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Modeling of Electron Sources for High Voltage Glow Discharge Forming Profiled Electron Beams

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Abstract—The methodology of simulation of high voltage glow discharge electrodes' systems with anode plasma, formed the profile electron beams with the ring-like focus, is considered in the article. The universality of proposed model is confirmed by the fact, that the slope angle of generatrix line of conic anode surface correspond to the axis of the electrodes system have been chosen as one of model parameters. Such novel solution has led to simulating the different types of electrodes' systems, which form the profiled electron beams. The position of plasma boundary in considered high voltage glow discharge electrodes' systems is firstly calculated on the basis of one-dimensional model of discharge gap, and after that recalculated with taking into account the real geometry of electrodes' system. Such approach allows to significantly simplify the proposed mathematical model and to avoid using the sophisticated iteration numerical methods for defining plasma boundary form and position without decreasing the accuracy of simulation results. The results of simulation of plasma boundary position are compared with obtained experimental data and disagreement between simulation and experimental results is nearly 10–15%. The results of simulation of distribution of electric field in the cathode-fall region as well as results of simulation of distribution of beam current density in the ring-like focus of formed hollow electron beam are presented in the article. The simplicity and universality are the main advantages of the proposed model.

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INTRODUCTION

Technological sources of electrons of high-voltage glow discharge (HGD) are widely used for welding, soldering, annealing of small-sized products, for composite coatings of complex chemical composition, as well as for vacuum remelting of refractory metals to clean them from harmful impurities [1–7]. Sources of electrons HGD have a number of technical and economic advantages in comparison with traditional electron guns with glowing cathodes, among which the following should be noted [2, 7].

1. Stable and steady operation of electron guns on the basis of HGD in a low and medium vacuum in the environment of various process gases, including inert and active.

2. The relative simplicity of the design of HGD electron guns and the possibility of their restoration and repair, in particular, replacement and restoration of wearing knots, for example, the cold cathode of HGD.

3. The relative simplicity and low cost of the vacuum process equipment, ensuring functioning of the HGD electron guns.

4. Simplicity of the hydrodynamic and electric control of the electron beam current.

Due to the specified technical and economic advantages, HGD electron guns are increasingly used in the electronics industry, instrument making industry, mechanical engineering, metallurgy and aircraft industry. In particular, they are used in such modern directions of development of electron beam technologies as the three-dimensional press on metal and obtaining nanomaterials with new physical properties [3–7].

Recently, the theoretical and experimental studies related to the analysis of the physics of the HGD burning under various physical conditions have been constantly carried out [9–14], and the pulsed mode of the HGD burning, allowing to increase the power of the formed electron beam in an impulse, is of particular

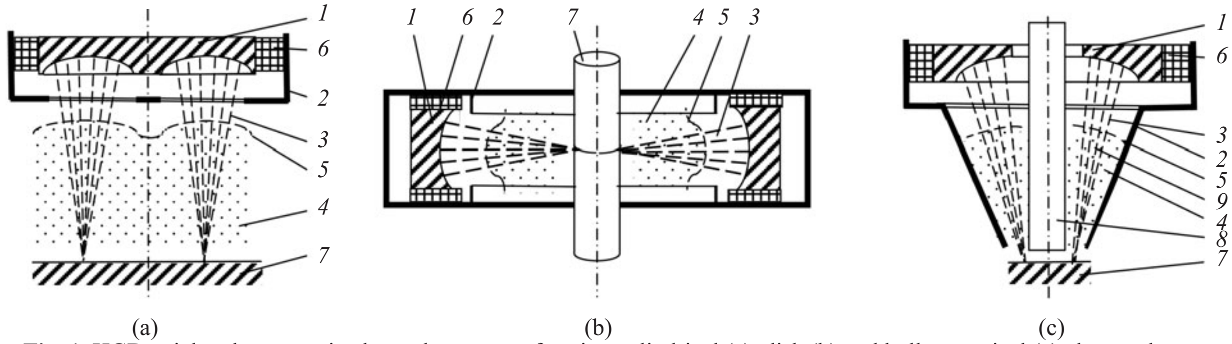

Fig. 1. HGD axial and symmetric electrode systems, forming cylindrical (a), disk (b) and hollow conical (c) electron beams. Notations: cathode (*1*), anode (*2*), electron beam (*3*), anode plasma (*4*), plasma boundary (*5*), high-voltage insulator (*6*), workpiece (*7*), auxiliary cylindrical electrode (*8*).

interest [10–13]. Separate studies are devoted to the possibility of using HGD for the formation of pulsed high-energy flows of neutral atoms [14].

