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Estimation of Ultrawideband Quasi-Radio Signal Duration[1]

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Abstract—Quasi-likelihood and maximum likelihood algorithms of duration estimation for ultra-wideband quasi-radio signal of arbitrary shape with unknown amplitude and initial phase, influenced by additive Gaussian white noise, are synthesized. It was considered that conditions of relatively narrow band of received signal are not satisfied and its duration can constitute only several periods or a fraction of period of harmonic oscillation. It is shown that the structure of the algorithm for duration estimation of ultra-wideband quasi-radio signal is significantly different from the structure of duration estimation algorithm for narrowband radio signal. Relative bias and variance are determined as the statistical characteristics of synthesized duration estimates. The influence of unknown amplitude and initial phase on the accuracy of duration estimation is investigated. Quantitative limits for relation of signal bandwidth to its center frequency are formulated, such that the classical solution of the problem of duration estimation for narrowband radio signal possesses the required accuracy.

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Ultra-wideband (UWB) signals finds ever broader applications in many practical cases of modern radioelectronics, which are evidenced by large number of publications, including considerable number of monographs [1–6]. Implementation of UWB signal into telecommunication systems allows increasing the information transfer rate due to large spectral width. Application of UWB signals in measurement systems, radars and positioning devices unveils the possibilities of enhancement of measurement accuracy and resolution.

The problems of processing of UWB signals with unknown time of arrival are studied in detail in modern literature [1]. This is related to the necessity of signal delay measurement in radars and also to the active utilization of temporal-impulse modulation in the UWB systems. At the same time, there are plenty of applications that require processing of UWB signals with unknown duration. In this case duration can serve both as an informative signal parameter and as a non-informative parameter, which is undefined at the receiving side due to specifics of UWB signal propagation.

The meaning of UWB signals is wide and includes large number of various mathematical models [1–6]. Obtaining of constructive results from processing algorithms of UWB signals of any type poses significant difficulties. Therefore among UWB signals we separate a sub-type of such signals that have structure similar to narrowband signals, but the narrowband conditions are not satisfied. Such signals are termed as UWB quasi-radio signals (QRS) [1]. This paper investigates algorithms for duration estimation of UWB QRS. Such narrowing of the type of signals under consideration allows more in-depth and informative results of synthesis and analysis of the duration estimation algorithms.

The problem of signal duration estimation in the noise background for different signal types has been considered multiple times [7–13], but for UWB signals this problem remains mainly unsolved. Algorithms for duration estimation of video impulses of rectangular [7], arbitrary shape [8] and signals of arbitrary shape and unknown amplitude [9] have been investigated previously. It has been shown that accuracy of maximum likelihood (ML) duration estimation asymptotically does not depend on signal shape in case of large signal to noise ratio (SNR), but is determined only by the magnitude of back front of the signal. In addition, synthesis and analysis of the duration estimation algorithms for narrowband radio impulse with an arbitrary envelope shape having unknown initial phase [10] and simultaneously unknown amplitude and initial phase [11] have been performed. The accuracy of ML estimation of radio signal duration is asymptotically defined

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by the magnitude of the envelope back front in the case of large SNR values. However the known results for problems of duration estimation of narrowband radio signal cannot be applied to the UWB QRS. The algorithms for duration estimation of UWB QRS are presented below.

Signal under consideration can be written in the form:

$$s(t,\tau,a,\varphi) = af(t)I(t/\tau)\cos(\omega t-\varphi), \tag{1}$$

where $I(x)=\begin{cases}1, & 0\le x\le 1,\\ 0, & x<0,\ x>1,\end{cases}$ is an indicator of unit duration, $f(t)$ is a continuous modulation function, a, φ, ω, τ are amplitude, initial phase, frequency and duration respectively.

If the bandwidth $\Delta\omega$ and frequency ω of signal (1) satisfy the following condition:

$$\Delta\omega << \omega, \tag{2}$$

then signal (1) is considered as narrowband radio signal [12, 13]. If condition (2) is not satisfied than equation (1) describes UWB QRS [14]. Quantities a, φ, ω are the parameters of a harmonic oscillation used for formation of the signal (1). Nevertheless, further in the text a, φ, ω will be referred as amplitude, initial phase and frequency of UWB QRS (1), according to [14]. Signal bandwidth $\Delta\omega$ can be made approximately equal to frequency ω by appropriate selection of modulation function $f(t)$ [14]. In a similar manner, variation of modulation function $f(t)$ can describe both UWB QRS with large relative bandwidth and a narrowband radio signals that satisfy condition (2) by means of equation (1).

