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Simulation of Energetic Efficiency of Triode High Voltage Glow Discharge Electron Sources with Account of Temperature of Electrons and Its Mobility in Anode Plasma

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Abstract—The article proposes an iterative technique of calculation of energetic efficiency of triode high voltage glow discharge electron sources', based on taking into account the influence of heating of anode plasma by the accelerated electrons of electron beam and by the slow secondary discharge electrons, reflected from the anode. The increase of temperature of anode plasma influences its' volume, as well as the concentration and mobility of ions' in it. Therefore, the proposed model allows obtaining the corrected values of discharge current and energetic efficiency of electron sources by taking into account the thermodynamic parameters of anode plasma. In addition, one of the advantages of proposed iterative calculation technique is that the model is closed and self-consistent, therefore, it does not demand use of approximated data about the thermodynamic parameters of discharge plasma from references.

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INTRODUCTION

Technological sources of electrons based on the high-voltage glow discharge (HVGD) find broad application in the industry at realization of various thermal technological operations. This includes highly productive welding; soldering; annealing of small-sized products; application of composite coatings of the complex chemical composition in the environment of various gases, including inert and active; vacuum remelting of refractory metals and dielectric materials for the purpose of their purification from harmful impurities [1–7].

The main advantages of HVGD technological sources are the possibility of work in low and average vacuum, in the environment of various technological gases, the relatively simple design of the source and the technological vacuum equipment used and the ease of controlling the electron beam current [8].

The problem consists that gas-dynamic electron beam current control systems used in technological equipment based on HVGD electron sources often do not meet modern requirements of electron beam technologies because of their low speed [8–11]. In this regard now, there is a considerable interest in theoretical and applied research, connected with the development of low-inertia systems for electric control of the HVGD current [12, 13]. Theoretical studies have shown that in such systems the time of increase in discharge current ranges from hundreds of microseconds to tens of milliseconds [14] that fully meets the requirements of electron-beam technologies [9–11].

However, the complex and diverse physical processes proceeding in the triode electrode systems of HVGD during the ignition of a low-voltage control discharge have not been sufficiently studied at present. This constraints the development and deployment of promising triode sources of HVGD electrons in industry. In particular, the dependences of the discharge current and the energy efficiency of HVGD electron sources on the thermodynamic parameters of the anode plasma have not been studied yet. The formation of the corresponding iterative mathematical model of the triode discharge gap of a HVGD intended for the research of these dependencies is the purpose of this article.

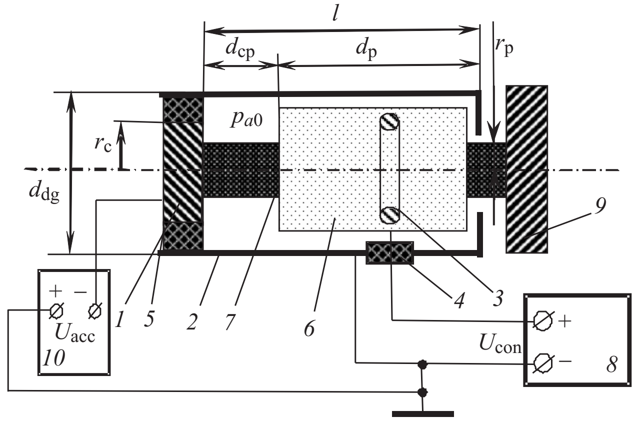

Fig. 1. Scheme of the modeled triode electron source HVGD with connected circuits of power and discharge current control [13]. Designations: cathode *1*, anode HVGD *2*, annular electrode for ignition of auxiliary discharge *3*, low-voltage insulator *4*, high-voltage insulator *5*, anode plasma *6*, electron beam *7*, low-voltage control voltage source *8*, workpiece *9*, high-voltage power supply *10*.

PROBLEM DEFINITION

In [13, 14] and a number of other publications, the energy and time parameters of the HVGD during the ignition of an auxiliary discharge were investigated. In particular, the authors of the articles solved the problem of determining the volume of the anode plasma and the position of its boundary for a quasi-one-dimensional axisymmetric electrode system with an annular auxiliary electrode. As a result, the ion concentration in the anode plasma, the currents of the main and auxiliary discharge are determined and the energy efficiency of the triode sources of HVGD electrons is estimated [13].

