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# Reliability of Supply Chains in a Random Environment[1]

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**Abstract**—A supply chain is considered that operates in a random environment.The last is described by continuous-time finite irreducible Markov chain. Each state of the environment (the Markov chain) has its own probability of fatal failure as a result of the breakdown of the supply. There are algorithms for calculating the reliability of the chain (a probability of a successful supply) and the distribution function of the successful performance time presented in this paper. The numerical example illustrates the suggested approach.

*Keywords*: Markov chain, nonstationary probabilities of states, failure intensity, polynomial approximation

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## 1. INTRODUCTION

Many technological processes contain sequentially performed operations. Supply chains are an example of this kind, and they have been the subjects of many investigations [2, 6]. This interpretation will be used for visualization in this paper.

A supply chain consists of sequential links of supply, which will be called a stage. Let us denote $c$ as the number of stages (links). The stages are performed sequentially; a new stage begins after the successful ending of the previous stage. Let $W_\eta$ be the time of the $\eta$th stage, $\eta = 1, ..., c$, $f_\eta(t)$, $F_\eta(t)$, $t \geq 0$, the corresponding density and cumulative distribution functions. It is supposed that the $\eta$th distribution concentrates on the finite interval $(0, b_\eta)$. The random variables $W_1, W_2, \ldots, W_n$ are independent.

The supply chain operates in the so-called external random environment, which is described [3] by the continuous-time Markov chain $J(t)$, $t > 0$, with the finite space of states $E = \{1, 2, \ldots, k\}$. Let $\lambda_{i,j}$ be the transition intensity from state $i$ to state $j$, $i, j \in E$, $\lambda_{i,i} = 0$. We denote a corresponding matrix by $(\lambda_{i,j})$ and transition intensity from state $i$ by $\Lambda_i = \sum_{j \in E} \lambda_{i,j}$.

This is the modulating process [4] for the random sequence $W_1, W_2, \ldots, W_n$. If the external environment $J(t)$ is in the state $i$ and the $\eta$th link is performed, then a fatal failure arises with the intensity $\gamma_{i,\eta}(t)$. The supply chain performs successfully if all $c$ links are performed without failures. It is necessary to calculate the corresponding probability, i.e., the probability of successful performance of all stages, and a distribution of the corresponding time.

The paper is organized as follows. We derive a system of differential equations for transition probabilities of chain states in the next section. Section no. 3 contains a procedure of a polynomial approximation for these probabilities. Survival function of one link is considered in Section no. 4. The chain reliability and performance time distribution are considered in Sections 5 and 6. Section no. 7 contains a numerical example. Section no. 8 ends the paper with final remarks.

## 2. DIFFERENTIAL EQUATIONS FOR STATE PROBABILITIES

First, we consider only one link that allows us to omit index $\eta$ in failure intensity $\gamma_{i,\eta}(t)$. Only two states take place, i.e., the working link and disturbed (absorbing) link. Let $A(t)$ be an event that the link continues to work (survive) at time moment $t$. Since early $\Lambda_i$ is the failure intensity at the time $t$ for state $i \in E$ of modulating process $J(t)$. We denote $P_{i,j}(t), i, j \in E$, the following probability:

$$P_{i,j}(t) = P\{A(t), J(t) = j \mid A(0), J(0) = i\}, \quad i, j \in E = \{1, 2, \ldots, k\}, \quad t > 0.$$

[1] The article was translated by the authors.

Let us calculate these probabilities. It allows us to calculate the probability that there is no failure until time moment $t$ as shown below:

$$S_i(t) = P\{A(t)|A(0), J(0) = i\} = \sum_{j=1}^{k} P_{i,j}\{t\}, \quad i, j \in E, \quad t > 0. \tag{1}$$

We have

$$P_{i,j}(t+\Delta) = P_{i,j}(t)(1 - (\Lambda_j + \gamma_j(t))\Delta) + \Delta \sum_{l \neq j} P_{i,l}(t)\lambda_{l,j}, \quad i, j \in E, \quad t \geq 0,$$

$$P_{i,j}(0) = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases}$$

It gives the following system of the linear differential equations:

$$\dot{P}_{i,j}(t) = -P_{i,j}(t)(\Lambda_j + \gamma_j(t)) + \sum_{l \neq j} P_{i,l}(t)\lambda_{l,j}, \quad i, j \in E, \quad t \geq 0. \tag{2}$$

The solution of this system will be performed via an approximation.

