
Yaw Moment Control Strategy for Four Wheel Side Driven EV¹

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Abstract—When four wheel side driven EV travels in steering or changes lanes in high speed, the vehicle is easy to side-slip or flick due to the difference of wheel hub motor and a direct effect of vehicle nonlinear factors on vehicle yaw motion, which would affect vehicle handling and stability seriously. To solve this problem, a joint control strategy, combined with the linear programming algorithm and improved sliding mode algorithm, which combines the exponential reaching law and saturation function was proposed. Firstly, the vehicle dynamics model and the reference model according with the structure and driving characteristics of four wheel side driven EV were set up. Then, introduced the basic method of the improved sliding mode variable structure control and complete the sliding mode variable structure controller design basic on vehicle sideslip angle and yaw velocity. The controller accomplish optimal allocation of vehicle braking force through a linear programming algorithm, according to yaw moment produced by the vehicle motion state. Single lane driving simulation results show that the proposed control strategy can not only control vehicle sideslip angle and yaw velocity well, but also accomplish good controlling of the vehicle yaw moment, so as to significantly improve the handling and stability of vehicle.

Keywords: wheel side drive, sliding mode variable structure, yaw moment control, optimal allocation

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1. INTRODUCTION

With the increasingly prominence of environmental and energy issues, wheel drive electric vehicles, as one of effective ways, can solve two major problems, and it developed rapidly in the automotive field in recent years. Among them, the electric vehicle yaw moment control technology has attracted researchers' attention, as it is one of the key technologies to improve traffic safety [1, 2]. Due to the complexity of the structure of electric vehicle, people usually considered the electric vehicle as a nonlinear system during the research. In addition, the vehicle nonlinearization problem would become more serious for influence from steering angle, road adhesion coefficient, speed and lateral wind and other factors during the process of driving, so it would be difficult to achieve the desired control effect with the conventional control theory.

In order to solve the nonlinear control problem better, the researchers achieved the direct yaw moment control of the vehicle with the help of the sliding mode variable structure control method, which has strong anti-disturbance and good robustness in recent years. The sliding mode variable structure controller takes the vehicle sideslip angle and lateral swing angle velocity as control variables and adopts the controlling methods of the constant reaching or index reaching to improve the dynamic quality of approach motion, achieve the correct tracking of the vehicle reference yaw velocity and sideslip angle [3–5]. With the research on vehicle yaw stability going deeply by scientific research scholars, they put forward the joint control strategy combing sliding mode variable structure and other control theories to design the sliding mode controller in the way of designing the switching function of sliding mode variable structure with the aid of LQ method effectively, realize the control of vehicle direct yaw moment by the sliding mode controller by using the exponential reaching law [6]; recently, a combined control strategy based on sliding mode variable structure and PI control is proposed [7]. The strategy force the system into the sliding surface through adaptive update of PI controller gain without the need to know in advance the controller parameters and the interference range, to achieve the goal of controlling the vehicle yaw stability by weak-

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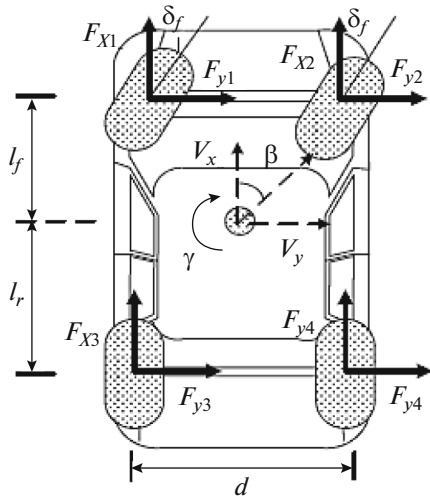


Fig. 1. 2-Degree of freedom nonlinear vehicle dynamics model.

ening the flutter phenomenon of the sliding mode controller. So far, scholars at home and abroad carried out a lot of research on vehicle yaw stability control combined with sliding mode control theory. Because the parameter perturbation of the system with sliding mode variable structure and external disturbance invariance is in exchange for the control of the high frequency chattering, in the practical application, the chattering problem has always been the main factor that affects the control performance of sliding mode variable structure [8–10].