The study of the dependence of the energy efficiency on the electrical parameters of the discharge, working pressure and the geometry of the discharge gap made it possible to formulate important practical engineering recommendations for designers of electron beam equipment. In [14] and a number of other studies, the estimates of the time of the increase in the HVGD current when the control pulses were applied to an additional electrode were made.

The drawback of the previously proposed physical and mathematical models is that in order to determine the position of the boundary of the anode plasma relative to the cathode, it is necessary to know the thermodynamic parameters of the electron gas in the anode plasma, in particular, the temperature and mobility of the plasma electrons. Therefore, quantitative estimates of the position of the boundary of the anode plasma in [13, 14] were carried out based on reference data [15–17]. Using this approach, the dependence of the temperature of the electron gas and the mobility of electrons in the anode plasma on the combustion regimes of the main, auxiliary discharge and on the operating pressure in the discharge gap can be taken into account only indirectly, through the internal parameters of the model. At the same time, the influence of these physical factors is decisive in the study of the power of HVGD electrode systems, since they determine the volume of the anode plasma and the concentration of ions in it [22].

Therefore, the purpose of this article is to describe the features of the formation of a more universal, iterative physical and mathematical model of triode electrode systems of HVGD. The proposed model allows taking into account the influence of the electron temperature and their mobility in the anode plasma on its volume, current of the main and auxiliary discharge, and on the power of the triode discharge gap in explicit form.

BASIC PHYSICAL AND MATHEMATICAL MODEL OF THE HVGD TRIODE ELECTRODE SYSTEM

A generalized structural layout of the HVGD triode electrode system with the indicated geometric dimensions and input electrical parameters of the electron source model is shown in Fig. 1 [13]. Depending on the functionality of the source of the control voltage *8*, triode electron sources can operate in a stationary and pulsed mode [13, 14].

The main geometric parameters of the simulated HVGD triode electrode system are: the length of the discharge gap l, the longitudinal length of the anode plasma d_p, the distance from the plasma boundary to the cathode surface d_{cp}, the crosscut dimensions of the discharge gap d_{dg} and the crosscut dimensions of the cathode r_c. An important output geometric parameter of the model is the radius of the generated electron beam r_b.

The main electrical parameters of the considered model of the discharge gap are the accelerating U_{acc} and the control voltage U_{con}. For pulsed electron sources, two values of the control voltage are considered: the voltage in the pulse U_{con1} and in the pause between the pulses U_{con0} [14].

Important internal parameters of the simulated HVGD electrode system are the reduced pressure p_{a0}, which is calculated as the product of the gas pressure in the discharge gap p by its longitudinal dimension [13, 21, 22]:

$$p_{a0} = pl. \tag{1}$$

For pulse sources of HVGD electrons, two values of the position of the boundary of the anode plasma relative to the cathode are also considered. The values of d_{p1} and d_{cp1} correspond to the position of the boundary of the anode plasma in the pulse, and the values of d_{p0} and d_{cp0} correspond to its position in the pause between the pulses of the control voltage [14].

Calculation of the position of the boundary of the anode plasma in the quasi-one-dimensional triode electrode system HVGD in accordance with the technique of [13, 14, 22] was carried out on the basis of the ion balance equation in the plasma volume, taking into account ionization of the gas by fast beam electrons and slow electrons reflected from the anode. At the same time, the diffusion of ions from the plasma into the HVGD anode and the influence of the auxiliary discharge are taken into account.

The model of the discharge gap HVGD is based on the equation of the balance of charged particles and is generally written as follows [13, 22]:

$$z_f + z_s + z_d = z_{dif},$$

$$z_f = \frac{j_{ec} d_p p_{a0} A_i U_{acc}^{-a_i}}{e}\left(1 + \eta_i\left(1 - f(1 - d_p p_{a0} Q_{esc0})\right)\eta_U^{-a_i} k_e\right),$$

$$z_s = \frac{d_p p_{a0} n_e}{4}\sqrt{\frac{8kT_e}{\pi m_e}}\,\overline{Q}_{esc0}, \qquad \overline{Q}_{esc0} = N_0 \alpha_i \left(U_i + \frac{2kT_e}{e}\right)\exp\left(-\frac{eU_i}{kT_e}\right),$$