One of the drawbacks of electron sources based on HGD is the low current density of electrons from the surface of the cold cathode, which is approximately 0.01 A/cm^2, which requires the use of cathodes with a developed emission surface to obtain the required electron beam power [2]. In this regard, from the point of view of the industrial application, HGD electron guns with a toroidal surface of the cathode, forming electron beams with a circular focus are of special interest [7, 8, 15]. Among such guns, the most widely used in the industry are guns that form tubular, disk electron beams, as well as hollow conical beams. The corresponding electrode systems of HGD electron sources are shown in Fig. 1 [7, 15].

The advantage of electron beams with an circular focus, formed by HGD electrode systems (Fig. 1), is that they allow instant and uniform heating of cylindrical products around the perimeter without their local overheating that increases the speed of technological operations and improves the quality of processed products. For example, such beams can be effectively used for pulsed automatic welding of electronic device cases without overheating of semiconductor crystals [2, 7, 15].

However, due to the complexity of the physical processes occurring in the HGD, and also because of the complexity of the geometry of the HGD electrode systems (Fig. 1), at present there are no universal mathematical models of the HGD electrode systems that form the profiled electron beams. In [15], generalized methodological principles for constructing such models were formed and nonlinear equations for the position of the anode plasma boundary were obtained, which were solved by numerical methods. Analytical relations for determining the position of the anode plasma boundary in the HGD electrode systems were not obtained in [15], and the electrode system shown in Fig. 1c was not considered in this work.

The purpose of this article is to form a generalized mathematical model for the HGD electrode systems forming profiled electron beams with a circular focus, as well as the analysis of the obtained simulation results and their comparison with experimental data.

STATEMENT OF PROBLEM

As it was noted in [2, 15, 16], for the analysis of electron-optical and energy characteristics of the HGD electrode systems, the most important is to determine the position and shape of the anode plasma boundary, which is considered as a source of ions and as transparent for electrons with a given potential.

Generally, the geometric position of the anode plasma boundary points is defined as the equilibrium position of the electric field pressure from the side of the cathode potential drop area, and the kinetic pressure of the electron gas from area of the anode plasma. Mathematically, this equilibrium condition is described by a rather complex system of algebraic differential equations [2]:

$$\left.\left(\frac{\partial\varphi}{\partial n}\right)\right|_{r\in\Gamma} = \sqrt{\frac{n_e k T_e}{\varepsilon_0 A(\varphi^*)}},$$

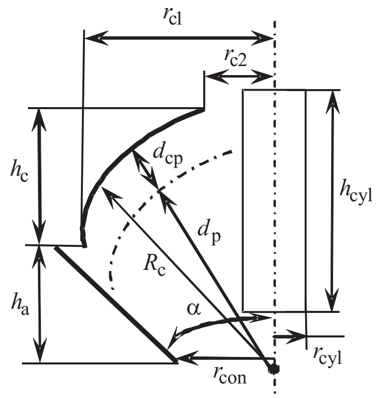

Fig. 2. Geometric parameters of conical electrode system HGD (Fig. 1c).

$$A(\varphi^*) = \frac{1}{4}\left(\sqrt{1+\frac{2q\varphi^*}{kT_e}} + e^{-\frac{2q\varphi^*}{kT_e}} - 2\right), \tag{1}$$

where n_e is the concentration of electrons in plasma, T_e is their temperature, q is the charge of plasma ions, ε_0 is the dielectric constant, k is the Boltzmann constant, φ^* is the near-electrode potential, depending on the composition of the used gas. Typically, the value of the parameter φ^* is within several volts.

The iterative solution of equation (1) for real geometry of HGD electrode systems, even when using modern computer technologies, is extremely difficult [2, 17]. Therefore, when solving practical engineering problems often to determine the position and shape of the plasma boundary in HGD electron sources, a complex theoretical and experimental method is used, based on photography of the discharge gap, analyzing dark and bright areas of photographs obtained using computerized image analysis methods, and then fitting the resulting experimental data [16].