It should be noted that (1) applies also to special cases of signal models [7–11]. If in (1) we assume that $\omega=0$, $\varphi=0$, $f(t)=1$, than the model of quasi-determined rectangular video impulse as investigated in (7) is obtained. For $\omega=0$, $\varphi=0$ we get a video impulse of arbitrary shape $f(t)$ with known [8] or unknown [9] amplitude. The model of narrowband radio signal studied in [10, 11] is also described by (1) if condition (2) is satisfied. However, relative narrowband condition in the form of (2) is only of qualitative nature. Synthesis and analysis results of processing algorithms of UWB QRS allow defining quantitative characteristics of relative narrowband meaning. As such, model of UWB QRS in the form of (1) is a generalization and development of signal models [7–11].

Assume that additive mix of signal (1) and Gaussian white noise $n(t)$ with spectral density N_0 is observed during time interval $t\in[0,T]$:

$$\xi(t) = s(t,\tau_0,a_0,\varphi_0)+n(t), \tag{3}$$

where τ_0, a_0, φ_0 are the actual values of unknown parameters. We assume that signal duration can be any value from the a priori interval $\tau\in[T_1,T_2]$. Having an observed realization (3) it is necessary to form an estimate of useful signal duration (1), assuming that unknown amplitude and initial phase are non-informative parameters, which do not need an estimate.

Method of maximum likelihood (ML) [7, 12, 13] is used for the purpose of synthesis of estimation algorithm. According to this method duration estimate coincides with the position of absolute maximum of logarithm of likelihood relation functional (LRF). However, for unknown amplitude and initial phase, LRF logarithm depends on three unknown parameters

$$L(\tau,a,\varphi) = \frac{2a}{N_0}\int_0^{\tau}\left(\xi(t)-\frac{af(t)\cos(\omega t-\varphi)}{2}\right)f(t)\cos(\omega t-\varphi)\mathrm{d}t. \tag{4}$$

Therefore, a priory parametric uncertainty takes place for amplitude and initial phase. Two ways of overcoming this uncertainty is considered according to [11]: application of quasi-likelihood (QL) estimate algorithm, where certain expected values of a^* and φ^* are used in (4) instead of unknown amplitude and initial phase respectively; and application of ML algorithm, where unknown amplitude and initial phase in (4) are replaced by their ML estimates.

Firstly, QL algorithm for duration estimation is considered. Receiver forms logarithm of LRF (4) for expected values of amplitude and initial phase and all possible duration values in the interval $\tau\in[T_1,T_2]$:

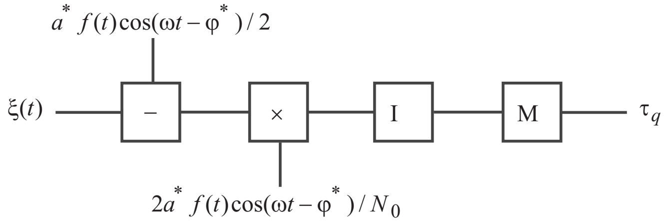

Fig. 1.

$$\tau_q = \mathrm{argsup} L_q(\tau),$$

$$L_q(\tau) = L(\tau, a^*, \varphi^*) \tag{5}$$

and defines QL duration estimate as a position of absolute maximum of deciding statistics (5). Expressions (4), (5) determine the structure of receiving device. Fig. 1 shows the block diagram of QL duration measurement, where “I” is integrator device operating in the interval $[0,\tau]$, $\tau \in [0,T_2]$, “M” is a finder of absolute maximum position of input signal at the time interval $[T_1, T_2]$.