## 3. APPROXIMATION PROCEDURE

We use the following polynomial approximation [1] of the transition probabilities of interest:

$$P_{i,j}(t) = \sum_{\nu=0}^{n} z_{i,j,\nu} t^{\nu}, \quad i, j \in E, \quad t \geq 0. \tag{3}$$

Obviously,

$$z_{i,j,0} = \delta_{i,j}, \tag{4}$$

where $\delta_{i,j}$ is the Kroneker symbol: $\delta_{i,j} = 1$ for $i = j$ and equals 0 otherwise.

We suppose that the failure intensity also accepts a polynomial approximation as follows:

$$\gamma_j(t) = \sum_{\mu=0}^{m} w_{j,\mu} t^{\mu}, \quad j \in E, \quad t \geq 0. \tag{5}$$

The values $n$, $m < n$ and $\{w_{j,\mu}\}$ in (3) and (5) are known, but values $\{z_{i,j,\nu}\}$ must be determined.

The substitution (3) and (5) in (2) gives

$$\sum_{\nu=1}^{n} z_{i,j,\nu} \nu t^{\nu-1} = -\sum_{\nu=0}^{n} z_{i,j,\nu} t^{\nu} \left( \Lambda_j + \sum_{\mu=0}^{m} w_{j,\mu} t^{\mu} \right) + \sum_{l \neq j} \sum_{\nu=0}^{n} z_{i,l,\nu} t^{\nu} \lambda_{l,j}, \quad i, j \in E, \quad t \geq 0.$$

Temporally we omit the index of the initial state $i$, which gives the following system of linear algebraic equations with respect to $\{z_{j,\nu}\}$:

$$z_{j,\nu}\nu = -z_{j,\nu-1}\Lambda_j - \sum_{\mu=0}^{\min\{m,\nu-1\}} w_{j,\mu} z_{j,\nu-1-\mu} + \sum_{l \neq j} z_{l,\nu-1}\lambda_{l,j}, \quad i, j \in E, \quad \nu \in \{0, \ldots, n\}, \quad t \geq 0. \tag{6}$$

Let us introduce the following vector-matrix designations:

$$z^{(\nu)} = (z_{j,\nu})_{k \times 1}, \quad \nu = 0, \ldots, n, \quad \Lambda = (\Lambda_1 \ \Lambda_2 \ \ldots \ \Lambda_k).$$

Then, system (6) can be rewritten for initial state $i$ as

$$\nu z^{(\nu)} = -\mathrm{diag}(\Lambda) z^{(\nu-1)} - \sum_{\mu=0}^{\min\{m,\nu-1\}} \mathrm{diag}(w_{1,\mu}, w_{2,\mu}, \ldots, w_{k,\mu}) z^{(\nu-1-\mu)} + \lambda^T z^{(\nu-1)}, \quad \nu = 1, \ldots, n,$$

$$z^{(0)} = (1 \ 0 \ \ldots \ 0)^T.$$

Therefore,

$$z^{(1)} = -\mathrm{diag}(\Lambda)z^{(0)} - \mathrm{diag}(w_{1,0}, w_{2,0}, \ldots, w_{k,0})z^{(0)} + \lambda^T z^{(0)}$$
$$= -(\Lambda_1 + w_{1,0} \;\; 0 \; \ldots \; 0)^T + (\lambda_{1,1} \;\; \lambda_{1,2} \; \ldots \; \lambda_{1,k})^T = (\lambda_{1,1} - \Lambda_1 - w_0 \;\; \lambda_{1,2} \; \ldots \; \lambda_{1,k})^T,$$

and so on, recurrently.

The obtained solution allows one to calculate the probability of interest (3) for the partial ηth stage.

## 4. SURVIVAL FUNCTION OF THE STAGE

Now, we can consider a survival function (1) of the ηth link by the condition that the initial state $i$ of the external environment is fixed:

$$S_i^{(\eta)}(t) = \sum_{j=1}^{k} P_{i,j}^{(\eta)}(t), \quad i \in E, \quad \eta = 1, \ldots, c, \quad t > 0, \tag{7}$$

where $P_{i,j}^{(\eta)}(t), i, j \in E$, is calculated by formula (3) for the ηth link.

