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Research on Adaptive Sliding Synchronization of Rikitake Chaotic System with Single Unknown Control Coefficient[1]

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Abstract—The control of second order system with uncertain parameters and single unknown control coefficient was investigated to solve the synchronization problem of Rikitake chaotic with reduced number of active inputs. In addition, a kind of adaptive strategy was hybrid with sliding mode method, where the adaptive strategy was used to cope with uncertain parameters produced in the process of sliding mode controller design. At last, detailed numerical simulations with both second order systems and synchronous chaotic system were done to testify the rightness of the proposed method and also multi-time random simulations were done to testify the robustness of the controller. In addition, the main conclusion is that the sliding mode control has very good consistency since the strategy formation is almost the same as the controller for system with known control coefficient, and high gain is necessary for system with single uncertain control coefficient.

Keywords: synchronization, stability, backstepping control, chaotic system, uncertainty, adaptive control

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1. INTRODUCTION

The theory of synchronization [1–3] is a recent research area extensively investigated by researchers from many countries in many fields including communication, mechanical systems, robotics, chemical reactions and biological systems. Especially chaos synchronization [4–7] has attracted a lot of attention from theoretical and practical viewpoints and several approaches had been proposed [8–11]. Application of chaos synchronization in secure communication is certainly the main reason why it attracted so many scholars not only in civilian and in army. But in the past, most research works were focused on synchronization of chaotic systems with different structure or systems with all kinds of uncertainties [12–14], or new synchronization strategies. Only a few researchers thought using less active inputs to realized synchronization, in fact this new format is not only make synchronization more difficult to be realized, but also make it more safe and more difficult to be deciphered in secure communication. So in this paper, a kind synchronization of Rikitake system with reduced number of active inputs was investigated And to make the design can be applied to a more widely range of other kind of chaotic systems, the design process was based on a control strategy of general second order system [15–17] since the synchronization problem can be changed to be a control problem of special second order system, so the stability of the whole chaotic synchronous system should be analyzed in the design process.

Stability is a main problem [18–21] that affect a control method to be meaningful or not, but robustness [22–26] is a key problem that affect a control method can be applied in real control engineering or not. To solve system uncertainties, many kinds of nonlinear control methods [27–29] were proposed and applied in many control examples. Sliding mode control strategy is loved and applied by many engineers for its strong robustness. In fact, the strong robustness [30–32] was achieved by the easily setting of high gain in the controller. So many papers shows the strong robustness but the essential reason of robustness

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is not revealed. Why it has strong robustness? It is an interesting problem. In this paper, a kind of adaptive sliding mode controller was designed for a second order system and synchronous chaotic systems with single unknown control coefficient. Moreover, what is most important is that it pointed out that the high robustness, which is showed in the final part of multi-time random simulation, and simulation of chaotic synchronous system, still depended on the using of high gain feedback in the control law design.

2. MODEL DESCRIPTION

A novel Rikitake chaotic system can be described as

$$\begin{aligned} \dot{z}_{a1} &= -2z_{a1} - z_{a2}z_{a3}, \\ \dot{z}_{a2} &= -2z_{a2} - (z_{a3} - 5)z_{a1}, \\ \dot{z}_{a3} &= 1 - z_{a1}z_{a2}, \end{aligned} \tag{1}$$

where z_{a1}, z_{a2} and z_{a3} are system state variables, and $a = 1$, $b = 0.46$ and $c = 0.46$. The above chaotic system was supposed to be driven system, and the response chaotic system was also chosen as the same chaotic system as

$$\begin{aligned} \dot{z}_{b1} &= -2z_{b1} - z_{b2}z_{b3}, \\ \dot{z}_{b2} &= -2z_{b2} - (z_{b3} - 5)z_{b1}, \\ \dot{z}_{b3} &= 1 - z_{b1}z_{b2}. \end{aligned} \tag{2}$$

In the next sections, we will research on the synchronous problem between driven chaotic system and response chaotic system with only two synchronous inputs.