In particular, in [2, 16], it was suggested and justified that for large values of the discharge current, the boundary of the anodic plasma is parallel to the surface of the cathode. On basis of this assumption, in [15] a method was proposed for calculating the volume of anode plasma in the HGD electrode systems that form the profiled electron beams, based on the fact that the volume occupied by the plasma depends only on the operating pressure and does not change with the geometry of the electrode system. In accordance with this method of calculation for a conical electrode system (Fig. 1c), it is necessary to calculate the height of the volume of a sphere segment located in the conical cavity of the anode. It is also important that since the electrode system shown in Fig. 1c, is axisymmetric, for calculating the distribution of the electric field, particle trajectories and space charge the previously obtained correlations for calculating the space charge in axially symmetric HGD systems recorded in a cylindrical coordinate system can be used [16].

GEOMETRIC PARAMETERS OF SIMULATED ELECTRODE SYSTEM

The main feature of the proposed mathematical model is that all its geometric parameters are considered in relation with the angle of generatix of the conical anode surface relative to the axis of the electrode system α, and in order to form these relation, trigonometric equations are used. From a practical point of view, this allows to create a simple universal mathematical model, in which the parameter α is considered as variable for optimization.

In particular, the electrode system shown in Fig. 1a, corresponds to value $\alpha = 0°$, and the electrode system shown in Fig. 1b is for $\alpha = 90°$. This parameter is also important from a technological point of view, since the thermal effect of technological electron beams depends not only on their specific power, but also on the angle of beam incidence on the surface of the workpiece [12, 13].

The basic geometric dimensions of the simulated electrode system are shown in Fig. 2. The main ones are the following: height of the anode plasma d_p, position of the anode plasma boundary relative to the cathode

d_{cp}, cathode thickness h_c, minimum radius of the anode base, which corresponds to the output region of the electron beam in the process chamber r_{con}, cathode sphere radius R_c, maximum transverse size spherical the cathode surface near the anode r_{c1}, the minimum transverse size of the spherical surface of the cathode near the cylindrical electrode r_{c2}, the cylinder radius r_{cyl} and the height of the cylinder h_{cyl}.

The height of the anode plasma d_p and the distance from the plasma boundary to the cathode surface d_{cp} are important internal parameters of the considered model. This is due to the fact that, in accordance with the considered basic principles of the theory of HGD, the plasma boundary is a source of ions and an electrode is transparent for electrons with a given potential [16]. Another important internal parameter of the model is the reduced residual gas pressure in the discharge gap p_{a0}, which is calculated as

$$p_{a0} = pL, \tag{2}$$

where p is the gas pressure, L is the height of the discharge gap.

An important electrical parameter of the model under consideration is accelerating voltage U_{acc}. The discharge current I_d depends on the accelerating voltage and the working pressure of the gas, and in a one-dimensional system the HGD is determined from the relation [2]:

$$I_d = AU_{acc}^m p^k, \tag{3}$$

where A, m, k are semi-empirical constants.

The principle of equality of plasma volumes in a one-dimensional linear electrode system V_L and in a simulated electrode system with a conical anode V_{con} (Fig. 1c), or in mathematical form is used to determine the height of the anodic plasma d_p:

$$V_L = V_{con}. \tag{4}$$

This hypothesis is also one of the basic conditions of the HGD combustion theory [2, 16]. In particular, in [15], on the basis of solving equation (4), nonlinear equations were obtained describing the dependence of the anodic plasma height on the discharge current in the HGD electrode systems forming tubular and disk electron beams (Fig. 1a, b). Since the analytical expressions for the dependence of the anode plasma height on the discharge current were not obtained in [15], the height of the anode plasma d_p was calculated using numerical methods. The accuracy of simulation in relation to experimental data was 10–15%.

