ISSN 0735-2727, Radioelectronics and Communications Systems, 2016, Vol. 59, No. 3, pp. 117–127. © Allerton Press, Inc., 2016.
Original Russian Text © A.P. Trifonov, M.B. Bespalova, P.A. Trifonov, I.V. Gushchin, 2016, published in Izv. Vyssh. Uchebn. Zaved., Radioelektron., 2016, Vol. 59, No. 3, pp. 30–39.

Quasi-Likelihood Estimation of the Time Parameters of Ultrawideband Signal Sequence of Unknown Shape under the Influence of Narrowband Interferences[1]

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Received in final form April 29, 2015

Abstract—In this article we investigate the characteristics of a quasi-likelihood estimation of the time of arrival and the repetition period of ultrawideband signal of unknown form, which is received on the background of narrowband interferences with unknown parameters and of Gaussian white noise.

DOI: 10.3103/S0735272716030031

Currently, the application of ultrawideband signals (UWBS) is the innovative and one of the most promising technologies [1–6], which is widely used in various radioelectronic systems, including military systems.

The physical basis of the expedience of implementation of ultrawideband signals is obvious, namely the amount of information, that is transmitted per unit time, is directly proportional to the frequency band utilized. An alternative to this is to increase the data transfer time, but in radiolocation the contact time with the target is always limited, thus there remains the problem of information capacity enhancement in the case of application of traditional approaches.

In many applied problems of radiolocation the radar receiver's objective is to measure the main time parameters of the reflected from the target UWBS sequence, namely the arrival time and repetition period. In [6] the authors considered the maximum likelihood estimation of these parameters under the influence of Gaussian white noise (GWN) only. In [3] the authors investigated the algorithms for the estimation of the time of arrival and repetition period of UWBS sequence on the background of noise, as the model of which they used Gaussian narrowband process (GNP) [7]. In that case the shape of UWBS was considered as known a priori.

Under real conditions the form of the received signal is unknown, because it changes after the reflection from the object (in the case of radiolocation), during the propagation in various media (navigation, communications), and in the process of radio monitoring the form of the signal is always unknown. In [5] the authors considered the problem of estimation of time parameters for the video pulses sequence (a special case of UWBS) of unknown shape, but the influence of interferences had not been taken into account. In this paper we consider the problem of estimation of the arrival time and repetition period for UWBS of unknown form on the background of the GNP and GWN. In this case the characteristics of GNP are also unknown.

The purpose of this investigation is to determine the deterioration in accuracy of measurement of the arrival time and the repetition period due to the differences in the form of expected and received UWBS, and also due to the influence of narrowband interferences.

Let us assume that on the time interval $t \in [0,T]$ we observe the realization of the following type:

$$x(t) = s_{0N}(t,\lambda_0,\theta_0) + n(t) + \xi(t),$$

[1] The investigation was carried out with the support of Russian Science Foundation (project No. 15-11-10022).

where

$$s_{0N}(t,\lambda,\theta)=\sum_{k=0}^{N-1}s_0\left[t-(k-\mu)\theta-\lambda\right] \tag{1}$$

denotes the desired signal, whose form is unknown, one known only that it is an ultrawideband one, λ_0 stands for the unknown arrival time of the signal, θ_0 designates an unknown repetition period, μ determines the sequence point, with which its arrival time λ is connected, $n(t)$ is the implementation of the GWN with single-side spectral density N_0, $\xi(t)$ stands for the narrowband interference. As a model of the narrowband interference we used the most universal [7] narrowband stationary Gaussian process $\xi(t)$ with zero mathematical expectation $\langle\xi(t)\rangle=0$ and the correlation function

$$B_\xi(\Delta)=\langle\xi(t)\xi(t+\Delta)\rangle. \tag{2}$$

The spectral density of narrowband Gaussian noise can be written as follows

$$G_\xi(\omega)=\int_{-\infty}^{\infty}B_\xi(\Delta)\exp(-j\omega\Delta)d\Delta=\frac{\gamma_\xi}{2}\left[g_\xi\left(\frac{\omega-\omega_{0\xi}}{\Omega_\xi}\right)+g_\xi\left(\frac{\omega+\omega_{0\xi}}{\Omega_\xi}\right)\right], \tag{3}$$

where $\Omega_\xi=\int_0^\infty G_\xi^2(\omega)d\omega/\max G_\xi^2(\omega)$ designates the equivalent bandwidth of the interference, $\omega_{0\xi}$ stands for the central frequency.