$$z_{dif} = \frac{\pi^2 n_i \mu_{e0} kT_e}{d_p p_{a0} e}, \qquad z_d = \frac{\pi^2 n_i \mu_{e0} \gamma d_{dg} U_{con}(eU_{con} + kT_e)}{d_p p_{a0} \lambda},$$

$$\lambda_e = \frac{9kT_e(eU_{con} + kT_e)^2}{4\pi e^4 p_{a0} \ln\left(1.5 - \sqrt{\dfrac{kT_e(eU_{con} + kT_e)^3}{\pi p_{a0}}}\right)}, \qquad n_i = \frac{j_{ec}\sqrt{\dfrac{kT_e}{2m_e}}}{e(A_i U_{acc}^{-a_i} + 1)}, \tag{2}$$

where z_f is the degree of ionization of the gas by fast electrons, z_s is the degree of ionization of the gas by slow electrons, z_{dif} is the efficiency of the diffusion scattering of ions in the anode plasma, z_d is the degree of ionization of the gas in the auxiliary discharge, η_i and η_U are the coefficients of electron reflection from the anode of the HVGD with respect to the current and energy, respectively, k_e is the coefficient of elongation of electron trajectories, f is the transparency coefficient of the HVGD anode, Q_{esc0} is the cross section of electron scattering at the ions of residual gas, λ_e is the electron mean free path, γ is the coefficient of secondary ion-electron emission from surface of HVGD anode, n_i is the ion concentration in the anode plasma, T_e is the electron gas temperature, j_{ec} is the electron current density from the cathode surface, μ_{e0} is the electron mobility in the anode plasma, U_i is the ionization potential of the gas, k is the Boltzmann constant, N_0 is the Loschmidt constant, α_i is the semi-empirical constant that is a constant for a given gas, A_i, a_i are semi-empirical coefficients.

Solving the system of equations (2), we write down the equation of the ion balance in the anode plasma with the fact that the combustion of the discharge is independent in the form [13, 22]:

$$\frac{\pi^2 \mu_{e0}(kT_e + eU_{\text{con}})}{(d_{\text{p}} p_{a0})^2}\left(1+\frac{\gamma d_{\text{dg}}}{\lambda}\right) - 3(kT_e + eU_{\text{con}})$$

$$\times N_0 \alpha_i \sqrt{\frac{e(kT_e + eU_{\text{con}})}{2\pi m_e}} \exp\left(-\frac{U_i}{\frac{kT_e}{e} + U_{\text{con}}}\right)$$

$$= A_i U_{\text{acc}}^{-a_i}(A_i U_{\text{acc}}^{-a_i} + 1)\sqrt{\frac{kT_e}{2\pi m_e}}\Big(1 + \eta_i\big(1 - f(1 - d_{\text{p}} p_{a0} Q_{e\text{sc}0})\big)\eta_U^{-a_i} k_e\Big). \qquad (3)$$

Equation (3) is then considered as a cubic equation with respect to the parameter d_{p}, which has an analytic solution [13, 22]:

$$R_1 = A_i U_{\text{acc}}^{-a_i}(A_i U_{\text{acc}}^{-a_i} + 1)\sqrt{\frac{kT_e + eU_{\text{con}}}{2\pi m_e}}, \qquad R_2 = f\eta_i \eta_U^{-a_i} k_e,$$

$$R_3 = 3(kT_e + eU_{\text{con}}) N_0 \alpha_i \sqrt{\frac{kT_e + eU_{\text{con}}}{2\pi m_e}} \exp\left(-\frac{U_i}{\frac{kT_e}{e} + U_{\text{con}}}\right),$$

$$R_4 = \mu_{e0}(kT_e + eU_{\text{con}})\left(\frac{\pi}{p_{a0}}\right)^2\left(1 + \frac{\gamma d_{\text{dg}}}{\lambda_e}\right), \qquad R_5 = R_1 R_2 p_{a0} Q_{e\text{sc}0},$$

$$C_{\text{e}} = -\frac{R_1 + R_3 + R_1 R_2}{R_5}, \qquad D_{\text{e}} = \frac{R_4}{R_5}, \quad p = -\frac{C_{\text{e}}^2}{3}, \quad q = \frac{2C_{\text{e}}^3}{27} + D_{\text{e}},$$

$$D_{\text{eq}} = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2, \quad u = \sqrt[3]{-\frac{q}{2} + \sqrt{D_{\text{eq}}}}, \qquad v = \sqrt[3]{-\frac{q}{2} - \sqrt{D_{\text{eq}}}}, \quad y = u + v,$$

$$d_{\text{p}} = y - \frac{C_{\text{e}}}{3}, \qquad d_{\text{cp}} = l - d_{\text{p}}, \qquad (4)$$

where C_{e}, D_{e} are the coefficients of the solved equation, D_{eq} is the discriminant of equation, p, q, u, v, y are the auxiliary variables.