ANALYTICAL EXPRESSION FOR PLASMA HEIGHT CALCULATION

The volume of the anode plasma in a conical electrode system (Fig. 1c) is calculated as the volume of a spherical segment [18]:

$$V_{con} = \pi\left(\left(\frac{4}{3}(R_c - d_p)^3 - r_{c1}^3\right)\sin\left(\frac{\alpha}{2}\right) - r_{cyl}^2(R_c - d_p) - \frac{4}{3}r_{cyl}^2\sin\left(\frac{\alpha}{2}\right) + r_{cyl}^2(r_{con} - d_p)\right). \tag{5}$$

On the other hand, an analytical relation is known to allow finding the volume of the anode plasma in a linear electrode system [2]:

$$V_L = \pi r_a^2\left(L - \frac{I_L}{Q_{e0}\left(\chi\gamma + \sqrt{\dfrac{m_i}{m_e}}\right) - \dfrac{5\mu_{i0}kT_e}{R_c^2 p_{a0}^2 e}\sqrt{\dfrac{m_i}{kT_e}}}\right), \tag{6}$$

where I_L is the discharge current in the linear electrode system, r_a, R_c, L are the geometric dimensions of the one-dimensional electrode system, m_e and m_i are the masses of the electron and gas ions, accordingly, Q_{e0} is the average cross-section of ion charge exchange on residual gas atoms, χ is the average coefficient of electron trajectory extension in HGD, T_e is the electron temperature in the plasma, γ is the coefficient of electrons reflection from the anode surface, μ_{i0} is the electron mobility in the anode plasma.

Equating relations (5) and (6) with (4), we get:

$$\left(\frac{4}{3}\sin\left(\frac{\alpha}{2}\right)\right)d_{\mathrm{p}}^3+4R_{\mathrm{c}}\sin\left(\frac{\alpha}{2}\right)d_{\mathrm{p}}^2-\left(4R_{\mathrm{c}}^2\sin\left(\frac{\alpha}{2}\right)-r_{\mathrm{con}}^2\right)d_{\mathrm{p}}-\frac{4}{3}R_{\mathrm{c}}^2\sin\left(\frac{\alpha}{2}\right)$$

$$+r_{\mathrm{cyl}}^2(R_{\mathrm{c}}+r_{\mathrm{con}})+\frac{4}{3}r_{\mathrm{con}}^3+(r_{\mathrm{c1}}-r_{\mathrm{c2}})^2\left(L-\frac{I_{\mathrm{L}}}{Q_{e0}\left(\chi\gamma+\sqrt{\frac{m_i}{m_e}}\right)-\frac{5\mu_{i0}kT_e}{R_{\mathrm{c}}^2p_{a0}^2e}\sqrt{\frac{m_i}{kT_e}}}\right), \tag{7}$$

where

$$L=\left(R_{\mathrm{c}}-\frac{r_{\mathrm{con}}}{\tan(\alpha)}\right),$$

$$r_{\mathrm{c1}}-r_{\mathrm{c2}}=R_{\mathrm{c}}\left(\sin(\alpha)-\frac{r_{\mathrm{c2}}\sqrt{1-\frac{r_{\mathrm{c2}}}{h_{\mathrm{cyl}}}}}{1-\frac{r_{\mathrm{c2}}}{h_{\mathrm{cyl}}}}\right). \tag{8}$$

It is obvious that the obtained equation (7) is a cubic equation with respect to the parameter d_{p}, and it can be solved analytically using the Cardano formulas. The corresponding solution is written in the form [18]:

$$p=\frac{3abc-b^2}{3a^2},\quad q=\frac{2b^3-9abc+27a^2d}{3a^3},$$

$$a=\frac{4}{3}\sin\left(\frac{\alpha}{2}\right),\quad b=4R_{\mathrm{c}}\sin\left(\frac{\alpha}{2}\right),\quad c=r_{\mathrm{con}}^2-4R_{\mathrm{c}}^2\sin\left(\frac{\alpha}{2}\right),$$

$$d=r_{\mathrm{cyl}}^2(R_{\mathrm{c}}+r_{\mathrm{con}})+\frac{4}{3}r_{\mathrm{c}}^3+(r_{\mathrm{c1}}-r_{\mathrm{c2}})^2\left(L-\frac{I_{\mathrm{L}}}{Q_{e0}\left(\chi\gamma+\sqrt{\frac{m_i}{m_e}}\right)-\frac{5\mu_{i0}kT_e}{R_{\mathrm{c}}^2p_{a0}^2e}\sqrt{\frac{m_i}{kT_e}}}\right),$$

$$d_{\mathrm{p}}=y-\frac{b}{3a},\quad D=\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2,$$

$$u=\sqrt[3]{-\frac{q}{2}+\sqrt{D}},\quad v=\sqrt[3]{-\frac{q}{2}-\sqrt{D}},\quad y=u+v,\quad d_{\mathrm{cp}}=R_{\mathrm{c}}-d_{\mathrm{p}}, \tag{9}$$

where a, b, c, d are the coefficients of the original cubic equation (7), p, q are the coefficients of the modified cubic equation $y^3+py+q=0$, D is the discriminant of the cubic equation, u, v, y are auxiliary variables.