The analysis of the QL algorithm for duration estimation is given below. For this the logarithm of LRF (5) is rewritten in the following form:

$$L_q(\tau) = a^*(X(\tau)\cos\varphi^* + Y(\tau)\sin\varphi^*) - a^{*2}\frac{Q(\tau) + P_c(\tau)\cos(2\varphi^*) + P_s(\tau)\sin(2\varphi^*)}{2}, \tag{6}$$

where

$$X(\tau) = \frac{2}{N_0}\int_0^\tau \xi(t) f(t)\cos\omega t \mathrm{d}t,$$

$$Y(\tau) = \frac{2}{N_0}\int_0^\tau \xi(t) f(t)\sin\omega t \mathrm{d}t,$$

$$Q(\tau) = \frac{1}{N_0}\int_0^\tau f^2(t)\mathrm{d}t,$$

$$P_c(\tau) = \frac{1}{N_0}\int_0^\tau f^2(t)\cos(2\omega t)\mathrm{d}t,$$

$$P_s(\tau) = \frac{1}{N_0}\int_0^\tau f^2(t)\sin(2\omega t)\mathrm{d}t.$$

According to (6) the random process $L_q(\tau)$ is Gaussian. Therefore, it suffices to determine mathematical expectation and correlation function to fully statistically describe such process. By means of averaging, mathematical expectation can be found:

$$S_q(\tau) = \langle L_q(\tau)\rangle = a^* a_0\Big[Q(\min(\tau,\tau_0)) \times \cos(\varphi^* - \varphi_0)$$

$$+P_c(\min(\tau,\tau_0))\cos(\varphi^*+\varphi_0)+P_s(\min(\tau,\tau_0))\sin(\varphi^*+\varphi_0)\Big]$$

$$-\frac{a^{*2}}{2}\Big[Q(\tau)+P_c(\tau)\cos(2\varphi^*)+P_s(\tau)\sin(2\varphi^*)\Big], \tag{7}$$

as well as correlation function:

$$K_q(\tau_1,\tau_2)=\langle[L_q(\tau_1)-\langle L_q(\tau_1)\rangle][L_q(\tau_2)-\langle L_q(\tau_2)\rangle]\rangle$$

$$=a^{*2}[Q(\min(\tau_1,\tau_2))+P_c(\min(\tau_1,\tau_2))\cos(2\varphi^*)+P_s(\min(\tau_1,\tau_2))\sin(2\varphi^*)]. \tag{8}$$

Further we assume that the output SNR relation of the received signal is sufficiently high. As known from [7, 12, 13], if SNR is increased, the QL duration estimation (5) converges to the position of mathematical expectation maximum:

$$\tau_s=\arg\sup S_q(\tau).$$

If the maximum position τ_s of mathematical expectation $S_q(\tau)$ coincides with the actual value of duration $\tau_s=\tau_0$, than QL estimate (5) is consistent [12]. Formulation of consistency conditions in a general form poses difficulty. However, in a particular case of $f(t)=1$ it can be easily shown that QL estimate (5) is consistent if the following two conditions are satisfied:

$$1-\cos(2\omega\tau-2\varphi^*)>0, \quad \tau\in(\tau_0,T_2]$$

and

$$\frac{2a_0\cos(\varphi^*-\varphi_0)}{a^*}-1+\left(\frac{2a_0\cos(\varphi^*+\varphi_0)}{a^*}-\cos 2\varphi^*\right)\cos(2\omega\tau)$$

$$+\left(\frac{2a_0\sin(\varphi^*+\varphi_0)}{a^*}-\sin 2\varphi^*\right)\sin(2\omega\tau)>0,$$

$$\tau\in\big[T_1,\tau_0\big).$$

The combinations of expected and actual values of amplitude and initial phase are limited to only those which have the position of mathematical expectation maximum (7) coinciding with actual value of unknown duration, that is $\tau_s=\tau_0$ and QL estimate (5) is consistent. The deciding statistics (6) in the vicinity of actual duration value τ_0 is investigated below. Asymptotical expressions for mathematical expectation can be obtained by decomposing (7), (8) into Taylor series in τ in the vicinity of τ_0:

$$S_q(\tau)\simeq\alpha+(\tau-\tau_0)\begin{cases}\beta_1, & \tau\le\tau_0,\\ -\beta_2, & \tau>\tau_0,\end{cases} \tag{9}$$

and similarly for correlation function:

$$K_q(\tau_1,\tau_2)\simeq\lambda+2\beta_2\min(\tau_1-\tau_0,\tau_2-\tau_0), \tag{10}$$