Let $S_{i,j}^{(\eta)}, i, j \in E$, be the probability that the ηth link will be ended successfully in the state $j$ of the external environment, given the initial state $i$. Obviously, if the performance time of the ηth link is constant $b_\eta$ then $S_{i,j}^{(\eta)} = S_{i,j}^{(\eta)}(b_\eta)$. If one is a random variable, then let $F_\eta(t)$, $t \geq 0$ be its cumulative distribution function. Both cases can be united rewritten using the Lebesque integral

$$S_{i,j}^{(\eta)} = \int_0^{b_\eta} P_{i,j}^{(\eta)}(t) dF_\eta(t), \quad t \geq 0. \tag{8}$$

If the approximation coefficients of formula (3) are calculated, then the last expression can be written as

$$S_{i,j}^{(\eta)} = \int_0^{b_\eta} \sum_{\nu=0}^{n} z_{i,j,\nu}^{(\eta)} t^\nu dF_\eta(t) = \sum_{\nu=0}^{n} z_{i,j,\nu}^{(\eta)} \int_0^{b_\eta} t^\nu dF_\eta(t) = \sum_{\nu=0}^{n} z_{i,j,\nu}^{(\eta)} \mu_\nu^{(\eta)}, \quad t \geq 0, \tag{9}$$

where $\mu_\nu^{(\eta)}$ is the νth initial moment for the performance time of the ηth link:

$$\mu_\nu^{(\eta)} = \int_0^{b_\eta} t^\nu dF_\eta(t). \tag{10}$$

## 5. RELIABILITY OF THE CHAIN

Now we need to calculate the reliability of the whole chain. Let $R_{i,j}^{(\eta)}, i, j \in E$, be the probability (reliability) that the first η links will be ended successfully and $j$ be the last state of the external environment given the initial state $i$. These probabilities are calculated sequentially:

$$R_{i,j}^{(1)} = S_{i,j}^{(1)},$$
$$R_{i,j}^{(\eta)} = \sum_{\zeta=1}^{k} R_{i,\zeta}^{(\eta-1)} S_{\zeta,j}^{(\eta)}, \quad \eta = 2, \ldots, c. \tag{11}$$

Obviously, the reliability of the whole chain is calculated as

$$R_i = \sum_{j=1}^{k} R_{i,j}^{(c)}. \tag{12}$$

The last formula allows one to calculate the probability of the successful performance of the supply chain without restrictions on the duration of the performance of the partial stages. Now, we consider a case when this restriction takes place.

## 6. DISTRIBUTION OF THE PERFOFMANCE TIME

Let $T_i^{(\eta)}$ be the time of successful of the $\eta$th stage calculated under the condition that the initial state of the random environment is $i$. Note that this random variable is a degenerate one: probability $P\{T_i^{(\eta)} \le \infty\} = P\{T_i^{(\eta)} \le b_\eta\}$ can be less than 1 because a failure can take place.

Below, we use decomposition $T_i^{(\eta)}$ for various final states $j$ of random environment $T_i^{(\eta)} = T_{i,1}^{(\eta)} + T_{i,2}^{(\eta)} + \ldots + T_{i,k}^{(\eta)}$. The distribution function of $T_{i,j}^{(\eta)}$ has the form

$$G_{i,j}^{(\eta)}(t) = P\{T_{i,j}^{(\eta)} \le t\} = \int_0^t P_{i,j}^{(\eta)}(u)dF_\eta(u), \quad t \ge 0. \tag{13}$$

Analogously to formula (9), we have

$$G_{i,j}^{(\eta)}(t) = P\{T_{i,j}^{(\eta)} \le t\} = \int_0^t \sum_{\nu=0}^n z_{i,j,\nu}^{(\eta)} u^\nu dF_\eta(u) = \sum_{\nu=0}^n z_{i,j,\nu}^{(\eta)} \int_0^t u^\nu dF_\eta(u), \quad t \ge 0. \tag{14}$$

Below, we dilate our reasoning to many stages. Let $T_i^{(1,\eta)}, i \in E$, be a successful performing time of the first $\eta$ link calculating under the condition that initial state of the random environment is $i$ and as earlier $\{T_{i,j}^{(1,\eta)}, j \in E\}$, is the considered decomposition. The corresponding distribution functions are calculated recurrently:

$$\begin{aligned} \tilde{G}_{i,j}^{(1)}(t) &= P\{T_{i,j}^{(1,1)} \le t\} = G_{i,j}^{(1)}(t), \\ \tilde{G}_{i,j}^{(\eta)}(t) &= P\{T_{i,j}^{(1,\eta)} \le t\} = \sum_{\zeta=1}^k \int_0^t G_{\zeta,j}^{(\eta)}(t-u) d\tilde{G}_{i,\zeta}^{(\eta-1)}(u)du, \quad \eta = 2,\ldots,c. \end{aligned} \tag{15}$$

