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Backstepping Control with Speed Estimation of PMSM Based on MRAS[1]

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Abstract—This paper presents a backstepping control method with speed estimation of permanent magnet synchronous motor (PMSM) based on model reference adaptive system (MRAS). First, a comprehensive dynamical model of PMSM in d–q axis and its space state equations are established. Next, using Lyapunov stability theorem, based on the backstepping control theory, the PMSM rotor speed and current backstepping controllers are designed. Furthermore, using Popov stability theory, based on MRAS, the PMSM rotor speed observer is designed. Finally, Matlab/Simulink simulation results show that the backstepping control and speed observer are effective and feasible.

Keywords: permanent magnet synchronous motor, model reference adaptive system, backstepping control, speed observer, stability

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1. INTRODUCTION

In recent years, with the development of power electronic technology, AC servo system has been paid more and more attention and research. The PMSM has the advantages of small size, high efficiency and high power density, which plays an important role in AC servo system and which has been widely used in high performance drive system [1]. At present, field oriented control (FOC) or direct torque control (DTC) is usually used in the drive of PMSM for control strategy. However, no matter what kind of control strategy, the rotor speed and position angle are required. In a sensor scheme, it can directly obtain the position information through installing encoder or hall sensor. But this scheme will undoubtedly increase the cost of the design of the system [2], and the adaptability is also relatively weak. In recent years, sensorless technology of PMSM is developed increasingly. The first proposed and most simple method is back electromotive force [3], but at the low speed, the back electromotive force is small and the accuracy is not high. Another more mature method is signal injection [4, 5], this approach relies on external incentives and also requires the motor itself has a convex polarity, it doesn't apply at high speed. With the development of modern control theory, sliding mode observer, MRAS, Kalman filter, and other sensorless schemes are also developed. These methods are more and more concerned by researchers for their good robustness.

MRAS has the advantages of simple algorithm, easy to implement in the digital control system, and has the advantages of faster adaptive speed. It has been proposed and applied to the PMSM sensorless control [6]. There are two different models. One is the reference model and another is the adjustable model including the rotor speed. The deviation signal of the output of the two models send to the adaptive mechanism, and then the output of the adaptive mechanism is the rotor speed [7]. In [8–10], the basic theoretical knowledge of MRAS theory for estimating rotor speed was given, and the d–q axis equations of stator current for adjustable model was described. The adaptive law was constructed by Popov stability theory. These establish the theoretical foundation for this paper.

Backstepping control is a new type recursive and systematic design methodology for the feedback control of uncertain nonlinear system, particularly for the system with matched uncertainties [11]. First, we need to establish a comprehensive dynamical model of PMSM and the space state equations. Then, using Lyapunov stability theorem can design the speed and current tracking backstepping controllers [12]. In [13–15], the backstepping control theory was explained in detail. In [13], it designed a speed observer

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based on slide model observer and phase loop lock (PLL) was designed, however it was more complex than MRAS. In [15], it also designed a speed observer through Luenberger observer. But it was still no more simple than MRAS.

In this paper, we mainly investigate backstepping sensorless speed controller for PMSM based on MRAS. First, we design the rotor speed and current backstepping controllers using Lyapunov stability theorem and backstepping control theory after presenting the mathematical model of PMSM. Then, in order to save system cost, we also design a rotor speed observer using MRAS and Popov stability theory. Finally, simulation results show that the backstepping sensorless speed control method is effective and feasible.

The rest of this paper is organized as follows. In Sections 2, the mathematical model and the backstepping controller for PMSM are presented. The rotor speed observer of PMSM is designed by MRAS in Section 3. Section 4 presents simulation parameters, conditions and results. Finally, some conclusions are drawn in Section 5.