Aiming at the chattering problem that existed in the process of traditional sliding mode variable structure theory in control of four wheel side driven electric vehicle yaw moment and using saturation function and exponential reaching law, this paper can inhibit the sliding mode control flutter phenomenon, designed sliding mode variable structure controller combining saturation function and exponential reaching law. On this basis, distribute opti-

mally the braking force of each wheel with control algorithm of linear programming so as to realize the yaw stability control of the vehicle.

2. VEHICLE MODEL

For vehicle yaw stability analysis, consider the motion of the car in the horizontal plane, a 2-degree of freedom vehicle dynamics model is established, as shown in Fig. 1. The model regards longitudinal velocity μ as unchanged, ignoring the car rolling resistance, air resistance and steering system.

The vehicle dynamics equations are expressed as follows:

$$mV(\dot{\beta} + \gamma) = F_{y1} + F_{y2} + F_{y3} + F_{y4}, \quad (1)$$

$$J_z \dot{\gamma} = (F_{y1} + F_{y2})l_f - (F_{y3} + F_{y4})l_r + M_z, \quad (2)$$

$$M_z = 0.5d(F_{x1} - F_{x2} + F_{x3} - F_{x4}), \quad (3)$$

where M_z for Yaw moment of the centroid axis z around the vehicle produced by the longitudinal driving force of each wheel; m for weight with full equipment; V for speed; $F_{xi} (i = 1, 2, 3, 4)$, $F_{yi} (i = 1, 2, 3, 4)$ respectively as the longitudinal force and lateral force of the four wheels of the vehicle; l_f, l_r respectively as distance from the centroid to the front and rear axles; d for wheel tread; J_z for moment of inertia around the axis z ; β for vehicle sideslip angle, and when the β is small, $\beta = \arctan(V_y/V_x)$; γ is yaw velocity, when γ is small, the lateral force of the four wheels can be expressed as:

$$F_{y1} = F_{y2} = C_f(\delta_f - \beta - l_f \gamma/V), \quad (4)$$

$$F_{y3} = F_{y4} = C_r(l_r \gamma/V - \beta), \quad (5)$$

where C_f, C_r respectively as lateral stiffness of front and rear wheels of vehicle.

3. DESIGN OF YAW MOMENT CONTROLLER FOR VEHICLE

The vehicle sideslip angle and yaw velocity are two important state variables to measure the vehicle stability, in this paper, vehicle sideslip angle and yaw velocity are set as the sliding mode variable structure controller, on this basis, the vehicle yaw stability control model is constructed, as shown in Fig. 2. Among them, the reference state of the vehicle is calculated by the reference model according to the steering angle signal given by the driver as well as the real time speed of the vehicle. The sliding mode variable structure controller adjusts the yaw moment on the vehicle according to the error between the actual state and the reference state. The optimal allocation module assigns the yaw moment calculated by sliding mode con-

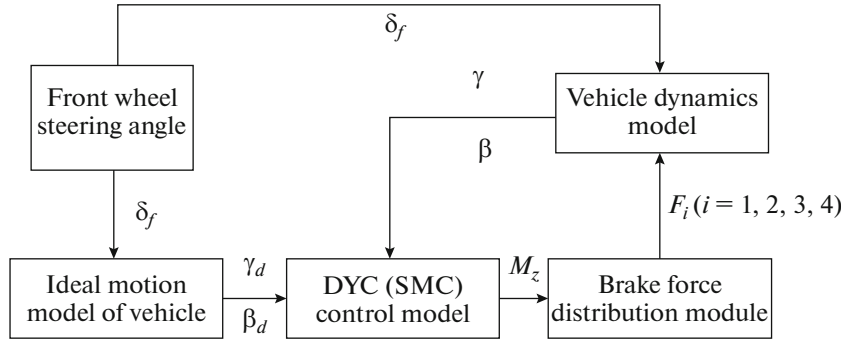


Fig. 2. DYC schematic diagram of structure.

troller to the four electric wheel reasonably. At the same time, considering the constraint of the working point of the tire and the electric wheel.