3. MODEL TRANSFORMATION

In order to realize the synchronization between above chaotic systems, which is very useful and can be applied in secure communication, we set the system (1), to be the driven system and set the system (2) to be the response system; and by adding active inputs the response system can be modified as

$$\begin{aligned} \dot{z}_{b1} &= -2z_{b1} - z_{b2}z_{b3}, \\ \dot{z}_{b2} &= -2z_{b2} - (z_{b3} - 5)z_{b1} + b_1u_1, \\ \dot{z}_{b3} &= 1 - z_{b1}z_{b2} + u_2, \end{aligned} \tag{3}$$

where u_1, u_2 are active control items which are used to set the control law to make the synchronization realized, and b_1 is a unknown control coefficient. The main innovation point of this paper is that we just use two active control items to realize the synchronization of two three order chaotic systems. Then the error system can be written as

$$\begin{aligned} \dot{e}_1 &= -2e_1 + z_{a2}z_{a3} - z_{b2}z_{b3}, \\ \dot{e}_2 &= -2e_2 + 5e_1 + z_{a1}z_{a3} - z_{b1}z_{b3} + b_1u_1, \\ \dot{e}_3 &= z_{b1}z_{b2} - z_{a1}z_{a2} + u_2, \end{aligned} \tag{4}$$

where errors are defined as $e_1 = z_{b1} - z_{a1}$, $e_2 = z_{b2} - z_{a2}$, $e_3 = z_{b3} - z_{a3}$.

Then the above system can be divided into two systems. The first system is a second order system, which can be written as follows:

$$\begin{aligned} \dot{e}_1 &= -2e_1 + f_1, \\ \dot{e}_2 &= -2e_2 + 5e_1 + f_2 + b_1u_1, \end{aligned} \tag{5}$$

where $f_1 = z_{a2}z_{a3} - z_{b2}z_{b3}$, $f_2 = z_{a1}z_{a3} - z_{b1}z_{b3}$, and the second system can be written as a first order system

$$\dot{e}_3 = f_3 + u_2, \tag{6}$$

where $f_3 = z_{b1}z_{b2} - z_{a1}z_{a2}$.

To make it is easy to understand for readers of this paper, we first do not consider f_2, then the synchronous law design for u_1 is simpler than the original system. But to make the discussion has a more universal and general application, we discuss the control problem of below second order system with single input coefficients first, then the synchronous law designed can be applied to any other kind of chaotic system.

4. PROBLEM DESCRIPTION

The second-order system with a single unknown control direction is a special situation in all of second-order systems. The control direction is the coefficient of the model input u, which is also called control coefficient. The model discussed in this paper can be written as:

$$\dot{x} = Ax + bu, \tag{7}$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ b_1 \end{bmatrix}$, where b_1 is an unknown constant. The parameters of model is unknown, the objective of adaptive sliding mode control is to design an adaptive sliding mode controller such that the system state x_1 can trace the expected value x_1^d.

5. ASSUMPTION

Assumption 1: $a_{12} \neq 0$, its direction is known, without loss of generality, assume $a_{12} > 0$.

Assumption 2: the expected x_1^d is a constant, then $\dot{x}_1^d = 0$.

Assumption 3: the sign of control coefficient is known, without loss of generality, assume $b_1 > 0$.

Assumption 4: the amplitude of control coefficient is unknown and it is a constant, so $\dot{b}_1 = 0$.

6. ADAPTIVE SLIDING MODE CONTROLLER DESIGN FOR UNCERTAIN SECOND ORDER SYSTEM WITH SINGLE UNKNOWN CONSTANT CONTROL COEFFICIENT

Consider the following the first order subsystem:

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2. \tag{8}$$

Define an error variable as $e_1 = x_1 - x_1^d$, then:

$$\dot{e}_1 = a_{11}x_1 + a_{12}x_2. \tag{9}$$

Then the second order derivative of error can be written as

$$\ddot{e} = a_{11}\dot{x}_1 + a_{12}\dot{x}_2 = a_{11}a_{11}x_1 + a_{11}a_{12}x_2 + a_{12}a_{21}x_1 + a_{12}a_{22}x_2 + a_{12}\,u, \tag{10}$$

and the sliding mode surface can be written as

$$s = \dot{e} + c_1 e + c_2\int e dt. \tag{11}$$

To make the sliding mode surface meaningful and stable, it is necessary to set parameters $c_1 > 0, c_2 > 0$ such that the differential equation is stable when the sliding mode surface is converged to 0.