where

$$\alpha = a^* a_0 [Q(\tau_0)\cos(\varphi^* - \varphi_0) + P_c(\tau_0)\cos(\varphi^* + \varphi_0) + P_s(\tau_0)\sin(\varphi^* + \varphi_0)] - \lambda / 2,$$

$$\beta_1 = \frac{a^* \rho_0^2}{T_2} \cos(\omega\tau_0 - \varphi^*)[2a_0 \cos(\omega\tau_0 - \varphi_0) - a^* \cos(\omega\tau_0 - \varphi^*)],$$

$$\rho_0^2 = \frac{f^2(\tau_0)}{N_0} T_2, \qquad \beta_2 = a^{*2} \rho_0^2 \cos^2(\omega\tau_0 - \varphi^*) / T_2,$$

$$\lambda = a^{*2} [Q(\tau_0) + P_c(\tau_0)\cos 2\varphi^* + P_s(\tau_0)\sin 2\varphi^*].$$

LRF logarithm (6) is approximated by Gaussian random process with mathematical expectation (9) and correlation function (10) in all a priory interval of possible duration values. Using expressions (9), (10) and Doob theorem [16] it is possible to show that deciding statistics (6) is asymptotically a Gaussian Markov process with the following coefficients of drift and diffusion [16]:

$$k_1 = \begin{cases} \beta_1, & T_1 \le \tau \le \tau_0, \\ -\beta_2, & \tau_0 < \tau \le T_2, \end{cases} \quad k_2 = 2\beta_2.$$

Probability density of maximum position (5) of random process $L_q(\tau)$ is determined below. According to notations [7, 15]

$$F_2(u, v, T) = P\{\sup_{T_1 \le \tau \le T} L_q(\tau) < u, \sup_{T \le \tau \le T_2} L_q(\tau) < v\} \tag{11}$$

—two-dimensional distribution function of maximum of random process $L_q(\tau)$. Then the probability density of random quantity τ_q (5) is governed by [7, 15]:

$$W(T) = \mathrm{d}F(T) / \mathrm{d}T,$$

$$F(T) = \int_{-\infty}^{\infty} \left[\frac{\partial F_2(u, v, T)}{\partial u} \bigg|_{v=u} \right] \mathrm{d}u. \tag{12}$$

Consequently, function (11) has to be found for the calculation of probability density (12).

For this additional random process is introduced

$$Z(\tau) = \begin{cases} u - L_q(\tau), & T_1 \le \tau \le T, \\ v - L_q(\tau), & T < \tau \le T_2, \end{cases}$$

which, according to (9), (10), is a Gaussian Markov process with drift and diffusion coefficients as follows:

$$k_1 = \begin{cases} -\beta_1, & T_1 \le \tau \le \tau_0, \\ \beta_2, & \tau_0 < \tau \le T_2, \end{cases} \quad k_2 = 2\beta_2. \tag{13}$$

Using $Z(\tau)$, expression (11) can rewritten as:

$$F_2(u, v, T) = P\{Z(\tau) > 0, \tau \in [T_1, T_2]\}. \tag{14}$$

Functions (11), (14) constitute the probability of random Markov process $Z(\tau)$ not reaching the limits $x=0$ and $x=\infty$ in the interval $\tau\in[T_1,T_2]$. The required probability (14) can be expressed using probability density $W(x,\tau)$ of random process $Z(\tau)$ instances, which did not reach the $x=0$, $x=\infty$ limits [16]:

$$F_2(u,v,T)=\int_0^\infty W(x,T_2)\mathrm{d}x. \tag{15}$$

Function $W(x,\tau)$ is a solution of Fokker–Planck–Kolmogorov equation [16]

$$\frac{\partial W(x,\tau)}{\partial\tau}+\frac{\partial}{\partial x}\left[k_1(x,\tau)W(x,\tau)\right]-\frac{1}{2}\frac{\partial^2}{\partial x^2}\left[k_2(x,\tau)W(x,\tau)\right]=0 \tag{16}$$

for the initial condition of

$$W(x,\tau)\big|_{\tau=T_1}=W(x,T_1)=W_L(u-x,T_1)$$

and the following boundary conditions:

$$W(x=0,\tau)=W(x=\infty,\tau)=0,$$

where $W_L(x,T_1)$ and $W(x,T_1)$ are probability density of random variables $L_q(T_1)$ and $Z(T_1)=u-L_q(T_1)$, respectively.