3.1. Reference Model Building

The two degree of freedom state space equation of vehicle can be expressed as:

$$\dot{x} = Ax + B_1 w + B_2 u. \quad (6)$$

In the formula, $x = [\beta \ \gamma]^T$ for vehicle state, $w = [\delta_f]$ for front wheel steering angle, $u = [M_z]$ for yaw moment of vehicle.

$$A = \begin{bmatrix} \frac{2(C_f + C_r)}{mV} & \frac{2C_r l_r - 2C_f l_f - mV^2}{mV^2} \\ -\frac{2C_f l_f - 2C_r l_r}{J_z} & -\frac{2C_r l_r + 2C_f l_f}{J_z V} \end{bmatrix}, \quad B_1 = \begin{bmatrix} \frac{2C_f}{mV} \\ \frac{2C_f l_f}{J_z} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ \frac{1}{J_z} \end{bmatrix}.$$

In the vehicle driving condition, yaw velocity and sideslip angle is an important index to evaluate the vehicle handling and stability. In the process of designing a reference model, a lot of literature designed sideslip angle to zero ($\beta_d = 0$), in order to maintain the stability of the vehicle is in the best condition. This paper uses the method of sideslip angle $\beta_d = 0$, reference model designed as [11]:

$$\dot{x}_d = A_d x_d + B_d w_d, \quad (7)$$

where $x_d = \begin{bmatrix} \beta_d \\ \gamma_d \end{bmatrix}$, $A_d = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{\tau_r} \end{bmatrix}$, $B_d = \begin{bmatrix} 0 \\ \frac{k_r}{\tau_r} \end{bmatrix}$, $w_d = [\delta_f]$, $k_r = \frac{2C_f V}{mV^2 + 2(C_f l_f - C_r l_r)}$, $\tau_r = \frac{J_z V}{2(C_f l_f^2 + C_r l_r^2)}$.

The above reference model is constructed based on the case that the road adhesion coefficient is large enough, in the actual driving process, the longitudinal force and lateral force of the wheel will be restrained by the maximum frictional force between the road and the tire, and the desired yaw velocity must meet the following formula [12]:

$$|\gamma_d| \leq |ug/V|. \quad (8)$$

3.2. Sliding Mode Variable Structure Controller Design

In order to improve the vehicle handling and stability, sideslip angle and yaw velocity of real vehicle in driving conditions must track output value of reference model very well. Considering the influence of the nonlinear factors such as the tire side slip, the axle load transfer, the sliding mode variable structure control can be used to force the system to make the sliding mode motion along the prescribed trajectory under

certain conditions, so as to realize the effective control of the direct yaw moment of the vehicle. The linear two degrees of freedom vehicle motion equations (6) are transformed:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} \frac{2(C_f + C_r)}{mV} & \frac{2C_r l_r - 2C_f l_f - mV^2}{mV^2} \\ -\frac{2C_f l_f - 2C_r l_r}{J_z} & -\frac{2(C_f l_f^2 + C_r l_r^2)}{J_z V} \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} + \begin{bmatrix} \frac{2C_f}{mV} & 0 \\ \frac{2C_f l_f}{J_z} & \frac{1}{J_z} \end{bmatrix} \begin{bmatrix} \delta_f \\ M_z \end{bmatrix}. \quad (9)$$

The error between vehicle actual sideslip angle and expectations sideslip angle, the actual yaw velocity and desired yaw velocity can be expressed as follows:

$$e_\beta = \beta - \beta_d. \quad (10)$$

$$e_\gamma = \gamma - \gamma_d. \quad (11)$$

According to the vehicle dynamics equation, sideslip angle and yaw velocity has a coupling relationship. Combined control of the two variables, the problem that the sideslip angle is too large when alone control yaw velocity and the ideal yaw velocity cannot be well tracked when separate control sideslip angle can be solved. The principle of sliding mode variable structure, the sliding surface can be defined as zero ($s = 0$). The sliding mode surface equation is:

$$s = e_\gamma + c e_\beta (c > 0). \quad (12)$$

In this formula, c for joint control parameters.