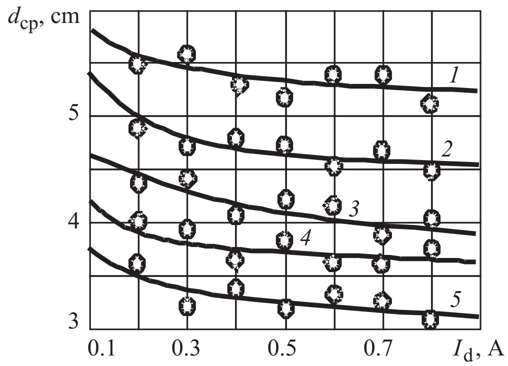

Fig. 3. Dependence of plasma–cathode distance on discharge current for different values of parameter α = 50° (*1*), 100° (*2*), 150° (*3*), 200° (*4*), 250° (*5*). Experimental data marked by dots.

Numerical calculations using (9) were carried out for the following parameters of the simulated HGD electrode system: the cathode material is aluminum, the working gas is nitrogen, $\gamma = 4.6$, $\chi = 4.3$, $Q_{e0} = 5.3\times10^{-19}$ m^{-2}, $T_e = 800$ K, $h_c = 0.01$ m, $r_{c1} = 0.25$ m, $r_{c2} = 0.15$ m, $r_{cyl} = 0.15$ m, $h_{cyl} = 0.5$ m; $R_c = 0.08$ m, $p_{a0} = 0.005$ Pa·m, $U_{acc} = 15$ kV. The angle of generatrix inclination of anode surface α was varied in the process of simulating within 5–25°.

The resulting dependence of the distance from the cathode surface to the anode plasma boundary d_{cp} for different values of the angle α is shown in Fig. 3. The relevant experimental data obtained using the equipment of the laboratory of electron-beam technological devices of the National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnic Institute” are also given there. Obviously, the resulting model is fairly reliable, since the values of the calculated and experimental data differ by no more than 15%.

From Fig. 3, it can be seen that the value of d_{cp} decreases with increasing discharge current and with increasing angle of inclination of the anode surface α. A decrease d_{cp} with an increase in the discharge current is associated with an increase in the volume of the anode plasma and is due to an increase in the degree of ionization of the residual gas by accelerated electrons in the beam. It should be noted that the dependencies of $d_{cp}(I_d)$ are not monotonously decreasing, but asymptotically. This is explained by the fact that under the conditions of burning HGD with an increase in the discharge current, the position of the anode plasma always stabilizes, and a further increase in current is caused only by an increase in the degree of gas ionization. Similar dependencies for the HGD electrode systems that form electron beams with a point focus were obtained in [2, 16], and for HGD electrode systems that form tubular and disk electron beams in [15].

DETERMINATION OF CURRENT GENERATED ELECTRON BEAM

In the general case, the difference between the current of an electron beam formed in a conical electrode system (Fig. 1c) and the current of an electron beam in a HGD system with flat electrodes is due to the divergence of the ion flow directed from the boundary of the anode plasma to the surface of the cold cathode. For such physical conditions, the ratio $\beta = I_p / I_c$ is defined as the ratio of the surface area of the plasma boundary S_p to the area of the cathode emission surface S_c, where I_p is the current from the surface of the anode plasma, I_c is the current from the surface of the spherical cathode.