2. BACKSTEPPING CONTROLLER DESIGN

2.1. Dynamic Model of PMSM

The mathematical model of a surface mounted PMSM can be given in d–q axis as follows:

$$\begin{bmatrix} \dot{\omega}_r \\ \dot{i}_q \\ \dot{i}_d \end{bmatrix} = \begin{bmatrix} -\frac{B}{J} & \frac{3p^2\psi_r}{2J} & 0 \\ -\frac{\psi_r}{L} & -\frac{R}{L} & -\omega_r \\ 0 & \omega_r & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \omega_r \\ i_q \\ i_d \end{bmatrix} + \begin{bmatrix} -\frac{T_L}{J}p \\ \frac{u_q}{L} \\ \frac{u_d}{L} \end{bmatrix}, \tag{1}$$

where R is the stator resistance, L is the d–q axis inductances, u_d, u_q, i_d, i_q are the stator voltages and currents of the motor in the d–q axis. p is the number of pole pairs of the PMSM, ω_r and ψ_r are rotor electric angular speed and flux. T_L is the load torque, J is the rotor inertia, and B is the viscous friction coefficient.

2.2. Speed Controller Design

The main control goal is to ensure global asymptotic convergence of the speed and current tracking errors to zero.

Backstepping control is an efficient method for nonlinear system. In the backstepping procedure, the first step is to define a virtual control state and then it is forced to become a stabilizing function. Consequently, by appropriately designing the related control input on the basis of Lyapunov stability theory, the error variable can be stabilized. Based on the backstepping design principle, the overall controller design can be established.

The controller for the rotor speed can be designed in three steps through backstepping control theory.

Step 1: To solve speed-tracking problem, the speed error variable can be defined as

$$e_1 = \omega_r^* - \omega_r, \tag{2}$$

where ω_r^* is the reference rotor electric angular speed and ω_r is the actual electric angular speed. For stabilizing the speed component, the speed tracking error dynamics derived from (1) and (2) can be obtained as

$$\dot{e}_1 = \dot{\omega}_r^* - \dot{\omega}_r = \frac{B}{J}\omega_r - \frac{3p^2\psi_r}{2J}i_q + \frac{T_L}{J}p. \tag{3}$$

Step 2: Choose the following candidate Lyapunov function as

$$V_1 = \frac{1}{2}e_1^2. \tag{4}$$

Step 3: The time derivative of (7) can be obtained as

$$\dot{V}_1 = e_1\dot{e}_1 = e_1\left(\frac{B}{J}\omega_r - \frac{3p^2\psi_r}{2J}i_q + \frac{T_L}{J}p\right). \tag{5}$$

According to Lyapunov stability definition, in order to allow the error to converge to zero, we need let $\dot{V}_1 < 0$. So, we can define as

$$\frac{B}{J}\omega_r - \frac{3p^2\psi_r}{2J}i_q + \frac{T_L}{J}p = -k_1 e_1, \quad k_1 > 0. \tag{6}$$

Therefore, when (6) is satisfied, the speed error approaches zero. Then we can earn as

$$i_q = \frac{2J}{3p^2\psi_r}\left(\frac{B}{J}\omega_r + \frac{T_L}{J}p + ke_1\right). \tag{7}$$

PMSM is a complicated high-order, nonlinear system with multiple variables and strong coupling characteristics. The exact thrust force to drive the motor is determined by the q-axis current. Moreover, it is advisable to make the d-axis current be zero according to FOC. For traditional FOC control, through the PI controller to adjust the speed and current, the output of the speed loop is the reference q-axis current. In addition, the output of the d–q axis currents loop is the reference d–q axis voltages. These can be shown in Fig. 4. So, we can earn the reference d–q axis currents as

$$i_q^* = \frac{2J}{3p^2\psi_r}\left(\frac{B}{J}\omega_r + \frac{T_L}{J}p + ke_1\right), \quad i_q^* = 0. \tag{8}$$

2.3. Current Controller Design

As same speed control, the controller for the d–q axis currents also can be designed in three steps. For q-axis current controller, it is described as follows.