According to (10), (11), (12) will be rewritten into:

$$s = [1 \ c] \begin{bmatrix} \gamma - \gamma_d \\ \beta - \beta_d \end{bmatrix} (c > 0). \quad (13)$$

According to (13), the dynamic characteristic equation of the system reaching the sliding mode plane is:

$$\dot{s} = [1 \ c] \begin{bmatrix} \dot{\gamma} - \dot{\gamma}_d \\ \dot{\beta} - \dot{\beta}_d \end{bmatrix} (c > 0). \quad (14)$$

When the sliding mode variable structure principle controls the system, the sliding mode reachability condition can only guarantee to reach the switching surface in a finite time from any moving point in the state space, without any restriction for the specific path of reaching movement. Dynamic quality of reaching motion can be improved by means of reaching law approach. In order to seek the higher control accuracy of the vehicle yaw moment and better weaken the chattering phenomenon of the sliding mode control, based on the analysis of the advantages of exponential reaching law control, the control method combined the exponential reaching law with saturation function is used to control the vehicle yaw moment. The modified exponential approximation law is:

$$\dot{s} = -ks - \varepsilon \text{sat}(s) (\varepsilon > 0, k > 0). \quad (15)$$

The saturation function $\text{sat}(s)$ can be expressed as the following form:

$$\text{sat}(s) = \begin{cases} 1 & s > \Delta \\ \frac{s}{\Delta} & |s| \leq \Delta (\Delta > 0) \\ -1 & s < -\Delta \end{cases} \quad (16)$$

Bring (9), (13), (14) into (15), we get:

$$\begin{aligned} 0 = & \left(\frac{2C_f l_f^2 + 2C_r l_r^2}{J_z V} - \frac{2C_r l_r - 2C_f l_f - mV^2}{mV^2} c - k \right) \gamma + \left(\frac{2C_f l_f + 2C_r l_r}{J_z} + \frac{2l_f + 2l_r}{mV} c - ck \right) \beta \\ & + k \gamma_d + ck \beta_d + \dot{\gamma}_d + \dot{\beta}_d - \frac{1}{J_z} M_z - \frac{2C_f l_f}{J_z} \delta_f - \frac{2C_f}{mV} c - \varepsilon \text{sat}(\gamma - \gamma_d + c\beta - c\beta_d). \end{aligned} \quad (17)$$

According to (17), we can get the Yaw moment expressions of yaw rate and sideslip angle-combined control:

$$\begin{aligned}
M_z = & J_z \left[\left(\frac{2C_f l_f^2 + 2C_r l_r^2}{J_z V} - \frac{2C_r l_r - 2C_f l_f - mV^2}{mV^2} c - k \right) \gamma \right. \\
& + \left(\frac{2C_f l_f + 2C_r l_r}{J_z} + \frac{2l_f + 2l_r}{mV} c - ck \right) \beta + k\gamma_d + ck\beta_d + \dot{\gamma}_d + \dot{\beta}_d \\
& \left. - \frac{2C_f l_f}{J_z} \delta_f - \frac{2C_f}{mV} c - \varepsilon \text{sat}(\gamma - \gamma_d + c\beta - c\beta_d) \right]. \quad (18)
\end{aligned}$$

3.3. Wheel Braking Force Distribution Control Strategy

When the vehicle is in the steering condition, the sliding mode variable structure controller calculates the yaw moment of the vehicle in real time according to the error between the reference state and the actual state so as to adjust the vehicle's travel state. What is worth mentioning is that, the direct yaw moment did not finish distributing the four wheel braking force. Therefore, another important issue in this paper is how to distribute the braking force according to the direct yaw moment.