From Fig. 2 it can be seen that the surface of the anode plasma boundary S_p is described by a spherical layer with a radius d_p, a solid angle α and a segment of the radius r_{cyl} cut out of it. Similarly, the cathode emission surface S_c is described by a spherical layer with a radius R_c, whose solid angle is also α, and the radius of the segment cut from it is r_{cyl}. According to this, the mathematical expression for the coefficient β is written as [18]:

$$\beta = \frac{d_p^2\left(\sin\left(\frac{\alpha}{2}\right) - \sqrt{1 - \frac{r_{cyl}}{d_p}}\right)}{R_c^2\left(\sin\left(\frac{\alpha}{2}\right) - \sqrt{1 - \frac{r_{cyl}}{R_c}}\right)}. \tag{10}$$

Then, taking into account that for a system with flat electrodes, the current of electrons from the cathode surface I_{efl} is calculated from the relation [2]:

$$I_{efl} = I_{\mathrm{p}} Q_{e0}\left(\chi\gamma + \sqrt{\frac{m_i}{m_e}}\right), \tag{11}$$

the value of the electron current from the cathode surface for the simulated electrode system with a conical anode I_{econ}, taking into account the above relations (10), (11), will be written as follows:

$$I_{econ} = I_{\mathrm{p}} Q_{e0}\beta\left(\chi\gamma + \sqrt{\frac{m_i}{m_e}}\right) = I_{efl}\beta. \tag{12}$$

Taking into account the fact that the height of the anode plasma d_{p} is calculated using relations (9), the system of equations (10), (12) can be used to calculate the value of the electron beam current in the conical electrode system of the HGD (Fig. 1c).

Consider the case of $\alpha = 0°$, corresponding to the cylindrical electrode system of the HGD (Fig. 1a). In this limiting case, the expression (10) obtained for the coefficient β is simplified:

$$\beta = \frac{d_{\mathrm{p}}^2}{R_{\mathrm{c}}^2}. \tag{13}$$

The results of the calculation of the electron beam current, obtained using relations (10), (12), (13), were used to calculate the focal parameters of the generated electron beam. Corresponding relations for simulating the electron beam trajectories taking into account the space charge, as well as the results of calculating the distribution of the electric field and the distribution of the electron beam current density in the circular focus for the considered electrode system (Fig. 1c), are given in the next section.

SIMULATION OF ELECTRIC FIELD DISTRIBUTION AND BEAM CURRENT DENSITY IN CIRCULAR FOCUS

To calculate the distribution of the electric field in the conical electrode system of the HGD (Fig. 1c), the Poisson equation was used, which was solved by the finite difference method. The corresponding finite-difference relations are written in the cylindrical coordinate system as follows [16, 17, 19, 20]:

$$U^n(i,k) = \omega\Big[C_a U^{n-1}(i+1,k) + C_b U^{n-1}(i,k+1)$$

$$+ C_c U^n(i-1,k) + C_d U^n(i,k-1) + \frac{\rho^{n-1}(i,k)}{\varepsilon_0}\Big] + (1-\omega)U^{n-1}(i,k), \tag{14}$$

where

$$C_a = C_c = \frac{1}{4h_z^2}, \quad C_b = \frac{1+\frac{1}{2k}}{h_r^2}, \quad C_d = \frac{1-\frac{1}{2k}}{h_r^2}, \tag{15}$$

for points not on the z axis, and

$$C_a = \frac{4h_r^2}{6}, \; C_c = 0, \; C_b = C_d = \frac{h_z^2}{6}, \tag{16}$$

for points on the z axis. Here n is the number of the current iteration over the potential, i and k are the numbers of the current nodes under consideration along the longitudinal and radial coordinates, respectively, ω is a numerical parameter that depends on the geometry of the electrode system and affects the convergence rate of the iteration process over the potential, h_r is the axis discretization step r, h_z is the sampling step along the z axis, $\rho^{n-1}(i,k)$ is the space charge in the node with the number (i,k) at the previous iteration. For the conical electrode HGD system (Fig. 1c), the optimal value $\omega = 1.35$ was chosen during the computational experiment.

To calculate the trajectories of charged particles in the considered axially symmetric HGD electrode system, the well-known electron-optical equation was used [16, 17, 19]:

$$\frac{d^2 r}{dz^2} = \frac{\left(\frac{\partial U(r,z)}{\partial r} - \frac{\partial U(r,z)}{\partial z}\frac{dr}{dz}\right)\left(1 + \left(\frac{dr}{dz}\right)^2\right)}{\left(\frac{m_s v_s}{2} + 2\big(U(r,z) - U_r\big)\right)}, \tag{17}$$

where m_s is the mass of moving particle, v_s is the particle velocity, U_r is the potential of particle emitter.