Step 1: To solve q-axis current tracking problem, the q axis current error variable can be defined as

$$e_2 = i_q^* - i_q. \tag{9}$$

For stabilizing the q-axis current component, the q-axis current tracking error dynamics derived from (1), (8) and (9) can be obtained as

$$\dot{e}_2 = \dot{i}_q^* - \dot{i}_q = \frac{2}{3p^2\psi_r}\left(\frac{B}{J} - k_1\right)\left(\frac{3p^2\psi_r}{2}i_q - B\omega_r - T_L p\right) + \frac{R}{L}i_q + \left(i_d + \frac{\psi_r}{L}\right)\omega_r - \frac{u_q}{L}. \tag{10}$$

Step 2: Choose the following candidate Lyapunov function as

$$V_2 = \frac{1}{2}e_2^2. \tag{11}$$

Step 3: The time derivative of (11) can be obtained as

$$\dot{V}_2 = e_2\dot{e}_2 = e_2\left(\frac{2}{3p^2\psi_r}\left(\frac{B}{J} - k_1\right)\left(\frac{3p^2\psi_r}{2}i_q - B\omega_r - T_L p\right) + \frac{R}{L}i_q + \left(i_d + \frac{\psi_r}{L}\right)\omega_r - \frac{u_q}{L}\right). \tag{12}$$

As the same, we also need let $\dot{V}_2 < 0$. So, we can define as

$$\frac{2}{3p^2\psi_r}\left(\frac{B}{J} - k_1\right)\left(\frac{3p^2\psi_r}{2}i_q - B\omega_r - T_L p\right) + \frac{R}{L}i_q + \left(i_d + \frac{\psi_r}{L}\right)\omega_r - \frac{u_q}{L} = -k_2 e_2, \quad k_2 > 0. \tag{13}$$

Therefore, when (13) is satisfied, the global asymptotic tracking of q-axis current can be achieved. Then we can earn as

$$u_q = L\left(\frac{2}{3p^2\psi_r}\left(\frac{B}{J} - k_1\right)\left(\frac{3p^2\psi_r}{2}i_q - B\omega_r - T_L p\right) + \frac{R}{L}i_q + \left(i_d + \frac{\psi_r}{L}\right)\omega_r + k_2 e_2\right). \tag{14}$$

Similarly, the d-axis current controller can be designed by the following steps.

Step 1: the d-axis current error variable can be defined as

$$e_3 = i_d^* - i_d. \tag{15}$$

Also, the d-axis current tracking error dynamics derived from (1), (8) and (15) can be obtained as

$$\dot{e}_3 = \dot{i}_d^* - \dot{i}_d = \frac{R}{L}i_d - i_q\omega_r - \frac{u_d}{L}. \tag{16}$$

Step 2: Choose the following candidate Lyapunov function as

$$V_3 = \frac{1}{2}e_3^2. \tag{17}$$

Step 3: The time derivative of (17) can be obtained as

$$\dot{V}_2 = e_2\dot{e}_2 = e_2\left(\frac{R}{L}i_d - i_q\omega_r - \frac{u_d}{L}\right). \tag{18}$$

As the same, we let $\dot{V}_2 < 0$, likewise. So, we can define as

$$\frac{R}{L}i_d - i_q\omega_r - \frac{u_d}{L} = -k_3e_3, \quad k_3 > 0. \tag{19}$$

Therefore, when (19) is satisfied, the global asymptotic tracking of the d-axis current can be achieved. Then we can earn as

$$u_d = L\left(\frac{R}{L}i_d - i_q\omega_r + k_3e_3\right). \tag{20}$$

So, we can use equation (14) and (20) as reference d–q axis voltages. The objective of backstepping control for PMSM is completed.