So the derivative of the sliding mode surface can be written as

$$\begin{aligned}\dot{s} = \ddot{e} + c_1\dot{e} + c_2 e = a_{11}a_{11}x_1 + a_{11}a_{12}x_2 + a_{12}a_{21}x_1 + a_{12}a_{22}\,x_2 \\ + c_2 e + c_1 a_{11}\,x_1 + c_1 a_{12}x_2 + a_{12}b_1 u.\end{aligned} \tag{12}$$

Then it holds

$$\frac{1}{a_{12}b_1}\dot{s} = a_{11}a_{11}\frac{1}{a_{12}b_1}x_1 + \frac{a_{11}}{b_1}x_2 + \frac{a_{21}}{b_1}x_1 + \frac{a_{22}}{b_1}x_2 + c_2 e\frac{1}{b_1 a_{12}} + c_1 a_{11}\frac{1}{b_1 a_{12}}x_1 + \frac{c_1}{b_1}x_2 + u, \tag{13}$$

and it can be arranged as

$$\frac{1}{a_{12}b_1}\dot{s} = \left(a_{11}a_{11}\frac{1}{a_{12}} + a_{21} + c_1 a_{11}\frac{1}{a_{12}}\right)\frac{x_1}{b_1} + (a_{11} + c_1 + a_{22})\frac{x_2}{b_1} + c_2\frac{1}{a_{12}}\frac{e}{b_1} + u. \tag{14}$$

Define

$$l_1 = \left(a_{11}a_{11}\frac{1}{a_{12}} + a_{21} + c_1 a_{11}\frac{1}{a_{12}}\right)\Big/ b_1, \tag{15}$$

$$l_2 = (a_{11} + a_{22} + c_1)/b_1, \tag{16}$$

$$l_3 = (c_2/a_{12})/b_1. \tag{17}$$

Then the control law can be designed as

$$u = -\hat{l}_1 x_1 - \hat{l}_2 x_2 - \hat{l}_3 e - k_1 s - k_2 \operatorname{sgn}(s) - k_3 \int s dt \tag{18}$$

and define variables as

$$\tilde{l}_1 = l_1 - \hat{l}_1, \tag{19}$$

$$\tilde{l}_2 = l_2 - \hat{l}_2, \tag{20}$$

$$\tilde{l}_3 = l_3 - \hat{l}_3. \tag{21}$$

Then it holds

$$\frac{1}{a_{12} b_1} \dot{s} = \tilde{l}_1 x_1 + \tilde{l}_2 x_2 + \tilde{l}_3 e - k_1 s - k_2 \operatorname{sgn}(s) - k_3 \int s dt, \tag{22}$$

and it can be transformed as

$$\frac{1}{a_{12} b_1} \dot{s} + k_3 \int s dt = \tilde{l}_1 x_1 + \tilde{l}_2 x_2 + \tilde{l}_3 e - k_1 s - k_2 \operatorname{sgn}(s). \tag{23}$$

Then it also can be expressed as

$$\frac{1}{a_{12} b_1} \dot{s} s + k_3 s \int s dt = \tilde{l}_1 x_1 s + \tilde{l}_2 x_2 s + \tilde{l}_3 e s - k_1 s^2 - k_2 \operatorname{sgn}(s) s. \tag{24}$$

Design the adjusting law of unknown parameters as

$$\dot{\hat{l}}_1 = \Gamma_1 x_1 s, \tag{25}$$

$$\dot{\hat{l}}_2 = \Gamma_2 x_2 s, \tag{26}$$

$$\dot{\hat{l}}_3 = \Gamma_3 e s. \tag{27}$$

Choose a Lyapunov function as

$$V_1 = \sum_{i=1}^{3} \frac{1}{2\Gamma_i} \tilde{l}_i^2. \tag{28}$$

Solve its derivative as

$$\dot{V}_1 = -\tilde{l}_1 x_1 s - \tilde{l}_2 x_2 s - \tilde{l}_3 e s. \tag{29}$$

Choose another Lyapunov function as

$$V_2 = \frac{1}{2a_{12} b_1} s^2 + \frac{k_3}{2} \left(\int s dt \right)^2. \tag{30}$$

Then its derivative can be commuted as

$$\dot{V}_2 = \frac{1}{a_{12} b_1} \dot{s} s + k_3 s \int s dt = \tilde{l}_1 x_1 s + \tilde{l}_2 x_2 s + \tilde{l}_3 e s - k_1 s^2 - k_2 \operatorname{sgn}(s) s. \tag{31}$$

Choose a big Lyapunov function for the whole system as

$$V = V_1 + V_2. \tag{32}$$

Solve its derivative as

$$\dot{V} = -k_1 s^2 - k_2 \operatorname{sgn}(s) s \le 0. \tag{33}$$

So the system is stable with the proposed adaptive sliding mode control law, and what is worthy pointing out is that the gain of the system is not necessary to be very big, which is very different from other backstepping methods.