A modified method of current tubes was used to calculate the space charge in a simulated axially symmetric HGD system. The corresponding iterative relationships [16], based on theoretical assumptions [17], can be written in general form as follows:

$$\rho_c^s = \frac{I_{\text{tb}}}{2\pi r_{\text{tb}} \Delta r}\sqrt{\frac{m_s}{2q_s}}\left(\frac{1}{\sqrt{U_z^{\text{in}}}} + \frac{1}{\sqrt{U_z^{\text{out}}}}\right),$$

$$U_r = U_{k,l} + \frac{U_{k,l} - U_{k-1,l}}{h_r}(kh_r - r),$$

$$\rho_\Sigma^c = \sum_{n=1}^{N_{\text{tb}}^i} \rho_{in} - \sum_{n=1}^{N_{\text{tb}}^e} \rho_{en}, \quad \Delta r = r_{\text{in}} - r_{\text{out}}, \tag{18}$$

where s is the sort of the particle, I_{tb} is the tube current, m_s/q_s is the specific charge of particles moving in the corresponding tube, U_z is the approximate potential value at the input and output of the cell, n is the sequence number of the current tube crossing the cell, ρ_c is the space charge in the finite-difference cell, ρ_{en} and ρ_{in} are the space charge of electrons and ions for the tube with number n, respectively, which makes the related tube. After all the iterations over current, the calculated values of the space charge are inserted into the system of iterative equations (14)–(16) and the calculations continue until the convergence of the iterative process for all the nodes of the cells is reached.

In order to take into account the influence of the ion charge exchange process on the residual gas atoms in the HGD electrode systems, relations (18) in [16] were modified as follows:

$$\Delta N_i = 2\pi N_{\text{A}}(1-\xi)U_{\text{avr}} Q_{in} \frac{\Delta r l_{\text{tb}} h_z p_0}{\Delta z p_{\text{atm}} r_{\text{tb}}^2},$$

$$U_{\text{avr}} = \frac{U_{k,l} + U_{k-1,l} + U_{k,l-1} + U_{k-1,l-1}}{4},$$

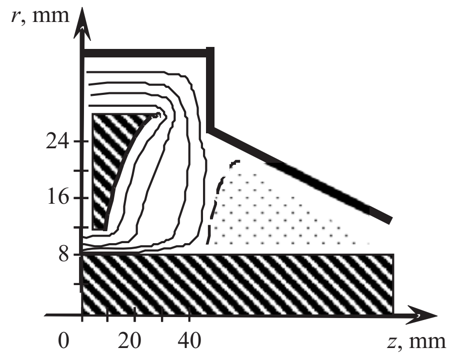

Fig. 4. Distribution of electric field in electrode system of HGD conical electron gun (Fig. 1c).

$$\rho_c^{i+1} = \frac{(\rho_c^i)^2}{\rho_c^i - 2\pi \Delta N_i r_{\text{tb}}^2 \dfrac{\Delta r l h_z}{\Delta z}},$$

$$I_{\text{tb}}^{i+1} = I_{\text{tb}}^i - \frac{\rho_c^i}{\rho_c^i - 2\pi \Delta N_i r_{\text{tb}}^2 \dfrac{\Delta r l h_z}{\Delta z}}, \tag{19}$$

where ΔN_i is the change in ion concentration within the finite-difference grid as a result of the resonant charge exchange process, N_A is the Avogadro constant, ξ is the specific ionization coefficient of gas atoms, p_{atm} is the atmospheric pressure, U_{avr} is the average value of the equivalent potential within the cell.

When calculating the space charge in accordance with relations (19), the degree of gas ionization ξ was determined using the well-known Margules equation for ionization cross sections [16, 21]:

$$\xi = \frac{a(U_e - U_i)\exp\big(-b(U_e - U_i)\big)}{p_0\sqrt{\Delta r^2 + \Delta z^2}}, \tag{20}$$

where a, b are empirical constants, U_e is the potential corresponding to the energy of primary electrons, U_i is the gas ionization potential.