Applying method of reflections with sign change [7, 16] enables finding solution (16) with coefficients (13) separately for intervals $\tau\in[T_1,\tau_0]$ and $\tau\in(\tau_0,T_2]$. Feeding these solutions into (15), then (15) into (12), expressions for probability density of random variable (5) can be obtained:

$$W(T)=\begin{cases} d_1^2\Psi(d_1^2(\tau_0-T),z_1^2,z_2^2,1/R), & T\le\tau_0,\\ d_2^2\Psi(d_2^2(T-\tau_0),z_2^2,z_1^2,R), & T>\tau_0,\end{cases} \tag{17}$$

where $d_1^2=R^2\beta_2$, $d_2^2=\beta_2$, $R=\beta_1/\beta_2$, $z_1^2=d_1^2(\tau_0-T_1)$, $z_2^2=d_2^2(T_2-\tau_0)$,

$$\Psi(l,l_1,l_2,l_3)=\frac{1}{2\sqrt{\pi}l^{3/2}}\left\{\frac{\exp[-(l_1-l)/4]}{\sqrt{\pi[l_1-l]}}+\Phi\left(\sqrt{\frac{l_1-l}{2}}\right)\right\}$$

$$\times\int_0^\infty x\exp\left\{-\frac{(x+l)^2}{4l}\right\}\left[\Phi\left(\frac{l_2+l_3x}{\sqrt{2l_2}}\right)-\exp(-l_3x)\Phi\left(\frac{l_2-l_3x}{\sqrt{2l_2}}\right)\right]\mathrm{d}x. \tag{18}$$

Asymptotical behavior of probability density (17) is analyzed below. For this purpose the following random variable is considered instead of estimate τ_q:

$$\mu_q=\begin{cases} d_1^2(\tau_q-\tau_0), & \tau_q\le\tau_0,\\ d_2^2(\tau_q-\tau_0), & \tau_q>\tau_0,\end{cases}$$

which has the probability function of the form:

$$W(\mu)=\begin{cases}\Psi(-\mu,z_1^2,z_2^2,1/R), & \mu\le 0,\\ \Psi(\mu,z_2^2,z_1^2,R), & \mu>0.\end{cases}$$

Note that quantities z_1 and z_2 are proportional to SNR at the output of the receiver $z_i^2 \sim a_0^2 f^2(\tau_0)T_2/N_0$. For $z_i \to \infty$, $i=1, 2$, second and third arguments of function (18) tend to infinity, and the function itself take the form of [15]:

$$W_0(x)=\Psi(x,\infty,\infty,1)=1-\Phi\{\sqrt{|x|/2}\}+3\exp\{2|x|\}\Big[1-\Phi\{3\sqrt{|x|/2}\}\Big], \tag{19}$$

where $\Phi(x)=(1/\sqrt{2\pi})\int_{-\infty}^{x}\exp(-t^2/2)\mathrm{d}t$ is a probability integral.

Using probability density (19) asymptotical expressions for bias and variance of QL estimate of duration τ_q of UWB QRS can be obtained:

$$b(\tau_q|\tau_0)=\frac{(R^2-1)T_2}{z_r^2 f^2(\tau_0)R^2\Delta_a^2\cos^2(2\pi\kappa-\varphi_0-\Delta_\varphi)}, \tag{20}$$

$$V(\tau_q|\tau_0)=\frac{2(2R^6+6R^5+5R^4+5R^2+6R+2)T_2^2}{z_r^4 f^4(\tau_0)R^4\Delta_a^4(R+1)^3\cos^4(2\pi\kappa-\varphi_0-\Delta_\varphi)}, \tag{21}$$

where

$$R=\frac{2}{\Delta_a}\frac{\cos(2\pi\kappa-\varphi_0)}{\cos(2\pi\kappa-\Delta_\varphi-\varphi_0)}-1, \tag{22}$$

$\Delta_a=a^*/a_0$, $\Delta_\varphi=\varphi^*-\varphi_0$ are quantities that characterize deviations of expected amplitude and initial phase of UWB QRS with respect to their actual values,

$$z_r^2=a_0^2T_2/N_0 \tag{23}$$

is SNR at the output of ML receiver for rectangular-shaped signal with amplitude a_0 and duration T_2, $\kappa=\omega\tau_0/(2\pi)$ is a quantity that characterize how narrowband UWB QRS is, which is equal to the number of periods of harmonic oscillation in (1) that can be fitted in the time interval equal to signal duration τ_0.