Knowing the longitudinal length of the anode plasma d_{p}, it is possible to determine the concentration of ions in it, the HVGD current, the auxiliary discharge current, and the energy efficiency of the electron source from the relationships [13, 22]:

$$S_1 = \frac{\pi^2 \mu_{i0}}{(d_{\text{p}} p_{a0})^2}\left(1 + \frac{\gamma d_{\text{dg}}}{\lambda}\right) - 3N_0 \alpha_i \sqrt{\frac{e(kT_e + eU_{\text{con}})}{2\pi m_e}},$$

$$S_2 = 1 + \eta_i \eta_u^{-a_i} \left(1 - f(1 - d_\text{p} p_{a0} Q_{\text{esc}0})\right),$$

$$n_i = \frac{A_i U_\text{acc}^{-a_i}(1 + A_i U_\text{acc}^{-a_i}) S_2}{S_1 (kT_e + eU_\text{con}) \exp\left(-\dfrac{U_i}{\dfrac{kT_e}{e} + U_\text{con}}\right)},$$

$$I_\text{d} = r_\text{c}^2 n_i (1 + A_i U_\text{acc}^{-a_i}) \sqrt{\frac{\pi e d_\text{p} (kT_e + eU_\text{con})}{2m_e}},$$

$$I_\text{con} = e n_i \pi r_\text{p} (1 + \gamma)(r_\text{p} + 2d_\text{dg}) \sqrt{\frac{2eU_\text{d}}{m_i}},$$

$$\eta_\text{D} = 1 - \frac{2 + ke(1 + 2ke\gamma\mu_{e0})}{2k_e(1 + \gamma k_e)},$$

$$k_e = l p_{a0} \overline{Q_{i0}}, \quad \eta_\text{T} = \frac{\eta_\text{D}}{1 + \dfrac{\eta_\text{D} U_\text{con} I_\text{con}}{U_\text{acc} I_\text{d}}}, \tag{5}$$

where S_1 and S_2 are auxiliary variables, I_d is the HVGD current, I_con is the auxiliary discharge current, η_T is the energy efficiency of a triode electron source, η_D is the energy efficiency of a diode source of electrons with the corresponding geometric dimensions and combustion conditions of the HVGD. Relations (5) follows from the current balance condition at the electrodes, while taking into account the geometry of the system being simulated (Fig. 1) [13].

On the one hand, the system of equations (4), (5) is closed and self-consistent. However, the problem of solving the assigned task of modeling is that the thermodynamic parameters of the anode plasma are included into the system of equations (4), namely, the electron gas temperature T_e and the electron mobility μ_{e0} that depend on the longitudinal length of the anode plasma d_p. The analytical relationships for calculating the electron temperature in the anode plasma and their mobility are given in the next section of the article.

CALCULATION OF THE ELECTRON GAS TEMPERATURE AND ELECTRON MOBILITY IN THE ANODE PLASMA

The electron temperature in the HVGD anode plasma is determined from the condition of the superheating of the electron gas for an ideally conducting plasma [15–18]:

$$\frac{e^2 n_e E^2}{m_e(\nu_{ea} + \nu_{ei})} = \frac{3kn_e}{2}\left((T_e - T_a)\delta_{ea}\nu_{ea} + (T_e - T_i)\delta_{ei}\nu_{ei}\right), \tag{6}$$

where ν_{ea} is the frequency of collision of electrons with gas atoms, ν_{ei} is the frequency of collision of electrons with gas ions, δ_{ea} is the fraction of energy transferred by electrons to atoms, δ_{ei} is the fraction of energy transferred by electrons to ions.