3. DESIGN OF SPEED OBSERVER

The MRAS estimators are designed to estimate the stator d–q axis currents and the rotor speed. The main idea behind MRAS method is that there is a reference model and an adjustable model. The mathematical model of d–q axis currents of PMSM as reference model can be given as

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_q \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega_r \\ -\omega_r & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{u_d}{L} \\ \frac{u_q - \psi_r\omega_r}{L} \end{bmatrix}. \tag{21}$$

According to MRAS, the mathematical model of d–q axis currents of PMSM as adjustable model can be given as

$$\begin{bmatrix} \dot{\hat{i}}_d \\ \dot{\hat{i}}_q \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \hat{\omega}_r \\ -\hat{\omega}_r & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} \hat{i}_d \\ \hat{i}_q \end{bmatrix} + \begin{bmatrix} \frac{u_d}{L} \\ \frac{u_q - \psi_r\hat{\omega}_r}{L} \end{bmatrix}. \tag{22}$$

After building adjustable and reference models, the adaptive mechanism will be built for MRAS method. Under ideal conditions, the error between the two models is zero. So, defined state error as

$$\varepsilon_d = i_d - \hat{i}_d, \quad \varepsilon_q = i_q - \hat{i}_q. \tag{23}$$

The state error equation of equation (21) subtracting equation (22) can be given as

$$\begin{bmatrix} \dot{\varepsilon}_d \\ \dot{\varepsilon}_q \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \hat{\omega}_r \\ -\hat{\omega}_r & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} \varepsilon_d \\ \varepsilon_q \end{bmatrix} + \begin{bmatrix} i_q \\ -i_d - \frac{\psi_r}{L} \end{bmatrix}\left(\omega_r - \hat{\omega}_r\right). \tag{24}$$

Therefore, the state error model of PMSM is given as follow

$$\dot{\varepsilon} = A\varepsilon + Bu. \tag{25}$$

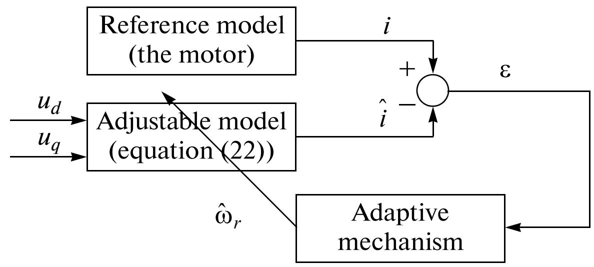

Fig. 1. MRAS basic block diagram of speed estimation.

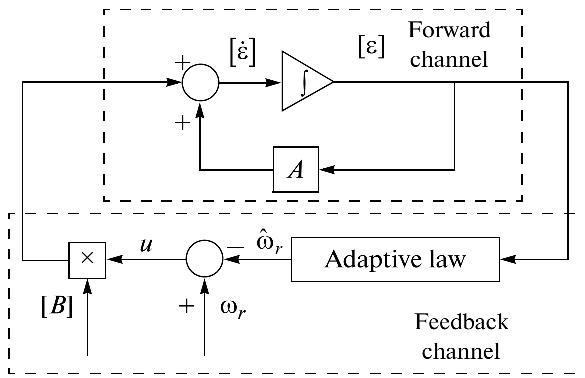

Fig. 2. Equation (22) structure diagram.

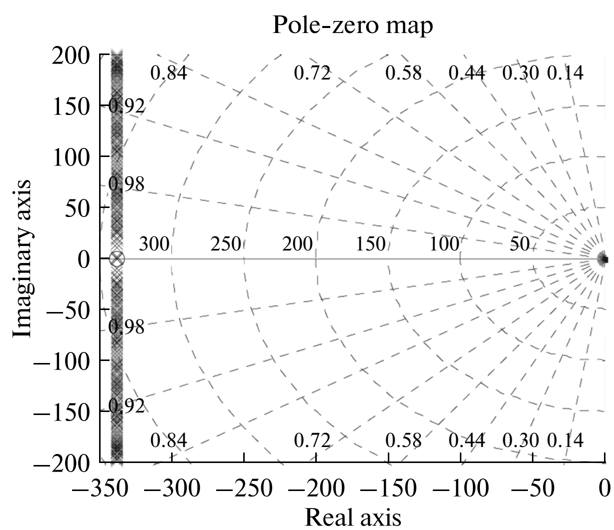

Fig. 3. The pole-zero loci of $H(s)$ about range of $\hat{\omega}_r$ starting at −400 up to 400 rad/s.