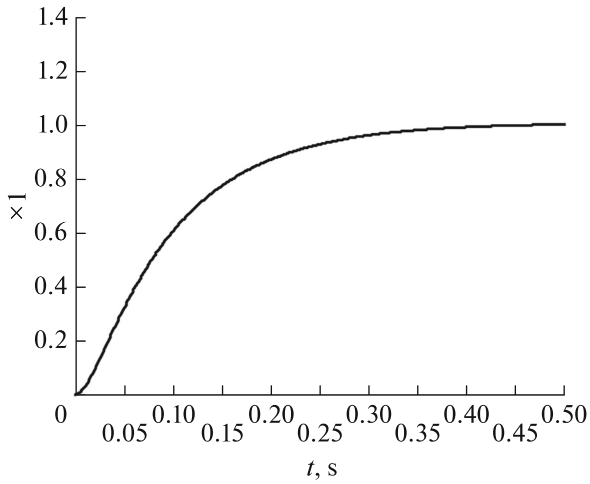

Fig. 1. The curve of state ×1.

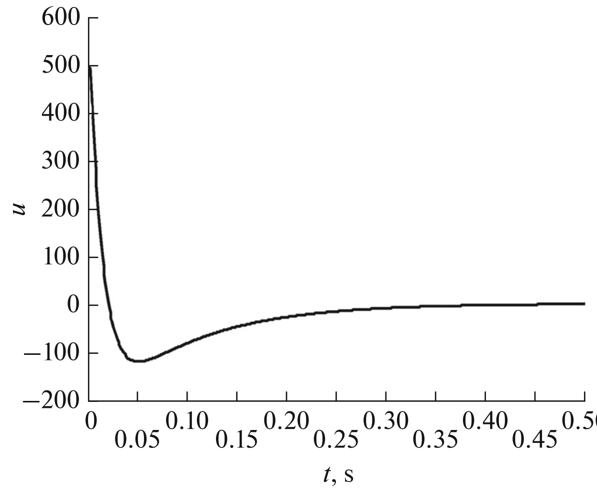

Fig. 2. The curve of control u.

Note 1: The design of adaptive sliding mode control law for system with uncertain control coefficient situation is almost the same as the situation of systems with unknown control direction, but there is one control item has different coefficient.

7. NUMERICAL SIMULATION

The simulation was done with the below second order system with single unknown control coefficient:

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2.$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + b_1u,$$

where unknown parameters are set as $a_{11} = 0.3$, $a_{12} = 0.7$, $a_{21} = 5.3$, $a_{22} = 15.2$, $b_1 = 2$, and the control law is designed as

$$u = -\hat{l}_1x_1 - \hat{l}_2x_2 - \hat{l}_3e - k_1s - k_2\,\mathrm{sgn}(s) - k_3\int sdt,$$

where $c_1 = 10$, $c_2 = 0.2$:

$$s = \dot{e} + c_1e + c_2\int edt,$$

$$e = x_1 - x_1^d.$$

Choose $\tau_1 = 0.1$, $\tau_2 = 0.1$, $\tau_3 = 0.1$:

$$\dot{\hat{l}}_1 = \Gamma_1x_1s,\quad \dot{\hat{l}}_2 = \Gamma_2x_2s,\quad \dot{\hat{l}}_3 = \Gamma_3es.$$

Assume all initial states are, and choose the expect value as $x_1^d = 1$, and set parameters for controller as

$$k_1 = 50,\quad k_2 = 1,\quad k_3 = 0.5.$$

The simulation results can be shown as following figures: Fig. 1 and Fig. 2.

Choose ten groups of random parameter to do the numerical simulation, and results can see the below Figs. 3, 4.

According to the above simulation result, the proposed method achieved good control performance, especially; the output is not affected by the change of random model parameters greatly. So it means that the control law is effective and has strong control ability for uncertainties, which is the main advantage of sliding mode control.