If amplitude and initial phase of UWB QRS are known a priory, then $a^*=a_0, \varphi^*=\varphi_0$, and QL duration estimate (5) coincides with ML estimate:

$$\tau_{qm}=\operatorname{argsup}L_0(\tau), \qquad L_0(\tau)=L(\tau,a_0,\varphi_0). \tag{24}$$

By substituting $\Delta_a=1$, $\Delta_\varphi=0$ into (20)–(22) asymptotical expressions for bias and variance of ML estimate of duration τ_{qm} (24) of UWB QRS can be obtained:

$$b(\tau_{qm}|\tau_0)=0, \tag{25}$$

$$V(\tau_{qm}|\tau_0)=\frac{13}{2}\frac{T_2^2}{z_r^4 f^4(\tau_0)\cos^4(2\pi\kappa-\varphi_0)}. \tag{26}$$

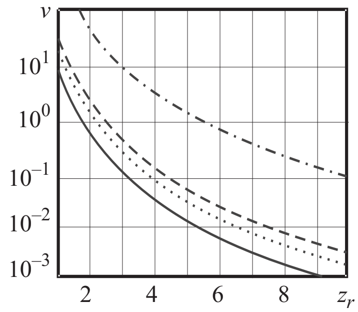

Fig. 2.

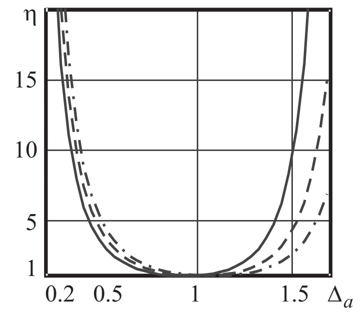

Fig. 3.

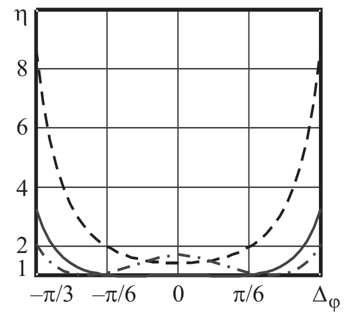

Fig. 4.

Note that similar to [8], asymptotical variance value of duration estimate is not dependent on signal shape, but is defined only by the magnitude of signal back front $a_0 f(\tau_0)\cos(\omega\tau_0 - \varphi_0)$. Equations (25), (26) can be derived directly from [8] by replacing signal shape $f(t)$ with $f(t)\cos(\omega t - \varphi_0)$. If harmonic filling of the impulse is absent ($\omega = 0$, $\varphi_0 = 0$), then equation (26) coincides with the variance of duration estimate of quasi-determined signal with shape $f(t)$ [8]:

$$V_0(\tau_{qm}|\tau_0) = \frac{13}{2}\frac{T_2^2}{z_r^4 f^4(\tau_0)}.$$

Duration estimation of UWB QRS with rectangular modulation function $f(t) = 1$ is considered below as an example. Fig. 2 shows dependences of normalized conditional variance $v = V(\tau_{qm}|\tau_0)/T_2^2$ of ML duration estimate of UWB QRS with rectangular modulation function and of narrowband radio signal with rectangular envelope on SNR z_r (23) for initial phase $\varphi_0 = 0$ and different values of parameter κ. Dependences of normalized conditional variance for signal without harmonic filling ($\kappa = 0$) and for narrowband radio signal ($\kappa >> 1$) [10] are marked by solid and dashed lines respectively. Dotted and dashed-dotted lines denote dependences of normalized conditional variance of ML duration estimate of UWB QRS for different parameters of $\kappa = 0.6$ and $\kappa = 2.8$, respectively, calculated using (26). Actual signal duration value is chosen in the middle of a priory interval $\tau_0 = (T_1 + T_2)/2$.