Since the interference is narrowband, then the condition $\Omega_\xi<<\omega_{0\xi}$ holds true. The function $g_\xi(x)$ describes the shape of spectral density of the interference and satisfies the conditions:

$$g_\xi(x)\ge 0,\quad g_\xi(x)=g_\xi(-x),$$

$$\max g_\xi(x)=\int_{-\infty}^{\infty}g_\xi^2(x)dx=1.$$

We assume that the processes $n(t)$ and $\xi(t)$ are statistically independent.

If the form of the received signal $s_{0N}(t)$ is known a priori, and the GNP is absent, then in order to estimate arrival time of the signal λ_0 and its repetition period θ_0 one can utilize the maximum likelihood technique [8]. For this purpose as an estimation we must be use the location of the highest maximum of the logarithm of likelihood ratio functional [8]

$$L_F(\lambda,\theta)=\frac{2}{N_0}\int_0^T x(t)s_{0N}(t,\lambda,\theta)dt. \tag{4}$$

If form of the signal $s_{0N}(t)$ is not known exactly, then as a reference signal in (4) we use some expected (predicted) signal

$$s_{1N}(t,\lambda,\theta)=\sum_{k=0}^{N}s_1\left[t-(k-\mu)\theta-\lambda\right],$$

$$s_{1N}(t)\ne s_{0N}(t). \tag{5}$$

Therefore, we obtain the following expression for the output signal of the measuring device (decisive statistics):

$$L(\lambda,\theta)=\frac{2}{N_0}\int_0^T x(t)s_{1N}(t,\lambda,\theta)\mathrm{d}t. \tag{6}$$

As the estimates $\hat{\lambda}, \hat{\theta}$ of the unknown arrival time λ_0 and repetition period θ_0 we take the values λ,θ, at which the decisive statistics (6) reaches its absolute maximum:

$$\left(\hat{\lambda},\hat{\theta}\right)=\operatorname{argsup}L(\lambda,\theta). \tag{7}$$

The obtained estimation (7) we will call a quasi-likelihood one [9]. Indeed, in the case of coincidence of the received signal $s_{0N}(t)$ and the expected signal $s_{1N}(t)$ under the absence of GNP the decisive statistics (6) coincides with the logarithm of the likelihood ratio functional (4). Consequently, the quasi-likelihood estimation (QLE) becomes the maximum likelihood estimation.

In order to determine the QLE characteristics of the arrival time and the repetition period let us rewrite (6) as the sum of signal and noise functions [8]

$$L(\lambda,\theta)=S(\lambda,\theta)+N(\lambda,\theta),$$

$$S(\lambda,\theta)=\frac{2}{N_0}\sum_{k=0}^{N-1}\int_{-\infty}^{\infty}s_0\left[t-(k-\mu)\theta_0-\lambda_0\right]s_1\left[t-(k-\mu)\theta-\lambda\right]\mathrm{d}t, \tag{8}$$

$$N(\lambda,\theta)=N_n(\lambda,\theta)+N_\xi(\lambda,\theta), \tag{9}$$

where $N_n(\lambda,\theta)$ denotes the noise function caused by the influence of GWN, $N_\xi(\lambda,\theta)$ stands for the noise function caused by the impact of GNP. Let us write these noise functions:

$$N_n(\lambda,\theta)=\frac{2}{N_0}\sum_{k=0}^{N-1}\int_{-\infty}^{\infty}n(t)s_1\left[t-(k-\mu)\theta-\lambda\right]\mathrm{d}t, \tag{10}$$

$$N_\xi(\lambda,\theta)=\frac{2}{N_0}\sum_{k=0}^{N-1}\int_{-\infty}^{\infty}\xi(t)s_1\left[t-(k-\mu)\theta-\lambda\right]\mathrm{d}t. \tag{11}$$

The noise function (9) is a realization of the Gaussian random field. In this case its first two moments are as follows

$$\langle N(\lambda,\theta)\rangle=0,$$

$$B(\lambda_1,\lambda_2,\theta_1,\theta_2)=\langle N(\lambda_1,\theta_1)N(\lambda_2,\theta_2)\rangle$$

$$=\langle N_n(\lambda_1,\theta_1)N_n(\lambda_2,\theta_2)\rangle+\langle N_\xi(\lambda_1,\theta_1)N_\xi(\lambda_2,\theta_2)\rangle$$

$$=B_n(\lambda_1,\lambda_2,\theta_1,\theta_2)+B_\xi(\lambda_1,\lambda_2,\theta_1,\theta_2), \tag{12}$$

where $B_n(\lambda_1,\lambda_2,\theta_1,\theta_2)$ designates the correlation function of the noise function (10), $B_\xi(\lambda_1,\lambda_2,\theta_1,\theta_2)$ defines the correlation function of the noise function (11).