For a weakly ionized quasineutral HVGD plasma while $|(n_e - n_i)/n_e| \ll 1$ the relation (6) takes the form [15–18]:

$$T_e = \frac{eU_{\text{con}}}{2k}\left(1+\sqrt{1+\frac{\pi e^2 m_i}{6m_e}\left(\frac{U_{\text{acc}}d_{\text{cp}}}{p_{a0}\overline{Q_{ea}}}\right)^2}\right), \tag{7}$$

where m_i is the mass of the gas ions.

The relationship between relations (6) and (7) is explained by the following physical considerations [18]. Suppose that the temperature of heavy particles, ions and atoms, and the fraction of energy transferred to them in the discharge by electrons, coincide, i.e. $\delta_{ea} = \delta_{ei}$ and $T_a = T_i$. Then the temperature difference between electrons and heavy particles is determined from the relation [18]:

$$T_e - T_a = \left(\frac{e^2 m_a}{3km_e^2}\right)\left(\frac{U_{\text{acc}}d_{\text{cp}}}{\nu_\Sigma}\right),$$

$$\nu_\Sigma = \nu_{ea} + \nu_{ei}. \tag{8}$$

For modelling $|(n_e - n_i)/n_e| << 1$ expression (8) is simplified. At low degrees of ionization under HVGD combustion conditions, which is 5–10% [21], it can be assumed that [18]:

$$\frac{\nu_{ei}}{\nu_{ea}} < 1, \quad \frac{p_e + p_i}{p_a} = \frac{n_e\left(1+\dfrac{T_e}{T_a}\right)}{n_a} < 1,$$

$$T_e >> T_i, \quad T_i = \frac{eU_{\text{con}}}{k}. \tag{9}$$

Substituting (9) into (8) we obtain expression (7). Expressing the crosscut dimensions of the region of the cathode drop of the potential d_{cp} through the temperature of the electron gas T_e in explicit form, relation (7) is written as follows:

$$d_{\text{cp}} = \frac{p_{a0}\overline{Q_{ea}}}{U_{\text{acc}}}\sqrt{\frac{6m_e}{\pi e^2 m_i}\left(\left(\frac{2kT_e}{eU_{\text{con}}}-1\right)^2 - 1\right)}. \tag{10}$$

The electron mobility in the anode plasma HVGD μ_{e0} depends on the temperature of the electron gas T_e and the average electron velocity v_e, which, taking into account (7), for the physical combustion conditions of the HVGD is determined from the relationship [15–17]:

$$\overline{v_e} = \sqrt{\frac{2kT_e}{m_e}} = \frac{U_{\text{acc}}d_{\text{cp}}}{\sqrt{\dfrac{3U_{\text{con}}m_e^2}{em_i}\left(\dfrac{1}{2}\left(1+\sqrt{1+\dfrac{\pi e^2 m_i}{6m_e}\left(\dfrac{U_{\text{acc}}d_{\text{cp}}}{p_{a0}\overline{Q_{ea}}}\right)^2}\right)-1\right)}}. \tag{11}$$

At a known electron temperature T_e and average electron velocity $\overline{v_e}$ that are determined from relations (8) and (9), assuming that the electron velocity distribution function in the HVGD anode plasma obeys Maxwell's law, we obtain the following analytical expression for the electron mobility [15–17]:

$$\mu_{e0} = \frac{e\int\limits_0^\infty v_e^4 \exp\left(-\frac{m_e v_e^2}{2kT_e}\right) dv_e}{3\nu_e m_e \int\limits_0^\infty v_e^2 \exp\left(-\frac{m_e v_e^2}{2kT_e}\right) dv_e} = \frac{eK_{\text{bp}}}{3\nu_e m_e}, \tag{12}$$

where the coefficient K_{bp} describes the process of thermodynamic interaction of the generated electron beam with the anode plasma and, in accordance with (10), is determined from the relation

$$K_{\text{bp}} = \frac{\int\limits_0^\infty v_e^4 \exp\left(-\frac{m_e v_{ea}^2}{2kT_e}\right) dv_e}{\int\limits_0^\infty v_e^2 \exp\left(-\frac{m_e v_{ea}^2}{2kT_e}\right) dv_e}. \tag{13}$$

The coefficient K_{bp}, determined by the relation (13), has the dimension of the square of the velocity and characterizes the change in the mobility of electrons in the anode plasma due to its heating in the region of passage of the electron beam [17, 18].