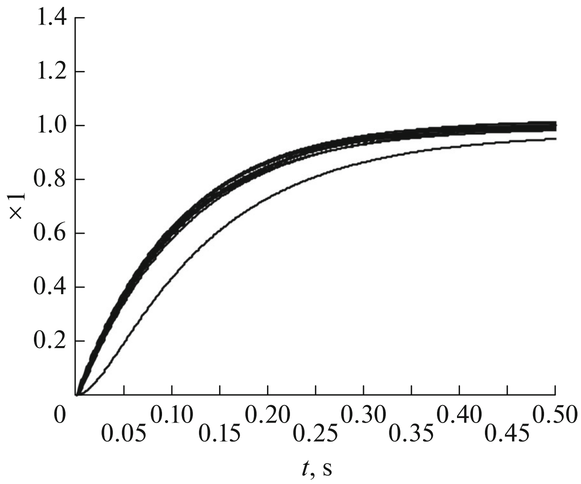

Fig. 3. The curve of state ×1.

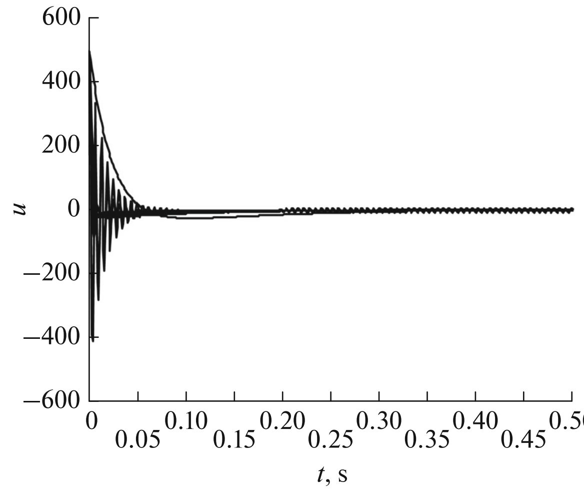

Fig. 4. The curve of control u.

8. SYNCHRONOUS CONTROL LAW DESIGN FOR ERROR SYSTEM

According to above control analysis, we first design the synchronous law for the first subsystem of error system with consideration of f_2. Define $z_{e1} = e_1 - e_1^d$, where $e_1^d = 0$, then design the sliding mode surface s_e is defined as

$$s_e = \dot{z}_{e1} + c_1 z_{e1} + c_2 \int z_{e1} dt. \tag{34}$$

Then the synchronous law can be designed as

$$u_1 = -\hat{l}_1 e_1 - \hat{l}_2 e_2 - \hat{l}_3 z_{e1} - k_1 s_e - k_2 \operatorname{sgn}(s_e) - k_3 \int s_e dt, \tag{35}$$

where

$$\dot{\hat{l}}_1 = \Gamma_1 e_1 s_e, \quad \dot{\hat{l}}_2 = \Gamma_2 e_2 s_e, \quad \dot{\hat{l}}_3 = \Gamma_3 z_{e1} s_e.$$

For the second subsystem of error system, we can design a simple feedback control law to realize the synchronization since it is a simple first order system. The feedback control law can be written as

$$u_2 = -k_{p3} e_3 - k_{s3} \int e_3 dt - k_{ps3} \frac{e_3}{|e_3| + \varepsilon_3} - f_3. \tag{36}$$

Then

$$\dot{e}_3 = -(k_{p3} + b) e_3 - k_{s3} \int e_3 dt - k_{ps3} \frac{e_3}{|e_3| + \varepsilon_3}. \tag{37}$$

So

$$e_3 \dot{e}_3 = -(k_{p3} + b) e_3 e_3 - k_{s3} e_3 \int e_3 dt - k_{ps3} \frac{e_3 e_3}{|e_3| + \varepsilon_3}, \tag{38}$$

and if we choose a Lyapunov function as

$$V = \frac{1}{2} e_3^2 + \frac{k_{s3}}{2} \left(\int e_3 dt \right)^2, \tag{39}$$

then solve its derivative, and it can be written as

$$\dot{V} = -(k_{p3} + b) e_3 e_3 - k_{ps3} \frac{e_3 e_3}{|e_3| + \varepsilon_3} \le 0. \tag{40}$$

So according to Lyapunov stability theorem, it is easy to prove that $e_3 \to 0$, so it means that the synchronization can be realized.