Having substituted (10), (11) into (12) and taking into account that $\langle n(t_1)n(t_2)\rangle=(N_0/2)\,\delta(t_1-t_2)$, we obtain

$$B_n(\lambda_1,\theta_1,\lambda_2,\theta_2)$$

$$=\left\langle \frac{2}{N_0}\sum_{k=0}^{N-1}\int_{-\infty}^{\infty} n(t)s_1\left[t-(k-\mu)\theta_1-\lambda_1\right]dt\,\frac{2}{N_0}\sum_{k=0}^{N-1}\int_{-\infty}^{\infty} n(t)s_1\left[t-(k-\mu)\theta_2-\lambda_2\right]dt\right\rangle$$

$$=\frac{4}{N_0^2}\int_{-\infty}^{\infty}\int\sum_{k=0}^{N-1}\sum_{m=0}^{N-1}\langle n(t_1)n(t_2)\rangle s_1\left[t_1-(k-\mu)\theta_1-\lambda_1\right]\times s_1\left[t_2-(k-\mu)\theta_2-\lambda_2\right]dt_1dt_2\rangle.$$

Using the filter property of δ-function, we find:

$$B_n(\lambda_1,\theta_1,\lambda_2,\theta_2)=\frac{2}{N_0}\sum_{k=0}^{N-1}\int_{-\infty}^{\infty} s_1\left[t-(k-\mu)\theta_1-\lambda_1\right]s_2\left[t-(k-\mu)\theta_2-\lambda_2\right]dt.$$

Similarly, for the correlation function of the noise function, which is caused by the influence of GNP, with the account of $\langle\xi(t_1)\xi(t_2)\rangle=B_\xi(t_2-t_1)$ we obtain

$$B_\xi(\lambda_1,\theta_1,\lambda_2,\theta_2)=\frac{4}{N_0^2}\sum_{k=0}^{N-1}\int_{-\infty}^{\infty}\int B_\xi(t_2-t_1)s_1\left[t_1-(k-\mu)\theta_1-\lambda_1\right]s_1\left[t_2-(k-\mu)\theta_2-\lambda_2\right]dt_1dt_2.$$

In the process of obtaining the expressions for the signal function (8) and the correlation function (12) of the noise function (9) it was assumed that their arguments satisfy the following condition:

$$\max\left(|\lambda-\lambda_0+(\theta-\theta_0)(k-\mu)|,|\lambda_1-\lambda_2+(\theta_1-\theta_2)(k-\mu)|\right)<\min(\theta,\theta_0,\theta_1,\theta_2)$$

for all $k=\overline{0,N-1}$. Therefore, (4) and (6) describe the central peaks of corresponding functions [8]. By the definition of QLE, at $\lambda=\hat{\lambda}$ and $\theta=\hat{\theta}$ the function $L(\lambda,\theta)$ (6) obtains its absolute maximum, and QLE $\hat{\lambda}$ and $\hat{\theta}$ are the solutions of the following equation system

$$\frac{\partial}{\partial\lambda}\left[S(\lambda,\theta)+N(\lambda,\theta)\right]_{\hat{\lambda},\hat{\theta}}=0,$$

$$\frac{\partial}{\partial\theta}\left[S(\lambda,\theta)+N(\lambda,\theta)\right]_{\hat{\lambda},\hat{\theta}}=0.$$

In this case if the noise function is absent ($N(\lambda,\theta)\equiv 0$), the function (8) reaches its maximum at some point $(\tilde{\lambda},\tilde{\theta})$, and in the general case $\tilde{\lambda}\neq\lambda_0$, $\tilde{\theta}\neq\theta_0$. The equation system for the determination of $\tilde{\lambda}$ and $\tilde{\theta}$ can be written in the following form:

$$\left[\frac{\partial S(\lambda,\theta)}{\partial\lambda}\right]_{\tilde{\lambda},\tilde{\theta}}=-\frac{2}{N_0}\sum_{k=0}^{N-1}\int_{-\infty}^{\infty}\frac{ds_1(t)}{dt}s_0(t-\Delta_k)dt=0,$$

$$\left[\frac{\partial S(\lambda,\theta)}{\partial\theta}\right]_{\tilde{\lambda},\tilde{\theta}}=-\frac{2}{N_0}\sum_{k=0}^{N-1}(k-\mu)\int_{-\infty}^{\infty}\frac{ds_1(t)}{dt}s_0(t-\Delta_k)dt=0, \quad (13)$$

$$\Delta_k=\Delta\lambda+(k-\mu)\Delta\theta,\quad \Delta\lambda=\tilde{\lambda}-\lambda_0,\quad \Delta\theta=\tilde{\theta}-\theta_0.$$

Since $\max S(\lambda,\theta)=S(\tilde{\lambda},\tilde{\theta})$, the signal-to-noise ratio [8] takes the form:

$$z^2 = S^2(\tilde{\lambda},\tilde{\theta}) / B(\tilde{\lambda},\tilde{\lambda},\tilde{\theta},\tilde{\theta}) = 2\left[\sum_{k=0}^{N-1}\int_{-\infty}^{\infty} s_1(t)s_0(t-\Delta_k)\mathrm{d}t\right]^2 \left[NN_0 \int_{-\infty}^{\infty} s_1^2(t)\mathrm{d}t\right]^{-1}, \quad (14)$$

where $S(\tilde{\lambda},\tilde{\theta})$ is determined by (8), and $B(\lambda,\lambda,\theta,\theta)$ can be obtained from (12).