The solution of equation (11) for many gases, for example, for nitrogen and air, when the electron collision integral with the atoms of the residual gas is written in the form [17, 18]:

$$J_{ea} = \frac{-v_e^{-2}\partial(v_e^2 \nu_{ea} f(v_e))}{\partial v_e},$$

where $f(v_e)$ is the electron velocity distribution function, can be written in the form of a simplified linear dependence [17]:

$$\mu_{e0} = \frac{a p_{a0} d_{\text{cp}}}{U_{\text{acc}} l} + b, \tag{14}$$

where a, b are semi-empirical coefficients.

The physical meaning of (14) is that the electron mobility in plasma is directly proportional to the gas pressure and inversely proportional to the magnitude of the electric field in the region of the cathode potential drop [17].

NUMERICAL ALGORITHM FOR MODELING OF TRIODE ELECTRODE SYSTEMS HVGD

It is obvious that the system of equations (4), (5), (7), (10), (14) does not have an analytical solution and can only be solved numerically. This is due to the fact that equations (4), (5) are written based on an analysis of the processes taking place in the region of the cathode potential drop, and equations (7)–(11) are based on analysis of the processes occurring in the anode HVGD plasma.

In the general case, the equilibrium position of the boundary of the anode plasma is determined from the relationships [2, 19]:

$$\left.\left(\frac{\partial\varphi}{\partial n}\right)\right|_{r\in\Gamma} = \sqrt{\frac{n_e k T_e}{\varepsilon_0 A(\varphi^*)}},$$

$$A(\varphi^*) = \frac{1}{4}\left(\sqrt{1 + \frac{2q\varphi^*}{kT_e}} + \exp\left(-\frac{2q\varphi^*}{kT_e}\right) - 2\right), \tag{15}$$

where q is the charge of plasma ions, φ^* is the near-electrode potential, that depends on the composition of the gas used and the ion temperature, the value of which lies within a few volts.

For the physical conditions of combustion of HVGD that are characterized by a low temperature of the ions and statistics of the Boltzmann distribution for the electron gas, the equilibrium equation for the plasma boundary (15) can be written in a simplified form [2, 19]:

$$\frac{\varepsilon_0 E_{\mathrm{p}}^2}{2} = \frac{\varepsilon_0 \left(\frac{U_{\mathrm{acc}}}{d_{\mathrm{cp}}} \right)^2}{2} = n_e k T_e, \tag{16}$$

where ε_0 is the dielectric constant, and E_{p} is the electric field strength near the plasma boundary from the side of the cathode drop area of the potential.

Taking into account (16), the values of the distance from the cathode surface to the anode plasma boundary, calculated using (4) and (10), should coincide. With this in mind, the following numerical iterative algorithm for solving the nonlinear system of equations (4), (7), (10), (14) is proposed, based on the golden section method [20].

1. The accuracy ε is specified for calculating the value of the crosscut dimensions of the region of the cathode potential drop d_{cp}.

2. From the reference literature, an approximate value of the temperature of the electron gas is determined. It is usually assumed that at room temperature $T_e \approx 800$ K [13, 21, 22].

3. Using (10), the value of the crosscut dimensions of the cathode drop area of the potential is calculated d'_{cp}.

4. For the given parameters a and b, using (14), the mobility of electrons in the anode plasma is calculated μ_{e0}.

5. For the given values of the electron gas temperature T_e and electron mobility in the anode plasma μ_{e0}, a new value of the crosscut dimensions of the region of the cathode drop of the potential d''_{cp} is calculated from (4).

6. From the values d'_{cp} and d''_{cp} found and the golden section method, new values d^{r1}_{cp} and d^{r2}_{cp} are determined and based on the following proportion:

$$\frac{d'_{\mathrm{cp}} - d''_{\mathrm{cp}}}{d'_{\mathrm{cp}} - d^{r1}_{\mathrm{cp}}} = \frac{d'_{\mathrm{cp}} - d''_{\mathrm{cp}}}{d^{r2}_{\mathrm{cp}} - d''_{\mathrm{cp}}} = \frac{1+\sqrt{5}}{2}. \tag{17}$$

7. The values found, d''_{cp}, d^{r1}_{cp}, and d^{r2}_{cp} are substituted in (7) and three average values of the temperature of the electron gas $T_e(d''_{\mathrm{cp}})$, $T_e(d^{r1}_{\mathrm{cp}})$, and $T_e(d^{r2}_{\mathrm{cp}})$ are calculated.