Finally, the performance time of the supply chain $T^{(1,c)}$ has the following distribution function:

$$P\{T^{(1,c)} \le t | J(0) = i\} = \tilde{G}_{i,\bullet}^{(c)}(t) = \sum_{j=1}^k \tilde{G}_{i,j}^{(c)}(t), \quad t \ge 0. \tag{16}$$

A defect of this distribution is equal to $1 - R_i$. Therefore, a conditional distribution function of successful performance time $T_{\text{succ}}$ given the successful performance takes place is calculated as

$$\tilde{G}_{i,\bullet}^{\text{succ}}(t) = \frac{1}{R_i} \tilde{G}_{i,\bullet}^{(c)}(t), \quad t \ge 0. \tag{17}$$

The conditional mean of the performance time is calculated as

$$E((T_{\text{succ}} | J(0) = i)) = \int_0^{b^*} (1 - \tilde{G}_{i,\bullet}^{\text{succ}}(t))dt,$$

where $b^* = b_1 + b_2 + \ldots + b_c$.

## 7. NUMERICAL EXAMPLE

Let us consider the following initial data. The external random environment has three states ($k = 3$) with the following matrix of transition intensities:

$$\lambda = \begin{pmatrix} \lambda_{1,1} & \lambda_{1,2} & \lambda_{1,3} \\ \lambda_{2,1} & \lambda_{2,2} & \lambda_{2,3} \\ \lambda_{3,1} & \lambda_{3,2} & \lambda_{3,3} \end{pmatrix} = \begin{pmatrix} 0 & 0.5 & 0 \\ 0.3 & 0 & 0.5 \\ 0.5 & 0.2 & 0 \end{pmatrix}.$$

The initial state of the random environment (at time $t = 0$) is the first one, $i0 = 1$.

The supply chain includes $c = 5$ stages (links). The failure intensities $\gamma_{i,\eta}(t)$ for the $\eta$th stage are the same for all stages and have linear dependences ($m = 1$) on the stage performance time $t$ as follows: $\gamma_{i,\eta}(t) = \gamma_i(t) = w_{i,0} + w_{i,1}t$. Corresponding matrices of the coefficients $w = (w_{i,\mu})_{3\times2}$ are as follows:

**Table 1.** Distributions of the random variables $\{W_\eta\}$

| Random variable | Distribution | Parameters |
|---|---|---|
| $W_2$ | Exponential, truncated at point $b$ | $\theta = 1$, $b = 2$ |
| $W_3$ | β-distribution | $u = 1$, $v = 3$, $\alpha = 1$, $\beta = 2.9$ |
| $W_4$ | Uniform | $a = 0.5$, $b = 1.5$ |

$$w = (w_{i,\mu})_{3\times 2} = \begin{pmatrix} 0.10 & 0 \\ 0.08 & 0.05 \\ 0.15 & 0.07 \end{pmatrix}.$$

The performance time of the first and the last links are constants: $W_1 = 2$, $W_5 = 3$. The distributions of the performance time of other links are presented in Table 1. In particular, the β distribution, which is less known, has the following density [5, 7]:

$$f(x) = \frac{1}{B(u,v)}\frac{1}{(\beta-\alpha)^{u+v-1}}(x-\alpha)^{u-1}(\beta-x)^{v-1}, \quad \alpha < x < \beta,$$

where $u$ and $v$ are the form parameters ($u > 0$, $v > 0$), $\alpha$ and $\beta$ bonds of the possible values ($\alpha < \beta$), $B(u, v)$ is the β function

$$B(u,v) = \int_0^1 t^{u-1}(1-t)^{v-1}dt.$$

It is known that, for the integer positive $u$ and $v$, $B(u,v) = (u-1)!(v-1)!/(u+v-1)!$

The maximal possible value of the considered random variables are the following: $b_1 = 2$, $b_2 = 2$, $b_3 = 2.9$, $b_4 = 1.5$, $b_5 = 3$. Therefore $b^* = b_1 + b_2 + b_3 + b_4 + b_5 = 11.4$. The minimum value of the performance time is 6.5.