Assume that the signal-to-noise ratio (14) is large enough, so that the QLE of time and repetition period posses high posteriori accuracy. Then the solution of equation system (13) can be found using the small parameter technique [8], as which we used the value $1/z$. Confining ourselves to the first approximation, we obtain the bias (systematic error) of QLE for the arrival time and the repetition period:

$$b_q(\lambda) = \langle\hat{\lambda} - \lambda_0\rangle = \tilde{\lambda} - \lambda_0 = \Delta\lambda,$$

$$b_q(\theta) = \langle\hat{\theta} - \theta_0\rangle = \tilde{\theta} - \theta_0 = \Delta\theta. \quad (15)$$

According to [8] the QLE variances can be represented in the following forms:

$$D_q(\lambda) = \langle(\hat{\lambda} - \tilde{\lambda})^2\rangle = (S_{\lambda\theta}^2 B_\theta - 2S_{\lambda\theta}S_\theta B_{\lambda\theta} + S_\theta^2 B_\lambda)(S_\lambda S_\theta - S_{\lambda\theta}^2)^{-2},$$

$$D_q(\theta) = \langle(\hat{\theta} - \tilde{\theta})^2\rangle = (S_\lambda^2 B_\theta - 2S_\lambda S_{\lambda\theta} B_{\lambda\theta} + S_{\lambda\theta}^2 B_\lambda)(S_\lambda S_\theta - S_{\lambda\theta}^2)^{-2}. \quad (16)$$

QLE correlation coefficient can be written as [8]:

$$R_q = \langle(\hat{\lambda} - \tilde{\lambda})(\hat{\theta} - \tilde{\theta})\rangle[D_q(\lambda)D_q(\theta)]^{-1/2} = \frac{S_\lambda S_\theta B_{\lambda\theta} - S_{\lambda\theta}S_\lambda B_\theta - S_{\lambda\theta}S_\theta B_\lambda + S_{\lambda\theta}^2 B_{\lambda\theta}}{(S_\lambda S_\theta - S_{\lambda\theta}^2)^2[D_q(\lambda)D_q(\theta)]^{1/2}}.$$

In (16) we introduced the following notations:

$$S_\lambda = \left[\frac{\partial^2 S(\lambda,\theta)}{\partial\lambda^2}\right]_{\tilde{\lambda},\tilde{\theta}} = \frac{2}{N_0}\sum_{k=0}^{N-1} A_k, \qquad S_{\lambda\theta} = \left[\frac{\partial^2 S(\lambda,\theta)}{\partial\lambda\partial\theta}\right]_{\tilde{\lambda},\tilde{\theta}} = \frac{2}{N_0}\sum_{k=0}^{N-1}(k-\mu)A_k,$$

$$B_\lambda = \left[\frac{\partial^2 B(\lambda_1,\lambda_2,\theta_1,\theta_2)}{\partial\lambda_1\partial\lambda_2}\right]_{\tilde{\lambda},\tilde{\theta}} = \frac{2N}{N_0}B_1,$$

$$B_{\lambda\theta} = \left[\frac{\partial^2 B(\lambda_1,\lambda_2,\theta_1,\theta_2)}{\partial\lambda_1\partial\theta_2}\right]_{\tilde{\lambda},\tilde{\theta}} = \frac{2B_1}{N_0}\sum_{k=0}^{N-1}(k-\mu),$$

$$B_\theta = \left[\frac{\partial^2 B(\lambda_1,\lambda_2,\theta_1,\theta_2)}{\partial\theta_1\partial\theta_2}\right]_{\tilde{\lambda},\tilde{\theta}} = \frac{2B_1}{N_0}\sum_{k=0}^{N-1}(k-\mu)^2,$$

$$A_k = \int_{-\infty}^{\infty} \frac{\mathrm{d}^2 s_1(t)}{\mathrm{d}t^2} s_0(t-\Delta_k)\mathrm{d}t, \qquad B_1 = \int_{-\infty}^{\infty} \left[\frac{\mathrm{d}s_1(t)}{\mathrm{d}t}\right]^2 \mathrm{d}t + B_\xi, \quad (17)$$

where $B_\xi = \frac{2}{N_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} B_\xi(t_2 - t_1) \frac{\mathrm{d}s_1(t_1)}{\mathrm{d}t_1} \frac{\mathrm{d}s_1(t_2)}{\mathrm{d}t_2} \mathrm{d}t_1 \mathrm{d}t_2$. According to (15) in general case the QLE of time and repetition period is biased.