8. The temperature differences $T_e(d''_{\mathrm{cp}}) - T_e(d^{r2}_{\mathrm{cp}})$ and $T_e(d'_{\mathrm{cp}}) - T_e(d^{r1}_{\mathrm{cp}})$ are calculated.

9. If $T_e(d''_{\mathrm{cp}}) - T_e(d^{r2}_{\mathrm{cp}}) > T_e(d'_{\mathrm{cp}}) - T_e(d^{r1}_{\mathrm{cp}})$ than it is considered that $d'_{\mathrm{cp}} = d^{r1}_{\mathrm{cp}}$, otherwise $d''_{\mathrm{cp}} = d^{r2}_{\mathrm{cp}}$.

10. If $|d'_{\mathrm{cp}} - d''_{\mathrm{cp}}| < \varepsilon$ the calculations are considered completed. In this case it is considered that $d_{\mathrm{cp}} = (d'_{\mathrm{cp}} + d''_{\mathrm{cp}})/2$. If $|d'_{\mathrm{cp}} - d''_{\mathrm{cp}}| \geq \varepsilon$ than it is time to transit to point 3 of the algorithm and the calculation for the new values d'_{cp} and d''_{cp}, calculated in paragraph 9, as well as the corresponding temperature values $T_e(d'_{\mathrm{cp}})$ and $T_e(d''_{\mathrm{cp}})$, calculated in paragraph 8.

After performing iterative calculations on the points of algorithm 1–10, the values of d_{cp} found are substituted into (5) to calculate the energy efficiency of the electron source.

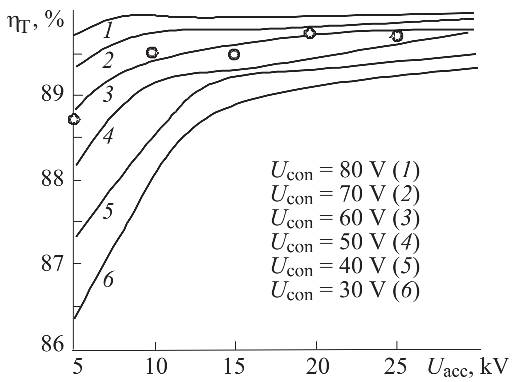

Fig. 2. Calculated dependences of the energy efficiency of the triode electron sources on the accelerating voltage for various auxiliary combustion voltages. Points indicate the corresponding experimental data for U_{con} = 60 V, p = 0.5 Pa.

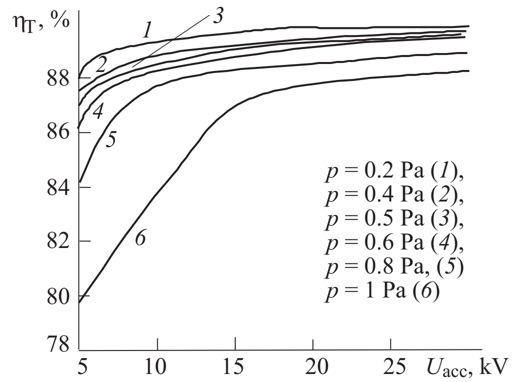

Fig. 3. Calculated dependences of the energy efficiency of the triode electron sources on the accelerating voltage for various pressures in the discharge gap at U_{con} = 50 V.

RESULTS OF MODELING AND THEIR COMPARATIVE ANALYSIS

The proposed algorithm was tested on a personal computer with the following technical characteristics: Intel Core I7 processor, 3 GHz clock speed, 2 GB RAM. For all the test tasks, the convergence of the proposed iterative process is achieved. Analysis of the stability of the convergence algorithm for the numerical methods used, as well as the possibilities of using other methods, are the subjects of separate studies.

Dependences of the energy efficiency of trioded electron sources of HVGD when operating in a stationary mode on the accelerating voltage for various pressures in the discharge gap and for auxiliary discharge voltages, obtained as a result of iterative calculations using relations (4), (5), (7), (10), (14), (17) are shown in Fig. 2 and Fig. 3, respectively. It can be seen from the above-calculated dependences that at a relatively high acceleration voltage, a large control voltage and a low pressure in the discharge gap, the energy efficiency of the triode electron sources of HVGD is close to the corresponding value for diode systems [2, 21] and is more than 80%.