The direct yaw moment distribution is essentially a constrained optimization problem, through the distribution of the four wheel tangential force, it will help to reduce the utilization rate of the tire and keep the tire with high adhesion coefficient, so as to improve the stability of the vehicle [13, 14]. Considering the limit of the vehicle brake and the working condition of the tire, to establish the objective function of braking force distribution with the lowest utilization rate of tire, so as to optimize the direct yaw moment calculated by the sliding mode controller to the four electric wheels.

The objective function can be defined as [15]:

$$\min J = \sum_{i=1}^4 \frac{\sqrt{F_{xi}^2 + F_{yi}^2}}{u F_{zi}} = \sum_{i=1}^4 \frac{F_i}{u F_{zi}}. \quad (19)$$

In the formula, i ($i = 1, 2, 3, 4$) for the four wheels, u for road adhesion coefficient, F_{zi} for four wheel vertical load and it can be described as $F_{zi} = \frac{mg l_r}{2(l_f + l_r)}$ ($i = 1, 2$), $F_{zi} = \frac{mg l_f}{2(l_f + l_r)}$ ($i = 3, 4$); $F_i = \sqrt{F_{xi}^2 + F_{yi}^2}$ ($i = 1, 2, 3, 4$) for road adhesion, can be expressed by F_i ($i = 1, 2, 3, 4$). The constraints of the wheels can be expressed as:

$$M_z = 0.5d(F_{x1} - F_{x2} + F_{x3} - F_{x4}), \quad (20)$$

$$F_i = \sqrt{F_{xi}^2 + F_{yi}^2} \leq \min \left(u F_{zi}, \left| \frac{T_{\max}}{r} \right| \right). \quad (21)$$

In the formula, T_{\max} for maximum driving torque of electric wheel, d for radius of electric wheel, the above optimization problems can be summarized as general linear programming problems:

—objective function

$$\min c^T x, \quad (22)$$

—constraint condition

$$\begin{cases} Ax \leq b \\ A_e x = b_e \\ lb \leq x \leq ub \end{cases} \quad \text{among them, } c = \frac{1}{u} \left[\frac{1}{F_{z1}} \quad \frac{1}{F_{z2}} \quad \frac{1}{F_{z3}} \quad \frac{1}{F_{z4}} \right]^T,$$

$$x = [F_1 \quad F_2 \quad F_3 \quad F_4]^T, \quad A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad b = u [F_{z1} \quad F_{z2} \quad F_{z3} \quad F_{z4}]^T,$$

Table 1. Vehicle simulation parameters

Parameter	Numerical value	Company
Weight with full equipment, m	600	kg
Acceleration of gravity, g	9.8	m/s ²
Moment of inertia of vehicle, J_z	2100	kg-m ²
Tread, d	1.430	m
The distance between centroid and front axle, l_f	0.61	m
The distance between centroid and rear axle, l_r	0.725	m
Lateral stiffness of front wheel, C_f	32000	N/rad
Lateral stiffness of rear wheel, C_r	32000	N/rad
Wheel radius, r	0.4	m
Maximum driving torque of electric wheel, T_{\max}	550	N-m

$$A_e = 0.5d [\cos(\delta_f + \beta) - \cos(\delta_f + \beta) \cos(\beta) - \cos(\beta)],$$