Note that with an increase in signal-to-noise ratio (14) the QLE variances (16) approach to zero. The estimations accuracy is also characterized by the value of dispersion (mean squared error) [8]:

$$V_q(\hat{\lambda}|\lambda_0, \theta_0) = \langle(\hat{\lambda} - \lambda_0)^2\rangle = b_q^2(\lambda) + D_q(\lambda),$$

$$V_q(\hat{\theta}|\lambda_0, \theta_0) = \langle(\hat{\theta} - \theta_0)^2\rangle = b_q^2(\theta) + D_q(\theta). \tag{18}$$

If the QLE of arrival time and repetition period are inconsistent ($\Delta\lambda \neq 0$, $\Delta\theta \neq 0$), then with increasing signal-to-noise ratio their dispersions tend to the values $\Delta\lambda^2$ and $\Delta\theta^2$, respectively. For the consistent estimations ($\Delta\lambda = 0$, $\Delta\theta = 0$) their dispersions tend to zero with increasing signal-to-noise ratio.

If the GNP is absent and the shape of single UWBS of the sequence (1) is known a priori, then it is possible to select the expected signal $s_1(t) \equiv s_0(t)$. In this case the QLE of $\hat{\lambda}$ and $\hat{\theta}$ are transformed into the maximum likelihood estimations λ_m and θ_m. These maximum likelihood estimations of the arrival time and repetition period for the UWBS sequence with known a priori forms, which is accepted on the background of purely GWN, possess biases and variances as follow [6]:

$$b_0(\lambda_m|\lambda_0, \theta_0) = \langle\lambda_m - \lambda_0\rangle = 0,$$

$$b_0(\theta_m|\lambda_0, \theta_0) = \langle\theta_m - \theta_0\rangle = 0,$$

$$\sigma_\lambda^2 = \langle(\lambda_m - \lambda_0)^2\rangle = \frac{N_0}{2F_0} \frac{N^2 - 1 + 12[(N-1)/2 - \mu]^2}{N(N^2 - 1)},$$

$$\sigma_\theta^2 = \langle(\theta_m - \theta_0)^2\rangle = 6N_0 / [F_0 N(N^2 - 1)], \tag{19}$$

where $F_0 = \int_{-\infty}^{\infty} \left[\frac{\mathrm{d}s_0(t)}{\mathrm{d}t}\right]^2 \mathrm{d}t$.

In this case the correlation coefficient of maximum likelihood estimations is as follows:

$$R_0 = \frac{(N-1)/2 - \mu}{\{(N^2 - 1)/12 + [(N-1)/2 - \mu]^2\}^{1/2}}.$$

Since the maximum likelihood estimations are unbiased, their dispersions coincide with the variances:

$$V_0(\lambda_m|\lambda_0, \theta_0) = \sigma_\lambda^2,$$

$$V_0(\theta_m|\lambda_0, \theta_0) = \sigma_\theta^2. \tag{20}$$

Comparing (16), (18), (19) we find the deterioration in QLE accuracy for the arrival time and the repetition period as a result of ignorance of the a priori UWBS form and the influence of GNP. In particular, from (16), (19) it follows that the QLE have variances, which exceed the variances of maximum likelihood estimations in ρ_1 times, with

$$\rho_{1\lambda} = D_q(\lambda) / \sigma_\lambda^2,$$

$$\rho_{1\theta} = D_q(\theta) / \sigma_\theta^2. \tag{21}$$

The increase in variance of estimations (21) does not depend on the amplitudes of the received and the expected signals.

In a number of problems the estimation dispersion is a more complete characteristic than the variance of the estimation. From (18), (20) it follows that QLE of the arrival time and the repetition period have dispersions that exceed the dispersions of the corresponding maximum likelihood estimations in κ_1 times, with:

$$\kappa_{1\lambda} = \frac{V_q(\hat{\lambda}|\lambda_0,\theta_0)}{V_0(\lambda_m|\lambda_0,\theta_0)} = \frac{\Delta\lambda^2}{\sigma_\lambda^2} + \rho_{1\lambda},$$

$$\kappa_{1\theta} = \frac{V_q(\hat{\theta}|\lambda_0,\theta_0)}{V_0(\theta_m|\lambda_0,\theta_0)} = \frac{\Delta\theta^2}{\sigma_\theta^2} + \rho_{1\theta}. \tag{22}$$

Consequently, the deterioration in accuracy of inconsistent QLE compared with the maximum likelihood estimations increases with increasing signal-to-noise ratio. Indeed, the increase in the received signal power and decrease in the spectral density of GWN lead to the increment of the first terms on the right sides of formulas (22).