Overall, the above graphs correspond to the results of analytical calculations that were carried out for the same discharge regimes with the use of relations (4), (5). The dependences of the electron gas temperature and electron mobility in the anode plasma on the accelerating voltage, the control voltage and the reduced pressure in the discharge interval that are determined by the relations (6)–(14) [13] were not taken into account. In this case, the averaged values of these parameters of the anode plasma from [15–18] were used.

In solving the modeling problem, nitrogen was considered as the working gas, aluminum as the cathode material, and copper as the anode material. Accordingly, in both cases, calculations were made for the following values of the geometric and internal parameters of the discharge gap model: $l = 7$ cm, $d_{dg} = 7$ cm, $r_c = 5$ cm, $U_i = 18$ V, $a_i = 0.343$, $\gamma = 4.6$, $Q_{ea} = 5.3\times10^{-19}$ m^{-2}, $\alpha_i = 1.452$, $\eta_i = 0.7$, $\eta_U = 0.95$, $f = 0.99$, $A_i = 3.8\times10^{-6}$, $a = 2.5\times10^4$ m/s, $b = 25.4$ m^2/(V·s).

The discrepancies between the results obtained and the results of [13] are no more than 10–15%. The greatest discrepancy between the dependencies shown in Figs. 2, 3 and [13] are observed for edge values of the reduced pressure in the discharge gap and the auxiliary discharge voltage. For the average values of these input parameters of the model, the results obtained practically coincide.

The discrepancy between the results obtained using different calculation techniques is explained by the fact that when using [13] for calculation, the values of the electron gas temperature and electron mobility in the anode plasma were taken for the mean values of the control voltages and the reduced gas pressure in the discharge gap.

It should be noted that the dependencies shown in Figs. 2, 3, are in better agreement with the experimental data for the investigated HVGD systems than the dependences [13]. For example, for curve *3* (Fig. 2), the discrepancy between the calculated and experimental data was no more than 10%, and for the results of [13] they amounted to more than 25%. The modeling error is also directly related to the determination of the values of the semiempirical coefficients a_i, A_i, Q_{ea}, γ, η_i, η_U, U_i, f, a, b that are the internal parameters of the model.

Thus, the iterative technique for calculating the position of the boundary of the anode HVGD plasma, based on the use of the relations (4), (5), (7), (10), (14), (17), proposed here, is more universal and adequate, than the simplified technique [13, 22], based on the solution of the system of equations (4), (5) without taking into account the relations (7), (10), (14), characterizing the temperature dependence of the electron gas in the plasma and the mobility of electrons in it from HVGD combustion modes.

The disadvantage of the technique proposed in this paper is that the use of the relation (17) with the iterative algorithm described above requires considerably more computer time to calculate the position of the anode plasma boundary than simplified analytical calculations using relations (4) and (5). For example, in the test computational experiments carried out, during the calculation of 30 values corresponding to curve *3* (Fig. 2), the counting time according to the method proposed in this work amounted to 5.2×10^7 processor cycles, and by the method [13]—4.3×10^7 processor cycles. However, taking into account that the Intel Core i7 processor, 3 GHz clock speed, 2 GB RAM size, the test task count time was less than 1 min, this disadvantage of the proposed model at the modern level of computer technology development is insignificant.

CONCLUSIONS

The proposed iterative algorithm for calculating the energy efficiency of triode HVGD sources is based on estimating the electron temperature, their mobility in the anode plasma, and on the numerical solution of equations describing the equilibrium condition for the plasma boundary. The algorithm also takes into account the complex of physical processes taking place in the anode plasma and in the region of the cathode potential drop.

In particular, the dependence of the temperature of the electron gas and the electron mobility in the anode plasma on the accelerating voltage, the control voltage, and on the gas pressure in the discharge gap.

The algorithm is sufficiently universal and does not require the use of reference data on the parameters of the HVGD anode plasma and their approximation.

The proposed model is accurate and adequate, since for the investigated discharge combustion regimes the discrepancy between theoretical and experimental data was only 10–15%.

With slight modification, the proposed algorithm can be used to analyze the operation features of triode electron sources of HVGD in the pulsed mode. The obtained results are of great theoretical and practical interest for a wide range of specialists engaged in the design of electron beam technological equipment.

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