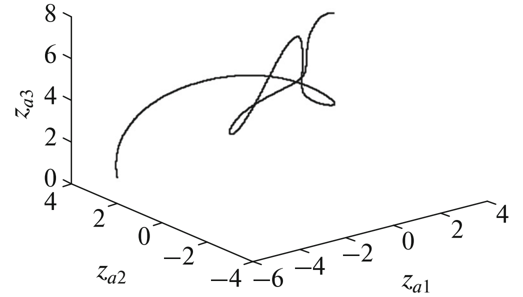

Fig. 5. The free movement of chaotic system with initial value 0.5, 0.5, 0.6.

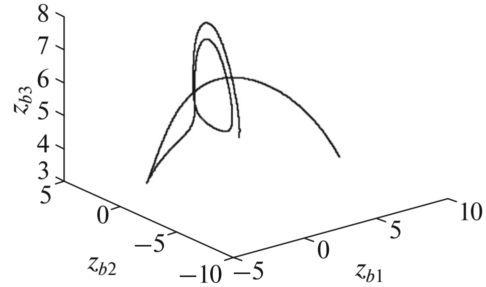

Fig. 6. The free movement of chaotic system with initial value 0.6, 0.3, 0.9.

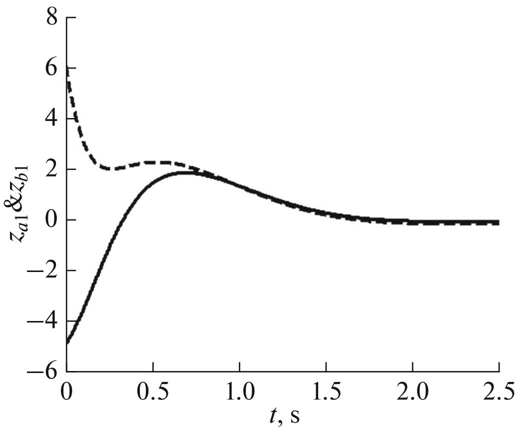

Fig. 7. The synchronous curve of z_{a1} and z_{b1}.

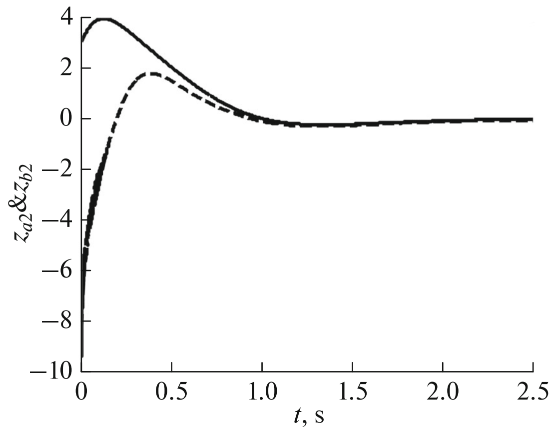

Fig. 8. The synchronous curve of z_{a2} and z_{b2}.

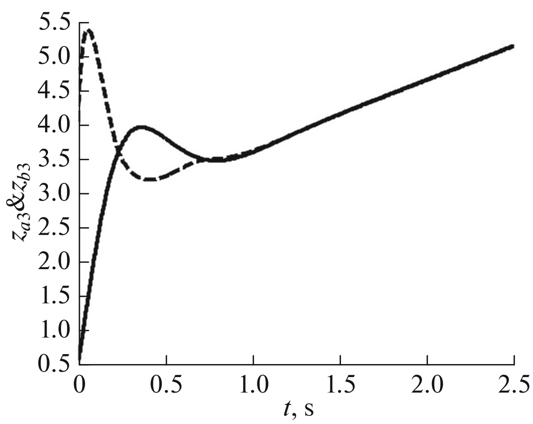

Fig. 9. The synchronous curve of z_{a3} and z_{b3}.

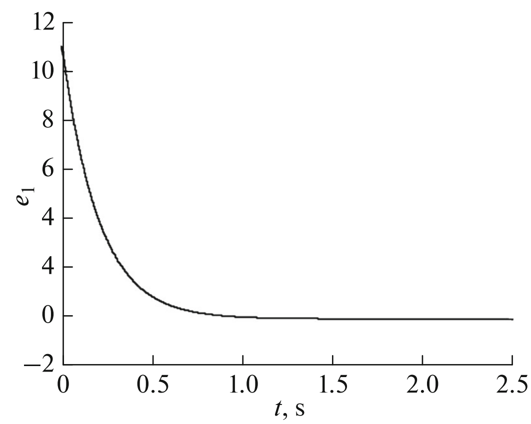

Fig. 10. The synchronous error e_1.