For a variety of forms of the received and expected UWBS in the sequences (1), (5), the QLE of the arrival time and the repetition period of the sequence can be justifiable. In particular, they are justifiable if UWBS of the received and the expected sequences are even functions of time:

$$s_0(t) = s_0(-t),$$

$$s_1(t) = s_1(-t), \tag{23}$$

or odd functions of time:

$$s_0(t) = -s_0(-t),$$

$$s_1(t) = -s_1(-t). \tag{24}$$

If (23) or (24) holds true, the solutions of equation system (13) coincide with the true values of the arrival time and the repetition period. Consequently, if (23), (24) hold true, the QLE of the arrival time and the repetition period are justified.

The expressions for the variances and correlation coefficient of QLE (16) are substantially simplified in the case, when (23) or (24) and QLE are justified. Indeed, assuming in (17) $\Delta_k = 0$ and substituting the result into (16), we obtain

$$D_q(\lambda) = \frac{N_0 B_1}{2F_1^2} \frac{N^2 - 1 + 12[(N-1)/2 - \mu]^2}{N(N^2 - 1)},$$

$$D_q(\theta) = 6N_0 B_1 / [F_1^2 N(N^2 - 1)],$$

$$\rho_q = \frac{(N-1)/2 - \mu}{\{(N^2 - 1)/12 + [(N-1)/2 - \mu]^2\}^{1/2}},$$

where $F_1 = \int_{-\infty}^{\infty} \frac{\mathrm{d}s_0(t)}{\mathrm{d}t}\frac{\mathrm{d}s_1(t)}{\mathrm{d}t}\mathrm{d}t.$

Now the variances (16) and dispersions (18) of QLE do match, and the deterioration in QLE accuracy for the arrival time and repetition period in comparison with the accuracy of maximum likelihood estimations is characterized by the value

$$\kappa_{01} = \rho_{01} = \kappa_{1\lambda} = \kappa_{1\theta} = \rho_{1\lambda} = \rho_{1\theta} = \chi_1 / R_s^2, \tag{25}$$

where

$$\chi_1 = 1 + \frac{\frac{2}{N_0}\int_{-\infty}^{\infty}\int B_\xi(t_2 - t_1)\frac{\mathrm{d}s_1(t_1)}{\mathrm{d}t_1}\frac{\mathrm{d}s_1(t_2)}{\mathrm{d}t_2}\mathrm{d}t_1\mathrm{d}t_2}{\int_{-\infty}^{\infty}\left[\frac{\mathrm{d}s_1(t)}{\mathrm{d}t}\right]^2\mathrm{d}t}, \tag{26}$$

and it describes the influence of GNP on the QLE accuracy, and

$$R_s = \frac{\int_{-\infty}^{\infty}\frac{\mathrm{d}s_0(t)}{\mathrm{d}t}\frac{\mathrm{d}s_1(t)}{\mathrm{d}t}\mathrm{d}t}{\sqrt{\int_{-\infty}^{\infty}\left[\frac{\mathrm{d}s_0(t)}{\mathrm{d}t}\right]^2\mathrm{d}t\int_{-\infty}^{\infty}\left[\frac{\mathrm{d}s_1(t)}{\mathrm{d}t}\right]^2\mathrm{d}t}} \tag{27}$$

stands for the cross-correlation coefficient between the derivative of received signal and the derivative of the expected signal. It is obvious that the value (27) describes the influence of difference between the forms of the received and the expected signals in variance of QLE of the arrival time and the repetition period. Note that, as before, the relative increase in the QLE variances (25) does not depend on the amplitudes of the received and the expected signals.

As a particular case, let us find the deterioration in accuracy of QLE in the absence of GNP, i.e. as a result of differences between the forms of the received the expected signal only. Indeed, assuming that $B_\xi(\Delta) \equiv 0$ from (25), (26) we obtain

$$\kappa_{01} = R_s^{-2}. \tag{28}$$

If the shapes of the received and the expected signals coincide, the deterioration in the accuracy of the estimations, that is caused by the influence of GNP, has the form

$$\kappa_{01} = \chi_0, \tag{29}$$

where $\chi_0 = 1 + \frac{\frac{2}{N_0}\int_{-\infty}^{\infty}\int B_\xi(t_2 - t_1)\frac{\mathrm{d}s_0(t_1)}{\mathrm{d}t_1}\frac{\mathrm{d}s_0(t_2)}{\mathrm{d}t_2}\mathrm{d}t_1\mathrm{d}t_2}{\int_{-\infty}^{\infty}\left[\frac{\mathrm{d}s_0(t)}{\mathrm{d}t}\right]^2\mathrm{d}t}.$

Indeed, in the case of coincidence of the shapes of received the expected signals $R_s \equiv 1$ and from (25) we obtain (29).