Formula (10) contains the νth initial moments $\mu_\nu^{(\eta)}$ of the random variables $\{W_\eta\}$. They are calculated as follows [4, 6]:

1. The exponential distribution with parameter θ, concentrated in interval (0, $b$) as follows:

$$\mu_\nu^{(2)} = \frac{1}{1-\exp(-\theta b)}\int_0^b x^v\theta\exp(-\theta x)dx = \frac{1}{1-\exp(-\theta b)}v!\theta^{-v}\left[1 - e^{-\theta b}\sum_{i=0}^{v}(\theta b)^i\frac{1}{i!}\right];$$

2. The β-distribution with parameters $u$, $v$, $\alpha$, $\beta$:

$$\mu_\nu^{(3)} = \alpha^\nu + \sum_{i=0}^{\nu-1}C_r^i\alpha^i(\beta-\alpha)^{\nu-i}\prod_{j=0}^{\nu-i-1}\frac{u+j}{u+v+j},$$

where $C_r^i = \frac{r!}{i!(r-i)!}$ is the binomial coefficient.

3. The uniform distribution on interval ($a$, $b$):

$$\mu_\nu^{(4)} = \frac{1}{(\nu+1)(b-a)}(b^{\nu+1} - a^{\nu+1}).$$

Below, we will consider and discuss some results. In addition the order $n$ of the approximation (3) will be changed to investigate its influence on the accuracy of the calculations.

Firstly, let us consider one link. Let $Z(n)$ be the matrix of approximation coefficients $\{z_{1,j,\nu}^{(1)}: j = 1,2,3;\ \nu = 0,\ldots,n\}$ in (3), calculated with respect to formula (6) recurrently. It is presented in Table 2 for $i0 = 1$ and $n = 8$.

Now, we can calculate the survival function of the ηth stage $S_1^{(\eta)}(t), i, j \in E$, using formulas (3) and (7). Table 3 contains corresponding values for the second stage and for various orders $n$ of the approximation polynomial (3).

**Table 2.** Matrix $Z(8) = (z^{(1)}_{1,j,\nu})_{3\times 9}$

| $\nu$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $j = 1$ | 1 | –0.6 | 0.255 | –0.067 | $9.267 \times 10^{-3}$ | $1.09 \times 10^{-4}$ | $-3.614 \times 10^{-4}$ | $9.215 \times 10^{-5}$ | $-1.42 \times 10^{-5}$ |
| $j = 2$ | 0 | 0.5 | –0.370 | 0.151 | –0.042 | $8.275 \times 10^{-3}$ | $-1.162 \times 10^{-3}$ | $1.055 \times 10^{-4}$ | $3.081 \times 10^{-6}$ |
| $j = 3$ | 0 | 0.0 | 0.125 | –0.097 | 0.037 | $-9.171 \times 10^{-3}$ | $1.553 \times 10^{-3}$ | $-1.799 \times 10^{-4}$ | $1.211 \times 10^{-5}$ |

**Table 3.** Survival function $S^{(2)}_1(t)$

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $n = 5$ | 1 | 0.901 | 0.785 | 0.626 | 0.321 | –0.400 | –2.072 | –5.590 |
| $n = 8$ | 1 | 0.901 | 0.788 | 0.653 | 0.398 | –0.553 | –4.560 | –17.171 |
| $n = 20$ | 1 | 0.901 | 0.788 | 0.655 | 0.542 | 0.429 | 0.332 | 0.254 |
| $n = 40$ | 1 | 0.901 | 0.788 | 0.655 | 0.542 | 0.429 | 0.332 | 0.250 |
| $n = 50$ | 1 | 0.901 | 0.788 | 0.655 | 0.542 | 0.429 | 0.332 | 0.250 |

**Table 4.** Reliabilities functions $R^{(\eta)}_{1,j}$

| $\eta$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $j = 1$ | 1 | 0.420 | 0.360 | 0.277 | 0.240 | 0.158 |
| $j = 2$ | 0 | 0.262 | 0.248 | 0.206 | 0.182 | 0.113 |
| $j = 3$ | 0 | 0.106 | 0.120 | 0.121 | 0.113 | 0.069 |
| $R_1$ | 1 | 0.788 | 0.728 | 0.604 | 0.535 | 0.340 |

Here, we can determine the necessary order of the approximation. We must increase one until a further increasing will not change the results of calculations. Because the first stage is performed at a constant time of 2, $n = 20$ is fully sufficient for the considered case.

Probabilities (reliabilities) $R^{(\eta)}_{i,j}, i, j \in \{1, 2, 3\}$, that the first $\eta$ stages will be ended successfully and $j$ is the last state of the external environment, are presented in Table 4. Here $\eta = 0, 1, \ldots, 5$, where $\eta = 0$ corresponds to the initial distribution of environment states. Presented data have been calculated using formulas (11). Note that the initial distribution of states for the given stage coincide with the final distribution for the previous stage. The last row of the table contains reliability (12), i.e., the sum of given probabilities over all final states $j$.