9. SIMULATION RESULT OF SYNCHRONIZATION OF TWO CHAOTIC SYSTEMS

To show the free movement of Rossler system, we use Matlab language to write a program to do the simulation with the proposed controller design method, and the simulation result is as bellows. The below Figs. 5 and 6 shows the free movement of chaotic system with different initial values by running above program.

By using the synchronous method proposed in section 6, we wrote a Matlab program to do the numerical simulation and set $b_1 = 1$, the simulation result of synchronization can see Figs. 7, 8 and 9. The synchronous error between driven chaotic system and response chaotic system was shown in Figs. 10, 11 and 12,

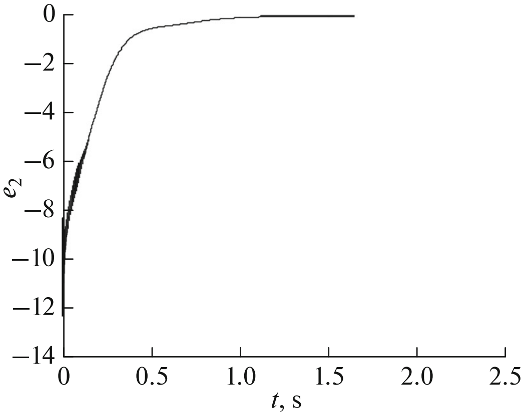

Fig. 11. The synchronous error e_2.

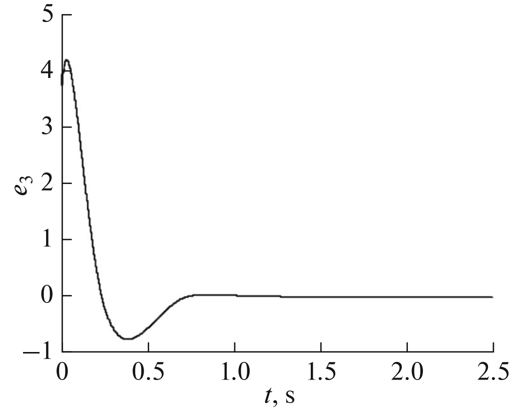

Fig. 12. The synchronous error e_3.

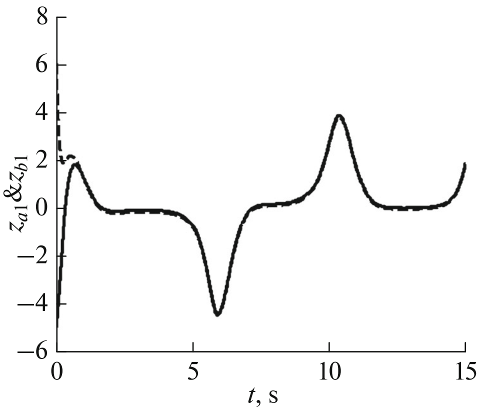

Fig. 13. The synchronous curve of z_{a1} and z_{b1}.

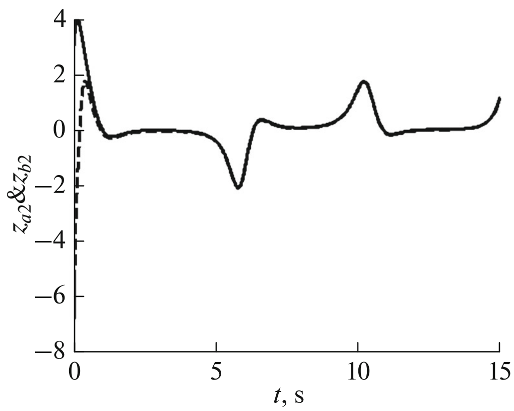

Fig. 14. The synchronous curve of z_{a2} and z_{b2}.