The MRAS basic block diagram of speed estimation is shown in Fig. 1.

Obviously, the stability and precision of the system is related to the construction of the adaptive mechanism. From Fig. 1, it can be seen that the adaptive mechanism is related to the state error equation (24). The structural diagram of the equation (24) is shown in Fig. 2.

For speed estimation, the adaptive law is constructed by Popov stability. So, the conditions about the stability of Fig. 2 are about two sides. One is the zero pole of the transfer function of the forward channel in the left half of the s domain. Another is feedback channel to satisfy Popov stability.

PMSM parameters

U_{dc}	p	L	R	ψ_r	B	J
310 V	2	8.5 mH	2.8758 Ohm	0.175 V s	1.349e-5 N m s	8e-4 kg m^2

For first condition, the transfer function of forward channel is given as

$$H(s) = \frac{s + \frac{R_s}{L_s}}{s^2 + 2\frac{R_s}{L_s}s + \left(\frac{R_s}{L_s}\right)^2 + \hat{\omega}_r^2}. \tag{26}$$

Its pole-zero loci is shown in Fig. 3 for a range of $\hat{\omega}_r$ starting at –400 up to 400 rad/s. From the graph, we can see that with the increase of the speed, the poles also have negative real parts. So this condition is confirmed.

For second condition, the derivation of Popov stability is introduced in above literature, and it's not described in this article. The adaptive law can be constructed by Popov stability.

$$\hat{\omega} = K_i \int\left(-\varepsilon_q i_d - \varepsilon_q \frac{\psi_r}{L_s}\right)dt + K_p\left(\varepsilon_d i_q - \varepsilon_q i_d - \varepsilon_q \frac{\psi_r}{L_s}\right) + \hat{\omega}_r(0)\Bigg). \tag{27}$$

4. SIMULATION RESULTS

Simulation works are carried out on the PMSM to prove the effectiveness of the proposed scheme. The simulations are performed using the Matlab/simulink simulation package. The structure diagram of the whole system is shown in Fig. 4. The PMSM parameters involved in this paper are shown in table.

The initial rotor speed (mechanical angular speed) of the PMSM is 10 rad/s, and the rotor speed is 200 rad/s at 0.6 s. The initial load torque of the motor is 1 N m and the load torque is 10 N m at 0.3 s.

The parameters of the backstepping controller and MRAS are selected as $k_1 = 3000$, $k_2 = 7000$, $k_3 = 9000$, $k_p = 10$, $k_i = 1000$.

Figure 5 indicates the given rotor speed and the actual rotor speed. From Fig. 5 (left), it can be seen that the rotor speed can rapidly track the given rotor speed with small stability error and fast response. Also, Fig. 5 (right) shows that the estimated speed also have small stability error and fast response. Figure 6 shows the variation of electromagnetic torque as the load torque changes, and it can be seen that the PMSM has fast torque response. Figure 7 shows the wave of stator currents in d–q axis, which satisfies the