In the case of using gas mixtures, which is quite characteristic in the industrial operation of HGD electron guns for the implementation of a specific technological process [1, 3–6], the total degree of ionization of the gas mixture ξ_Σ, in accordance with Margules law, is calculated as an additive value [16, 21]:

$$\xi_\Sigma = c_1\xi_1 + c_2\xi_2 + \ldots + c_n\xi_n, \tag{21}$$

where c_1, c_2, …, c_n are the respective relative concentrations of the individual components in the composition of the gas mixture used, ξ_1, ξ_2, …, ξ_n are the degree of ionization of these components.

The results of simulating the distribution of the electric field in the HGD electrode system using relations (3), (8)–(20) for the above-specified geometric and physical parameters of the model with accelerating voltage U_{acc} = 15 kV and the inclination angle of the anode cone of α = 15° are shown on Fig. 4.

Despite the electrons trajectories of the formed beam are also determined using (17), they can be continued to the plasma boundary region to determine the position of the circular focus of the beam and the distribution of the current density in the focus. The corresponding system of algebraic equations, which is based on models of discrete mathematics and minimax analysis, is given in [16]. When calculating the electron drift in the region of a quasi-neutral plasma, the space charge of the particles was considered compensated, but the effect of inelastic collisions with ions on the beam trajectory was taken into account in accordance with the Rutherford model.

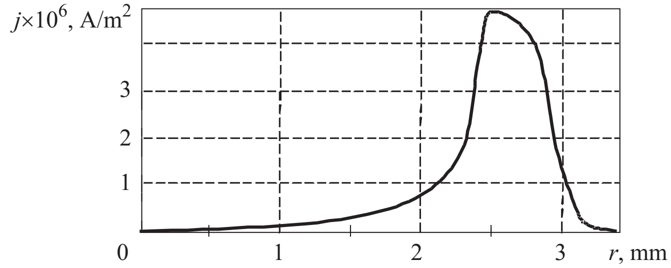

Fig. 5. Current density distribution of hollow conical electron beam in circular focus.

The results of the calculation showed that the circular focus of the beam is located at a distance of 10.73 mm from the outlet of the anode, and its diameter is 3.65 mm. The resulting graph of current density distribution in the focal plane of the beam is shown in Fig. 5.

ANALYSIS OF RESULTS AND CONCLUSIONS

The uneven distribution of the electron beam current relative to the central circular line of its circular focus is due to the fact that the beam falls at an angle to the surface of the object (Fig. 1c). However, in the ideal case, the current density distribution around the perimeter of the circle is uniform. In practice, the deviation from uniformity may be due to the inaccuracy of manufacturing the spherical surface of the cathode, or misalignment of the cathode position relative to the symmetry axis of the system [2].

Important advantages of the proposed mathematical model of the HGD electrode systems are its low cost, simplicity and universality. The simplicity and efficiency of the model is due to the fact that the position of the anodic plasma boundary in the HGD electrode system is not found by numerical solution of the nonlinear system of equations (1), but by a correct transition from the linear system model to a system with a spherical cathode and a conical anode (Fig. 1c) by solution of a simple equation (4) with regard to equations (5), (6).

Studies [8] showed the validity of this approach, and a comparison of the obtained simulation results with the experiment led to the conclusion that the discrepancy between the calculated and experimental data for different values of the discharge current does not exceed 15%. Such calculation accuracy for gas-discharge systems, with many of complex physical processes associated with elementary and collective interactions of beams of charged particles between themselves and with the surfaces of the electrodes is very high [2, 16].

The universality of the proposed mathematical model is due to the fact that the inclination angle of the forming anode surface to the axis of symmetry of the electrode system α is chosen as one of its input geometric parameters. This allowed us to connect the geometric parameters of the simulated electrode system through trigonometric relations, thereby reducing to a minimum the number of measured dimensional parameters.

In addition, the obtained analytical relations allow finding the position of the plasma boundary for any electrode systems that form a hollow conical electron beam, regardless of the value of the parameter α. This allows to select the values of the angle of the electron beam to the surface of the workpiece, depending on the requirements of the process.

The simulation results obtained in this work are of great practical interest for specialists engaged in the development of electron-beam process equipment and the introduction of modern electron-beam technologies in various industries.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

ADDITIONAL INFORMATION

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