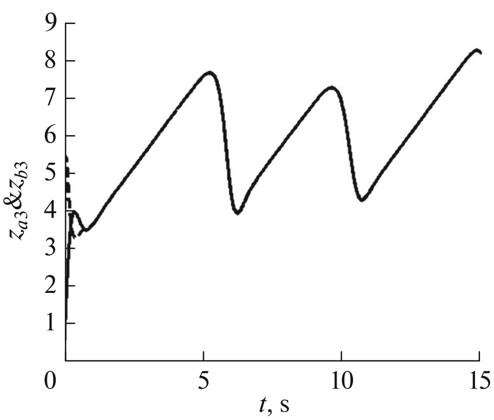

Fig. 15. The synchronous curve of z_{a3} and z_{b3}.

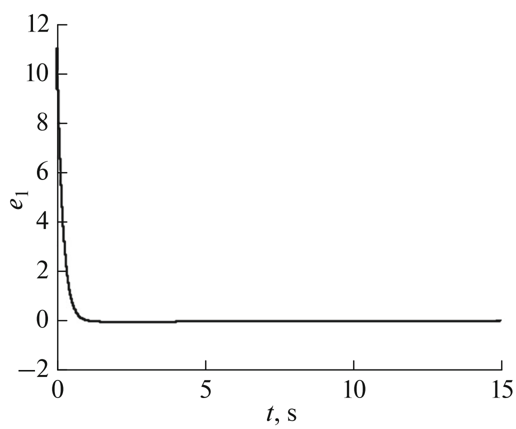

Fig. 16. The synchronous error e_1.

and it is clear that only two active inputs realized synchronization of three chaotic states. So we can make a conclusion that the proposed method is effective.

If we change $b_1 = 0.5$, the synchronization can also be realized, and simulation result can see below Figs. 13–18. We can find from simulation figures that the synchronization error is obviously increased as input coefficient decrease and the convergence of synchronous error is also become slower than previous

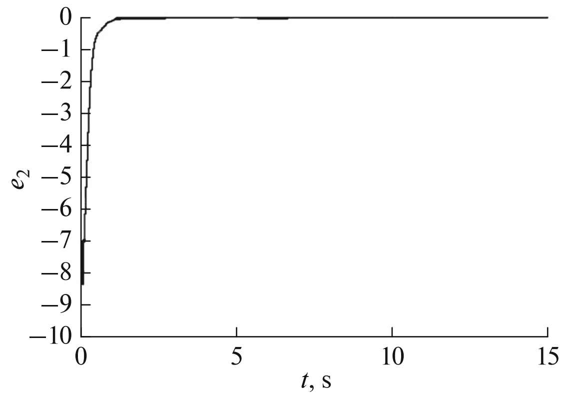

Fig. 17. The synchronous error e_2.

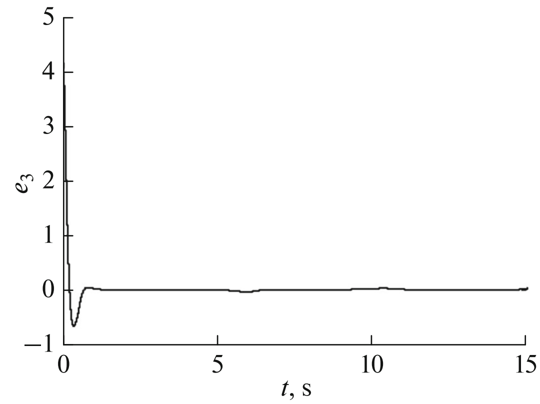

Fig. 18. The synchronous error e_3.

one. It means the accuracy of synchronization is related with the input coefficient and control gain. So we can choose a proper gain to control the accuracy of synchronization such that it can satisfy the requirement.

10. CONCLUSION

A new synchronization controller for Rikitake chaotic systems with reduced number of active inputs was designed by transforming it to be a control problem of special kind of second order systems with single control coefficient. It is obvious that the constructing of sliding mode control law for single unknown constant control coefficient situation can be the same as the situation with a known control coefficient. So the sliding mode control has very good consistency, which is very different from backstepping control method. In addition, the adopting of adaptive control item make the sliding mode control method is not necessary to use a high gain, but according to the simulation result of random parameters, it is still need to choose a big enough gain to fit all random situations, which will make the control law to be universal to cope with all kinds of disturbance or uncertainties. Generally speaking, control gain still needs to be big enough for sliding mode control to cope with all kinds of uncertainties. This conclusion was also testified in the final detailed simulation of second order system and complex synchronization of chaotic systems.

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