As seen from Fig. 2, asymptotical variance values of ML duration estimate of UWB QRS are larger than duration estimate variance of signal without harmonic filling for any SNR values. Indeed, if the harmonic filling is present and simultaneously signal is not narrowband, it can lead only to decreasing of the jump magnitude of its back front, and, accordingly, to rising of the estimate variance.

Consequently, utilization of UWB QRS with actual duration providing the largest magnitude of back front, i.e. $\cos(2\pi\kappa - \varphi_0) = \pm 1$ is viable in practical applications. This allows estimating of duration at receiving side with minimal uncertainty.

The following penalty coefficient is introduced for comparison of duration estimation accuracy by QL method (5) and by ML method (24):

$$\eta = V(\tau_q|\tau_0)/V(\tau_{qm}|\tau_0).$$

This penalty characterizes the influence of a priory unknown amplitude and initial phase on the accuracy of duration estimate.

Fig. 3 depicts the dependence of penalty η in accuracy of QL duration estimate of UWB QRS with rectangular modulation function on the deviations of amplitude Δ_a for $\kappa = 1$ and different deviations of initial phase. Solid curve correspond to $\Delta_\varphi = 0$, dashed is for $\Delta_\varphi = \pm\pi/8$ and dashed-dotted represent $\Delta_\varphi = \pm\pi/6$.

Fig. 4 depicts the dependence of penalty η in accuracy of QL duration estimate of UWB QRS with rectangular modulation function on the deviations of initial phase Δ_φ for $\kappa = 1$ and different deviations of amplitude. Solid curve corresponds to $\Delta_a = 1$, dashed is for $\Delta_a = 0.7$ (expected amplitude is lower than its actual value), dashed-dotted line is for $\Delta_a = 1.2$ (expected amplitude is larger than its actual value). For calsulation of curves in Figs. 3, 4 it was assumed that initial phase of received signal $\varphi_0 = 0$ and $z_r = 5$, and the actual value of signal duration is $\tau_0 = (T_1 + T_2)/2$.

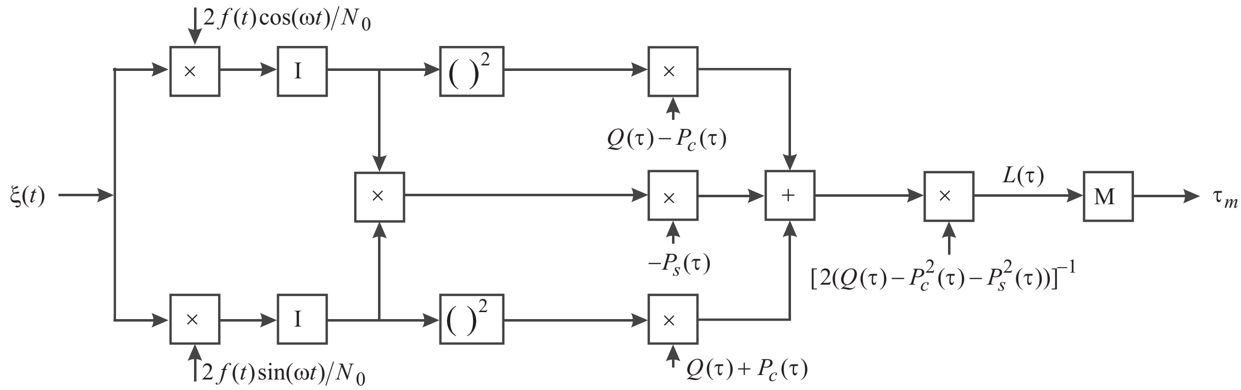

Fig. 5.

As seen from Fig. 2, asymptotical variance values of ML duration estimate of UWB QRS are larger than duration estimate variance of signal without harmonic filling for any SNR values, but can appear both lower and higher than duration estimate variance of narrowband radio signal, depending on the magnitude of parameter κ. Figs. 3, 4 show evidence that a priory unknown amplitude or phase of the signal can result in considerable degradation of duration estimation accuracy.