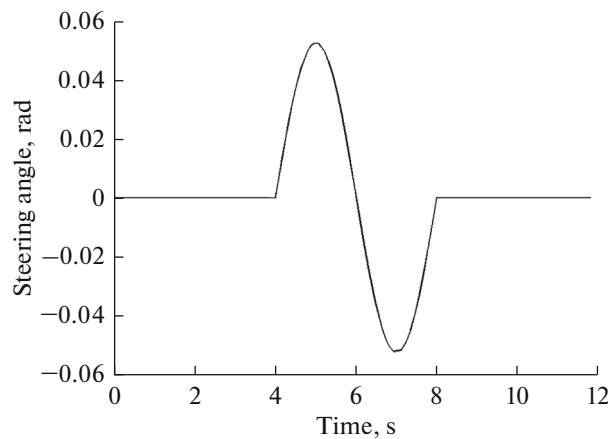
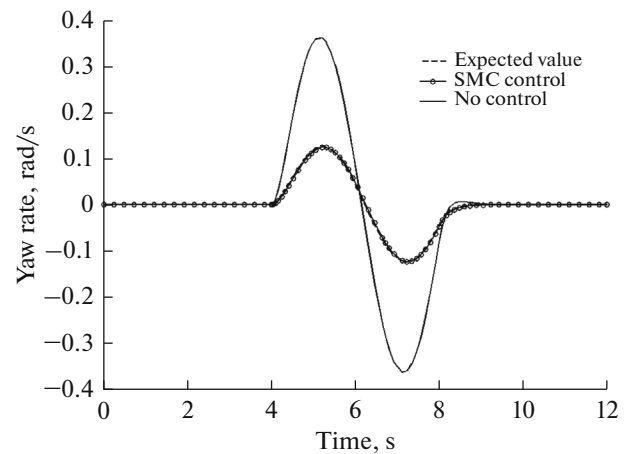
$$b_e = [M_z], lb = [0 \ 0 \ 0 \ 0]^T, ub = \frac{T_{\max}}{r} [1 \ 1 \ 1 \ 1]^T,$$

by using linear programming theory calculate wheel adhesion F_i ($i = 1, 2, 3, 4$), on this basis, combined tire longitudinal force calculation formula $F_{xi} = F_i \cos(\delta_f + \beta)$ ($i = 1, 2$), $F_{xi} = F_i \cos(\beta)$ ($i = 3, 4$), complete the distribution of four wheel braking force.

4. SIMULATION STUDY

In order to test the control effect of the designed vehicle yaw stability control system, a two degree of freedom vehicle dynamics model, a reference model, a sliding mode variable structure control model, and a braking force distribution model are established in the Matlab/Simulink environment. On this basis, use the single lane condition to simulate the actual vehicle driving condition, the simulation parameters are shown in Table 1.

When the road adhesion coefficient is 0.5, linear speed is 25m/s, input 30 sine steering angle at 4 s, as shown in Fig. 3. Figures 4 and 5 are respectively for the simulation graphics of vehicle yaw velocity and sideslip angle. The research and analysis shows that direct yaw moment control based on the improved sliding mode variable structure control and linear programming theory can make the electric vehicle yaw velocity and sideslip angle to track the expected value stably, and the error from the expected value is very

**Fig. 3.** Front wheel steering angle.**Fig. 4.** Yaw velocity of vehicle.

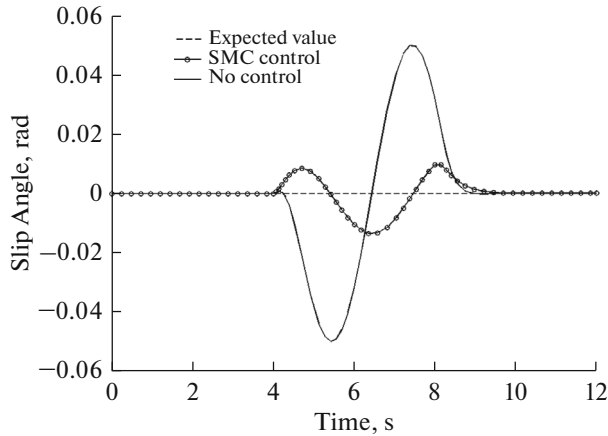


Fig. 5. Vehicle sideslip angle.

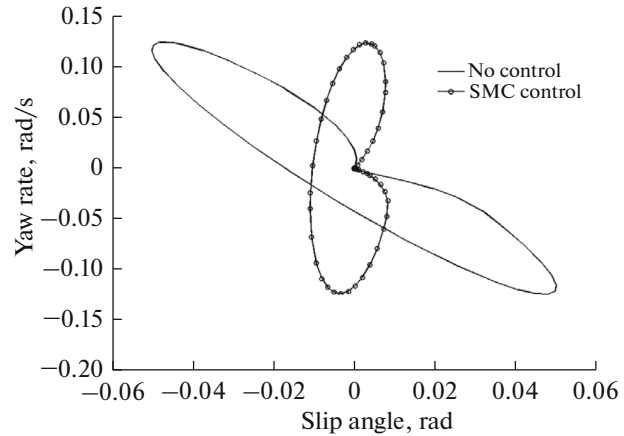


Fig. 6. $\gamma - \beta$ Phase diagram.

