S0165-1684(02)00288-8.
12. A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Trans. Signal Process.* **50**, No. 5, 1051 (2002). DOI: 10.1109/78.995062.
13. L. Collin, O. Berder, P. Rostaing, G. Burel, "Optimal minimum distance-based precoder for MIMO spatial multiplexing systems," *IEEE Trans. Signal Process.* **52**, No. 3, 617 (Mar. 2004). DOI: 10.1109/TSP.2003.822365.
14. Q.-T. Ngo, O. Berder, P. Scalart, "Minimum Euclidean distance based precoders for MIMO systems using rectangular QAM modulations," *IEEE Trans. Signal Process.* **60**, No. 3, 1527 (2012). DOI: 10.1109/TSP.2011.2177972.
15. S. Mahi, A. Bouacha, "Behavior of the minimum Euclidean distance optimization precoders with soft maximum likelihood detector for high data rate MIMO transmission," *Int. J. Advanced Computer Sci. App.* **9**, No. 2, 364 (2018). DOI: 10.14569/IJACSA.2018.090250.
16. C. E. Shannon, "A mathematical theory of communication," *Bell System Tech. J.* **27**, No. 4, 623 (1948). DOI: 10.1002/j.1538-7305.1948.tb00917.x.
17. A. A. Shpylka, S. Ya. Zhuk, "Decoding of convolutional codes on a sliding window during signal propagation in a multipath communications channel," *Radioelectron. Commun. Syst.* **53**, No. 9, 497 (2010). DOI: 10.3103/S0735272710090086.
18. R. Gallager, "Low-density parity-check codes," *IRE Trans. Inf. Theory* **8**, No. 1, 21 (1962). DOI: 10.1109/TIT.1962.1057683.
19. D. J. C. MacKay, R. M. Neal, "Near Shannon limit performance of low density parity check codes," *Electron. Lett.* **33**, No. 6, 457 (1997). DOI: 10.1049/el:19970362.
20. S. T. Brink, G. Kramer, A. Ashikhmin, "Design of low-density parity-check codes for modulation and detection," *IEEE Trans. Commun.* **52**, No. 4, 670 (Apr. 2004). DOI: 10.1109/TCOMM.2004.826370.
21. M. R. Islam, Md. M. Hossain, Md. A. Hoque, K. K. Islam, Md. S. Ullah, "Low density parity check code in cooperative MIMO communication at wireless sensor network," *Radioelectron. Commun. Syst.* **54**, No. 7, 359 (2011). DOI: 10.3103/S073527271107003X.
22. T. Deepa, R. Kumar, "Performance evaluation of LDPC coded MIMO transceiver with equalization," *Proc. of Int. Conf. on Recent Trends in Information Technology*, 25-27 Jul. 2013, Chennai, India (IEEE, 2013), pp. 147-151. DOI: 10.1109/ICRTIT.2013.6844196.
23. J. Hou, P. H. Siegel, L. B. Milstein, "Performance analysis and code optimization of low density parity- check codes on Rayleigh fading channels," *IEEE J. Selected Areas Commun.* **19**, No. 5, 924 (May 2001). DOI: 10.1109/49.924876.
24. S. Iniya, "Pergroup precoding for MIMO with LDPC coding using QAM modulation," *Int. J. Innovative Res. Technol. Sci. Eng.* **2**, No. 3 (Mar. 2016).
25. T. Chehade, L. Collin, P. Rostaing, E. Radoi, O. Bazzi, "Power allocation optimization: linear precoding adapted to NB-LDPC coded MIMO transmission," *Int. J. Antennas Propag.* **2015**, ID 975139 (2015). DOI: 10.1155/2015/975139.
26. F. Perez-Cruz, M. R. D. Rodrigues, S. Verdu, "MIMO Gaussian channels with arbitrary inputs: optimal precoding and power allocation," *IEEE Trans. Inf. Theory* **56**, No. 3, 1070 (2010). DOI: 10.1109/TIT.2009.2039045.
27. Digital Video Broadcasting (DVB); Second Generation Framing Structure, Channel Coding and Modulation Systems for Broadcasting, Interactive Services, News Gathering and other Broadband Satellite Applications (DVB-S2) [archive], ETSI EN 302 307, V1.2.1 (April 2009).