To improve the accuracy of duration estimation, ML algorithm based on a search of position of LRF logarithm absolute maximum can be applied:

$$\tau_m = \arg\sup L(\tau),$$

$$L(\tau) = \sup_{a,\varphi} L(\tau, a, \varphi) = \sup_{a} L(\tau, a),$$

$$L(\tau, a) = \sup_{\varphi} L(\tau, a, \varphi), \tag{27}$$

Here, instead of unknown amplitude and initial phase in (4) their respective ML estimates a_m and φ_m are used. This is equivalent to maximizing LRF logarithm (4) by unknown parameters a and φ. By performing analytical maximization of LRF logarithm (4), the following expression can be obtained:

$$L(\tau) = \frac{(Q(\tau) - P_c(\tau))X^2(\tau) + (Q(\tau) + P_c(\tau))Y^2(\tau) - 2X(\tau)Y(\tau)P_s(\tau)}{2(Q^2(\tau) - P_c^2(\tau) - P_s^2(\tau))}. \tag{28}$$

Equation (28) determines the structure of the receiving device. Receiver has to form a random process (28) for all possible values of duration and identify the ML duration estimate as a position of its absolute maximum. Fig. 5 presents the block schematics of ML duration measuring device, where "I" is integrator over time interval $[0, \tau]$, $\tau \in [0, T_2]$, "M" is a finder of absolute maximum position of input signal at the time interval $[T_1, T_2]$.

For narrowband radio signal in (29) it is possible to neglect the integrals of functions oscillating with double frequency, as $P_s(\tau) << Q(\tau)$, $P_c(\tau) << Q(\tau)$, and set $P_s(\tau) \approx 0$, $P_c(\tau) \approx 0$. In the case the expression for LRF is simplified and takes the form of:

$$L(\tau) = (X^2(\tau) + Y^2(\tau)) / (2Q(\tau)), \tag{29}$$

which agrees with [10, 12]. It follows from comparison of equations (28) and (29) that the structure of ML algorithm for duration estimation of UWB QRS appears to be significantly more complex than the structure of ML algorithm for duration estimation of narrowband signal.

For the purpose of analysis of ML algorithm for duration estimation (27) we examine in more detail the LRF logarithm (4). It represents a random field, that is differentiable by a and φ, and non-differentiable by variable τ. Consequently, amplitude and initial phase are regular parameters of the signal (1), and duration is a disruptive parameter [7]. Therefore, regularity conditions are partially violated.

According to [17], the accuracy of ML estimate of disruptive parameter (duration) asymptotically (as SNR increases) is not dependent on the presence of arbitrary number of unknown regular parameters (amplitude and initial phase). This means that drift and variance of ML duration estimate (27) asymptotically, as SNR increases, coincide with drift (25) and variance (26) of ML duration estimation of UWB QRS with a priory known amplitude and initial phase.

Consequently, in case of a priory known amplitude and initial phase the accuracy of ML duration estimate of UWB QRS coincides with accuracy of duration estimate of quasi-determined signal considering its harmonic filling [8]. To overcome possible a priory unknown amplitude and initial phase of UWB QRS it is feasible to use a fairly easy-to-implement QL estimation algorithm (5). However the difference of expected values of amplitude and initial phase from their actual values can result in a considerable reduction of duration estimation accuracy. Overcoming a priory unknown amplitude and initial phase without accuracy loss (for high SNR) is achievable by application of more complex ML estimation algorithm (27).

As a result, new structure of ML algorithm for duration estimation of UWB QRS, which substantially differs from the known structure of ML algorithm for duration estimation of narrowband radio signal, is found. Formulae for statistical characteristics of duration estimate are derived. Duration estimation variance of UWB QRS, in contrast to variance of duration estimate of narrowband radio signal, depends on the initial phase of the carrier φ_0 and on the magnitude of parameter κ representing how relatively narrowband the signal is. The structure of ML algorithm for duration estimation of UWB QRS appears to be considerably more complex in case if condition of relatively narrowband signal is not satisfied.

Obtained results for specified signal type allow defining quantitative limits for the ratio of signal bandwidth to its center frequency. These limits imply that classical solution of parameter estimation of narrowband radio signal possesses the required accuracy. The results also allow defining the effect of a priory unknown amplitude and initial phase of UWB QRS on its duration estimation accuracy. In addition obtained results enable us to justify the selection of duration estimation algorithm depending on available a priory information and requirements for estimation accuracy and the level of its technical implementation complexity.

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