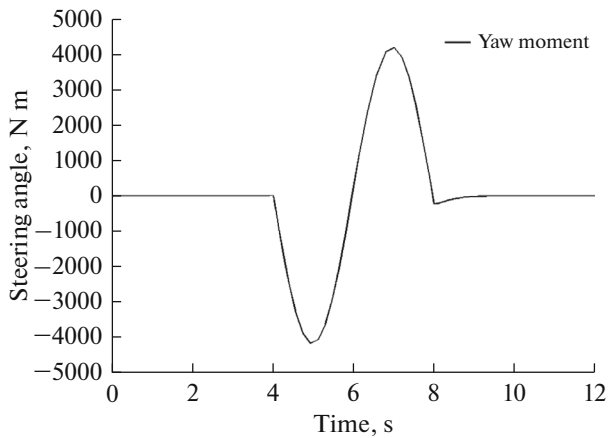


Fig. 7. Direct yaw moment.

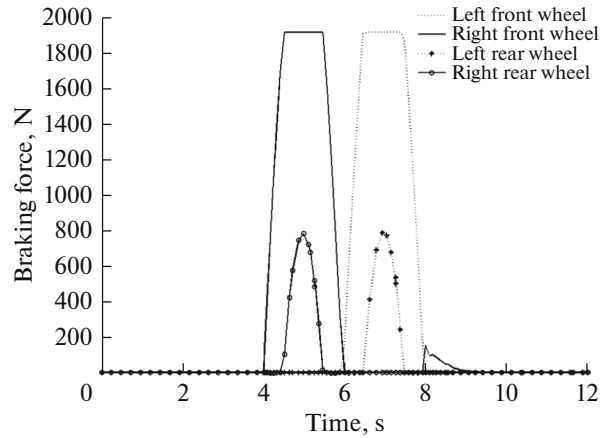


Fig. 8. Wheel brake force distribution.

small. In contrast, passive control to follow the effect is not ideal, when steering at 5 s and 7 s, the vehicle yaw velocity and sideslip angle increases rapidly, which may cause the vehicle skidding or turning sharply instability. The phase plane curve of Fig. 6 shows that the sideslip angle variation range by passive control was obviously larger than that of the improved sliding mode variable structure control. The overshoot is large, which greatly weakens the stability of vehicle steering. Through the comparative analysis, it can be concluded that the improved sliding mode variable structure and linear programming algorithm can effectively control the yaw moment of the vehicle, and significantly improve the vehicle handling stability.

Figure 7 is the simulation of direct yaw moment calculated by the improved sliding mode variable structure method, using the linear programming theory to optimize the yaw moment, and the optimal allocation results are shown in Fig. 8. When the vehicle enters the left turn (4 s~6 s), electric vehicle left front wheel braking force F_{x1} and left rear wheel braking force F_{x3} respectively output different values, and the right front wheel braking force F_{x2} and the right rear wheel braking force F_{x4} output are both zero; when the vehicle enters the left turn (6 s~8 s), the output of four wheel braking force system for electric vehicle is opposite to that of the left turn. The application of the linear programming theory in the direct yaw moment distribution can realize optimal allocation of four wheel braking force at different times and under the corresponding conditions, and effectively reduce the tire utilization rate, so as to raise the vehicle steering safety margin.

5. CONCLUSION

This paper aimed at the characteristics that each electric wheel driving force/braking force of the four wheel drive electric vehicle can be controlled independently, introduced the using of the improved sliding

mode variable structure control method combined exponential reaching law with saturation function to realize the vehicle yaw moment control. Using linear programming theory to complete the effective decomposition of yaw moment, so as to realize the optimal allocation of four electric wheel braking force. This method not only reduces the utilization rate of the tire, but also significantly improves the vehicle handling and stability performance.

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