Now, we consider the distributions of successful performance times $\{T^{(1,\eta)}_i\}$. First, we consider the first two stages only. The initial state of the random environment for the second stage is the state after performing the first link. It can be seen from Table 4 that the corresponding probabilities are as follows: $(0.420\ 0.262\ 0.106)^T$. Table 5 contains distributions $\tilde{G}^{(2)}_{1,j}(t), t \geq 0$, for $\{T^{(1,2)}_1\}$, calculated with respect to formulas (14) and (15) and Table 2. The last column of this Table 2 presents the reliabilities of the first links. We see that one coincides with the corresponding values from the second column of Table 4.

The distributions of the performance time of all supply chain $\{T^{(1,5)}_{1,j}\}, j \in E$, are calculated by formulas (15) and (16). As before, $j = 1, 2, 3$ is the final state of the chain. The corresponding results are represented in the Table 6. Let us remark that the reliability of whole supply chain, presented in the last columns of Tables 4 and 6, coincide in fact; i.e., the difference is less then 1%.

**Table 5.** Distribution functions $\tilde{G}^{(2)}_{1,j}(t)$, $t \geq 0$ of the successful performance time $\{T_1^{(1,2)}\}$

| $t$ | 2.3 | 2.6 | 2.9 | 3.2 | 3.5 | 3.8 | 4.1 |
|---|---|---|---|---|---|---|---|
| $j = 1$ | 0.121 | 0.204 | 0.262 | 0.302 | 0.331 | 0.350 | 0.360 |
| $j = 2$ | 0.078 | 0.135 | 0.176 | 0.206 | 0.226 | 0.241 | 0.248 |
| $j = 3$ | 0.033 | 0.060 | 0.081 | 0.096 | 0.108 | 0.116 | 0.120 |
| $\Sigma$ | 0.232 | 0.399 | 0.519 | 0.604 | 0.665 | 0.707 | 0.728 |

**Table 6.** Distribution functions $\tilde{G}^{(5)}_{1,j}(t)$, $t \geq 0$ of the successful performance time of the chain

| $t$ | 7 .2 | 7.4 | 7.6 | 7.8 | 8.0 | 8.2 | 8.4 | 8.6 | 8.8 | 9.0 | 9.2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $j = 1$ | 0.011 | 0.020 | 0.032 | 0.046 | 0.061 | 0.074 | 0.086 | 0.096 | 0.105 | 0.112 | 0.117 |
| $j = 2$ | 0.008 | 0.014 | 0.023 | 0.033 | 0.043 | 0.053 | 0.061 | 0.069 | 0.075 | 0.080 | 0.083 |
| $j = 3$ | 0.004 | 0.009 | 0.014 | 0.020 | 0.027 | 0.032 | 0.038 | 0.042 | 0.046 | 0.049 | 0.051 |
| $\Sigma$ | 0.023 | 0.043 | 0.069 | 0.099 | 0.131 | 0.159 | 0.185 | 0.207 | 0.226 | 0.241 | 0.251 |
| $t$ | 9.4 | 9.6 | 9.8 | 10.0 | 10.2 | 10.4 | 10.6 | 10.8 | 11.0 | 11.2 | 11.4 |
| $j = 1$ | 0.120 | 0.122 | 0.124 | 0.125 | 0.125 | 0.125 | 0.131 | 0.137 | 0.144 | 0.151 | 0.157 |
| $j = 2$ | 0.086 | 0.087 | 0.088 | 0.089 | 0.089 | 0.089 | 0.093 | 0.098 | 0.103 | 0.107 | 0.112 |
| $j = 3$ | 0.053 | 0.054 | 0.054 | 0.054 | 0.055 | 0.055 | 0.057 | 0.060 | 0.063 | 0.066 | 0.069 |
| $\Sigma$ | 0.259 | 0.263 | 0.266 | 0.268 | 0.269 | 0.269 | 0.281 | 0.295 | 0.310 | 0.324 | 0.338 |

## 8. CONCLUSIONS

A new approach to the reliability analysis of the supply chain has been suggested that takes into account the existence of a random environment. The latter is described as a continuous-time finite irreducible Markov chain. The chain is subjected by failures, the intensity of which depends on the state of the random environment. Methods of chain reliability are presented, as well as the distribution function of successful performance time computing in the paper. The numerical example illustrates a formal description of the methods.

The authors plan to use the presented approach to more complex structures of the supply, specifically to network structures in the future.

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