The calculation of deterioration in accuracy of QLE of the arrival time and the repetition period can be easier and more convenient if one uses the spectral characteristics of signals and interference. For this purpose we denote the spectra of the received and expected signals as follows

$$S_i(\mathrm{j}\omega)=\int_{-\infty}^{\infty} s_i(t)\exp(-\mathrm{j}\omega t)\mathrm{d}t,\quad i=0,1.$$

Then from (26), (27) we obtain

$$\chi_i=1+\frac{\frac{2}{N_0}\int_{-\infty}^{\infty}\omega^2 G_\xi(\omega)\left|S_i(\mathrm{j}\omega)\right|^2\mathrm{d}\omega}{\int_{-\infty}^{\infty}\omega^2\left|S_i(\mathrm{j}\omega)\right|^2\mathrm{d}\omega}, \tag{30}$$

$$R_s=\frac{\int_{-\infty}^{\infty}\omega^2 S_0(\mathrm{j}\omega)S_1^*(\mathrm{j}\omega)\mathrm{d}\omega}{\sqrt{\int_{-\infty}^{\infty}\omega^2\left|S_0(\mathrm{j}\omega)\right|^2\mathrm{d}\omega\int_{-\infty}^{\infty}\omega^2\left|S_1(\mathrm{j}\omega)\right|^2\mathrm{d}\omega}}. \tag{31}$$

Spectral representation is handier to use for the analysis of the GNP influence. By way of example, let us consider the influence of GNP with the rectangular shape of spectral density on the accuracy of quasi-likelihood estimations of the arrival time and the repetition period. In order to perform this let us assume (3) that $g_\xi(x)=1$ for $|x|<1/2$ and $g_\xi(x)=0$ for $|x|>1/2$. Substituting (3) into (30) we obtain the deterioration in accuracy of the estimations due to the influence of GNP in the following form:

$$\chi_i=1+\varepsilon_i q, \tag{32}$$

where $q=\gamma_\xi/N_0$ denotes the ratio of spectral densities of GNP and GWN, and

$$\varepsilon_i=\int_{\omega_{0\xi}-\Omega_\xi/2}^{\omega_{0\xi}+\Omega_\xi/2}\omega^2\left|S_i(\mathrm{j}\omega)\right|^2\mathrm{d}\omega\Big/\int_0^{\infty}\omega^2\left|S_i(\mathrm{j}\omega)\right|^2\mathrm{d}\omega$$

stands for the relative energy fraction of the signal's $s_i(t),\ i=0,1$ derivative within the frequency band that is affected by the GNP. From (32) it follows that the deterioration in accuracy of QLE increases with the increment of the GNP intensity and the relative energy fraction of the signal's derivative, which is affected by the GNP.

Quasi-rectangular pulses of the following form [10] are the examples of the received and the expected UWBS, for which the QLE of the arrival time and the repetition period are justifiable:

$$s_2(t,\tau,\delta)=\begin{cases}\exp\left[-\dfrac{\pi}{2\delta^2}\left(\dfrac{t}{\tau}-\dfrac{1-\delta}{2}\right)^2\right], & \dfrac{t}{\tau}\ge\dfrac{1-\delta}{2},\\ 1, & \dfrac{|t|}{\tau}\le\dfrac{1-\delta}{2},\\ \exp\left[-\dfrac{\pi}{2\delta^2}\left(\dfrac{t}{\tau}+\dfrac{1-\delta}{2}\right)^2\right], & \dfrac{t}{\tau}\le\dfrac{1-\delta}{2},\end{cases} \tag{33}$$

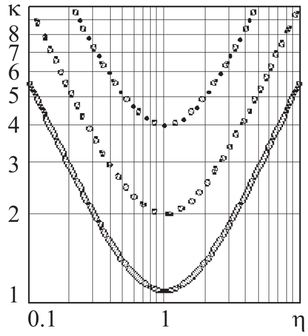

Fig. 1.

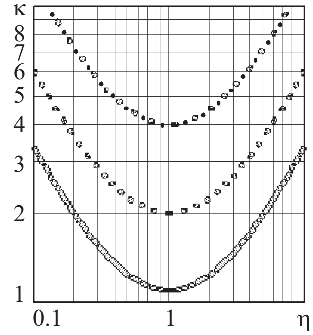

Fig. 2.