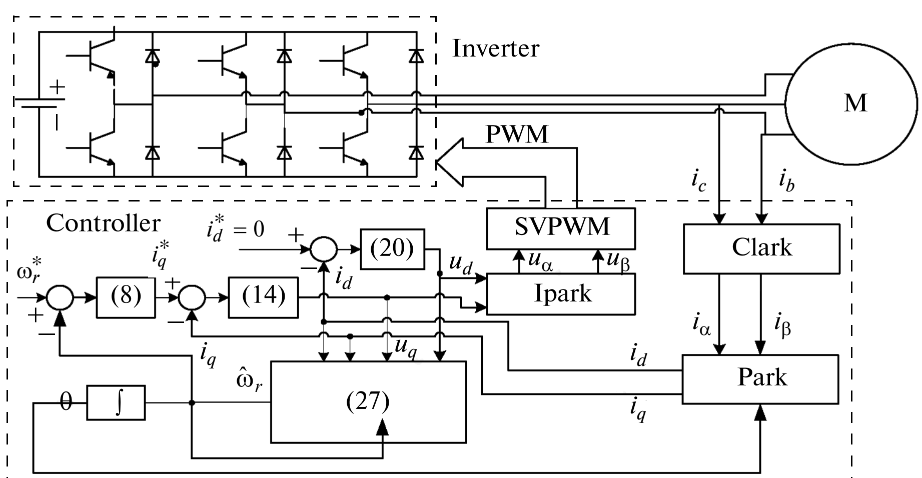

Fig. 4. Structure diagram of the whole system.

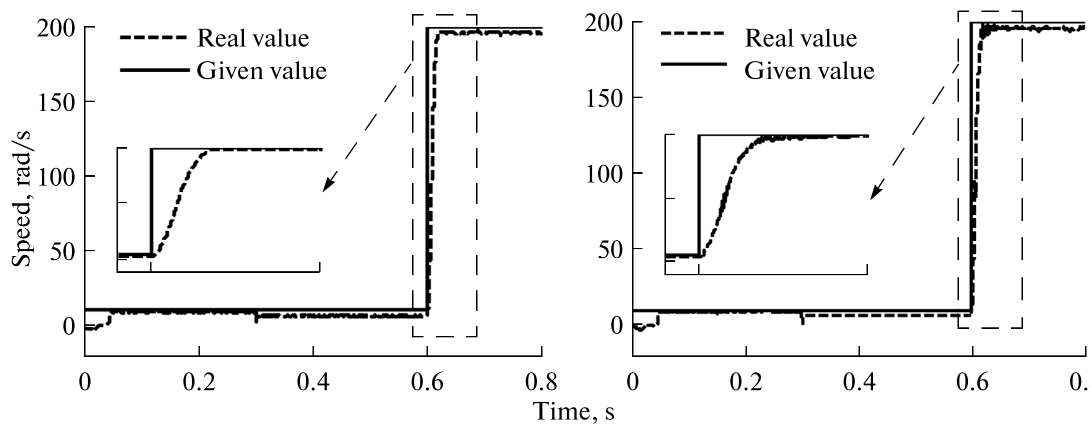

Fig. 5. The speed response under sensor (left) and sensorless (right).

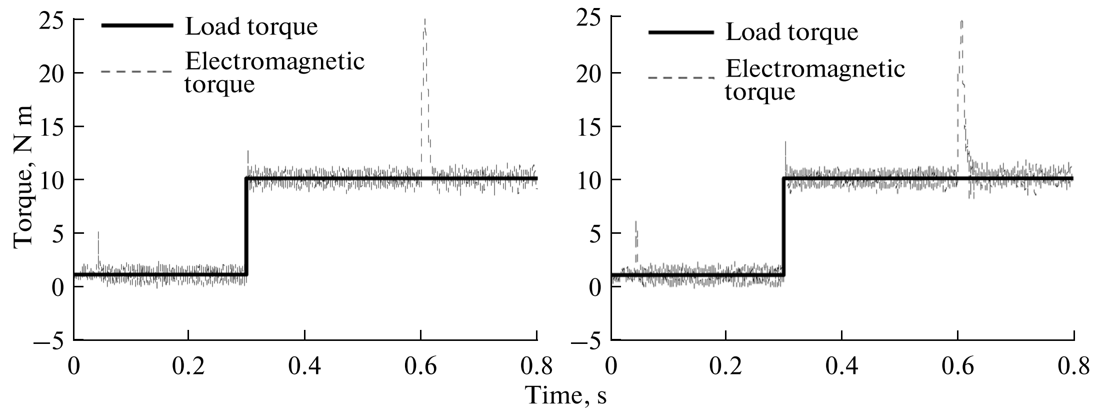

Fig. 6. The torque under sensor (left) and sensorless (right).