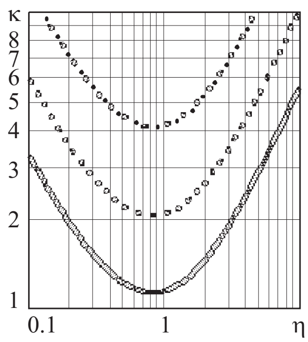

Fig. 3.

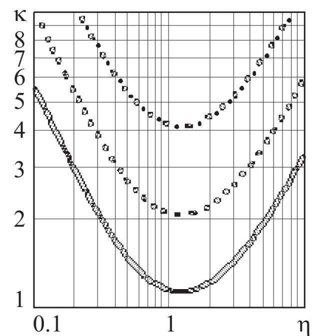

Fig. 4.

$$s_3(t,\tau,\delta)=\begin{cases}\left\{1+\left[\dfrac{\pi}{2\delta}\left(\dfrac{t}{\tau}-\dfrac{1-\delta}{2}\right)\right]^2\right\}^{-1}, & \dfrac{t}{\tau}\ge\dfrac{1-\delta}{2},\\ 1, & \dfrac{|t|}{\tau}\le\dfrac{1-\delta}{2},\\ \left\{1+\left[\dfrac{\pi}{2\delta}\left(\dfrac{t}{\tau}+\dfrac{1-\delta}{2}\right)\right]^2\right\}^{-1}, & \dfrac{t}{\tau}\le\dfrac{1-\delta}{2},\end{cases} \quad (34)$$

where $\tau=\int_{-\infty}^{\infty}s^2(t)\mathrm{d}t\,/\max s^2(t)$ denotes the equivalent pulse duration, $\delta\,(0<\delta\le 1)$ designates the relative energy fraction of the pulse, which is concentrated in its fronts.

The signals (33), (34) satisfy the condition (23) of the justifiability of QLE of the arrival time and the repetition period of the UWBS sequence. The examples of signals that satisfy the justifiability conditions are given in [4].

According to (28) the deterioration in accuracy of the justifiable QLE due to a priori ignorance of the shape of a single UWBS of the sequence is determined by the correlation coefficient (27), (31) of derivatives of the received signal and the expected one under the absence of systematic error $(\Delta\lambda\equiv 0, \Delta\theta\equiv 0)$. According to the formulas for signals (33), (34) we have calculated the deterioration (28) in the accuracy of QLE of the arrival time and the repetition period for different values of the duration τ_0 of the received signal and the duration τ of the expected signal.

In Figs. 1–4 we demonstrated the dependences of the deterioration $\kappa(\eta)$ in accuracy of QLE compared with the accuracy of maximum likelihood estimations versus the ratio $\eta=\tau/\tau_0$. The solid lines were calculated for the value of parameter $\delta=1$; the dashed ones were computed for $\delta=0.5$; the dash-dotted lines were calculated for $\delta=0.1$.

For the Fig. 1 we chose the received signal $s_0(t,\tau_0)=s_2(t,\tau_0,\delta=1)$ and the expected signal $s_1(t,\tau)=s_2(t,\tau,\delta)$. For the Fig. 2 we selected $s_0(t,\tau_0)=s_3(t,\tau_0,\delta=1)$ and $s_1(t,\tau)=s_3(t,\tau,\delta)$, respectively. For the Fig. 3 we chose $s_0(t,\tau_0)=s_2(t,\tau_0,\delta=1)$ and $s_1(t,\tau)=s_3(t,\tau,\delta)$, and for the Fig. 4 we selected $s_0(t,\tau_0)=s_3(t,\tau_0,\delta=1)$ and $s_1(t,\tau)=s_2(t,\tau,\delta)$.

As it follows from the rate of the curves (Figs. 1–4), the deterioration in accuracy of QLE of the arrival time and the repetition period of the UWBS sequence can be significant.

Examples of calculation of the deterioration in accuracy of QLE are also given in [4], where the repetition period was assumed known a priori.

Therefore, we have determined the deterioration in accuracy of measurement of the arrival time and the repetition period caused by both the differences between the forms of expected and received UWBS, and by the influence of narrowband interferences. Sufficient conditions of the QLE justifiability have been laid down. It has been demonstrated that the deterioration in accuracy of the estimations, which is caused by the differences between the forms of the expected and received UWBS, increases with decreasing correlation coefficient of the first derivatives of these signals. Consequently, the obtained QLE characteristics allow one to make a justified choice of the estimation algorithm and the expected signal form based on the available priori information and acceptable deterioration in the accuracy of estimation.

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