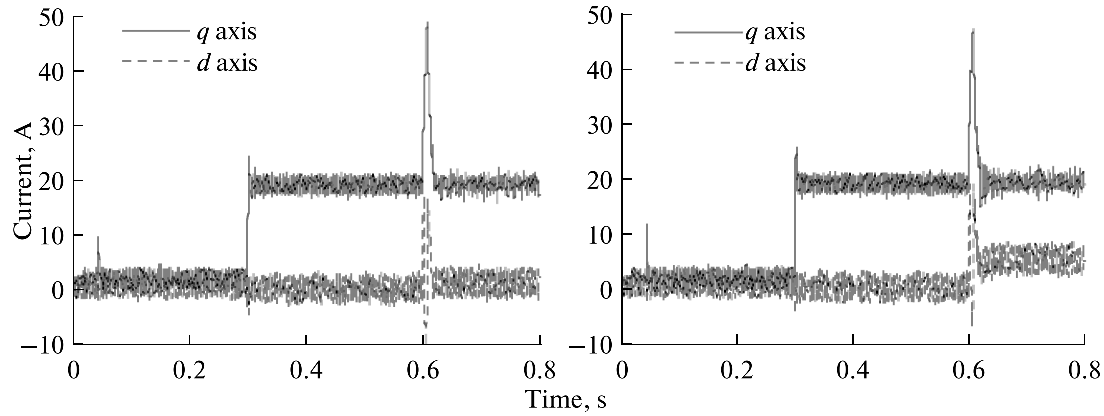

Fig. 7. The d–q axis current under sensor (left) and sensorless (right).

control scheme. Figure 8 shows the A and B phase currents of the stator. The current amplitude is proportional to the rotor speed and changes rapidly as the load torque varies.

Through the above diagram can be seen, the speed, current and torque response of PMSM are similar under sensorless and sensor. This fully shows the effectiveness of the proposed speed observer, and this is more economical than sensor.

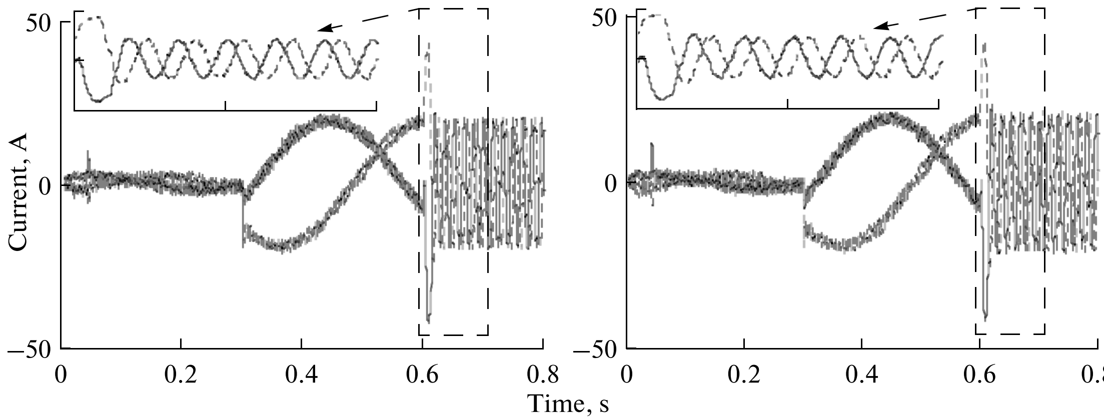

Fig. 8. A and B phase winding currents under sensor (left) and sensorless (right).

5. CONCLUSION

This paper proposes a backstepping control method with speed estimation of (PMSM) based on (MRAS). An efficient backstepping controller is first presented. Then, applying MRAS theory, the speed observer is designed. This Backstepping sensorless control is very convenient to be implemented in practice. Finally, simulation results verify the validity and feasibility of this approach.

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