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OPTICS AND SPECTROSCOPY. LASER PHYSICS

The State Vector of a Quantum System: Mathematical Fiction or Physical Reality?

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Abstract—A version of an experiment with a correlated pair of entangled particles is considered. This experiment demonstrates an interesting effect of variations in the entangled photon polarization that shows the reality of all of the various superposition components and the corresponding state vector of the quantum system. The possible consequences of this are analyzed.

Keywords: quantum state vector, entanglement, nonlocality, Copenhagen interpretation, hidden variables.
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INTRODUCTION

In contrast to classical measurements, quantum measurements are characterized by the principle that a physical quantity a priori has no any particular value prior to measurement if it is not under measurement (for example, [1] and references therein). This property, rather than the statistical nature of measurements, distinguishes quantum theory as an independent section of modern science. Otherwise, it would be just a subsection of statistical physics; this property is in full compliance with the Copenhagen interpretation of quantum theory.

RESULTS AND DISCUSSION

According to the projection postulate of von Neumann [2], the measurement moment involves vector state reduction, i.e., reduction in the vector dimension to the measured range of values of the measured quantity (for example, [1, 3] and references therein). Reduction in the quantum state of a pair or more of correlated particles in an entangled state is of special interest, because the measurement of one particle leads to an immediate change in the quantum state of another particle (or other particles) that traveled away from the first particle to an arbitrary and occasionally considerable distance. Strictly speaking, the von Neumann's projection postulate does not describe such a reduction. Its generalization to entangled states was proposed by F.Ya. Khalili [4]. However, the fundamental conclusion of instantaneous reduction remains in effect. This is the reason that researchers still make attempts to create superluminal communication lines based on this phenomenon (for example, [5] and references therein). For our investigation, it seems to us that a detailed study of this phenomenon is indisputable evidence for the nonlocal nature of quantum processes. This fact has been proven experimentally for single photons [6]. For a pair of entangled particles, we believe that the imaginary experiment described below is entirely sufficient as proof.

Figure 1 demonstrates the investigation chart of correlated photon pairs. Pairs of entangled photons are generated in a nonlinear crystal with a quadratic nonlinearity (commonly a piezoelectric crystal) under laser pumping. Signal "a" and idler "b" photons have mutually orthogonal polarization planes. They are sent to the respective observers, A and B, each of which has a Wollaston prism, which separates mutually orthogonal polarizations, and two detectors, "x" and "y." The angular orientation of the prism of the observers is the same and is controlled by rotation angles $\alpha = \beta$, respectively, around the photon-propagation direction. A phase half-wave polarization plate is installed in channel A. Polarization planes of its ordinary and extraordinary beams are oriented at an angle of $\pi/4$ relative to the respective planes of the piezoelectric crystal so that the plate rotates the photon polarization plane at an angle of $\pi/2$. The experimental time axis is given below. It shows the moments of the recording (measurements) of the polarization of photon b such as t_1 and t_2, which is always measured prior to a. The photon a path also contains conventional points, such as T_1 and T_2, where it occurs under these measurements. It is important to measure photon b so that in the first case a is situated before the $\lambda/2$ plate, while in the second case it is after it.

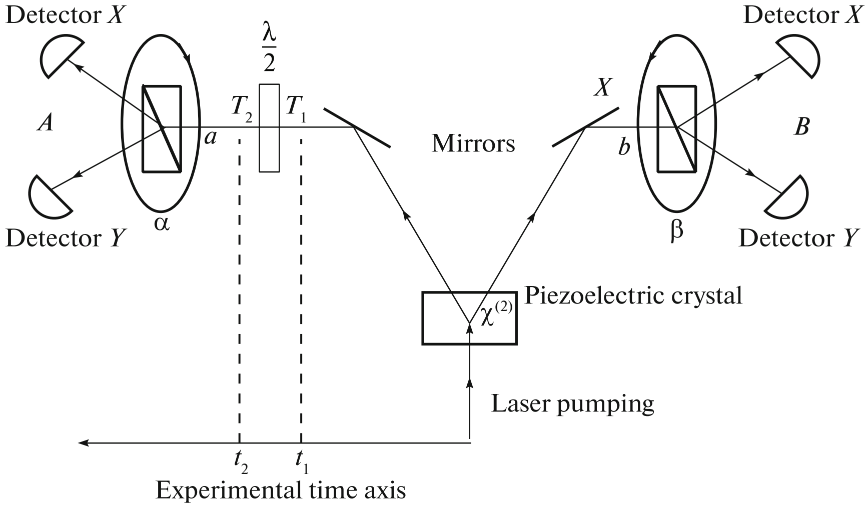

Fig. 1. The investigation of correlated photon pairs.

Let us consider two polarization anti-correlated entangled photons:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_x^a|1\rangle_x^b|1\rangle_y^a|0\rangle_y^b + |1\rangle_x^a|0\rangle_x^b|0\rangle_y^a|1\rangle_y^b). \quad (1)$$

In this case, the a and b indexes refer to the first and second photons of the entangled pair, respectively, while the mutually orthogonal transverse directions x and y define the orthogonal polarization directions. The structure of this state vector is as follows: although the polarization directions x and y of each photon a or b in the pair are equally probable they are strictly correlated between themselves, or more exactly, anti-correlated, because their polarization planes are mutually orthogonal.

Such conditions are commonly prepared by parametric light scattering under nonlinear interaction of the second type (for example, [7] and references therein). Let us install a phase half-wave polarization in one channel, for instance, a; this plate is oriented so that the polarization planes of its ordinary and extraordinary beams make up an angle of $\pi/4$ with corresponding planes of the piezoelectric crystal. The plate is used to rotate the polarization plane at $\pi/2$. Polarization variations in plane mirrors are not taken into account because they are not necessary in a real experiment. Thus, the phase plate will change the state vector (1) as follows:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle_x^a|1\rangle_x^b|0\rangle_y^a|0\rangle_y^b + |0\rangle_x^a|0\rangle_x^b|1\rangle_y^a|1\rangle_y^b). \quad (2)$$

Once photon a passes the phase plate, the measurement is carried out in channel b. Let us assume, for example, that after the Wollaston prism photon b occurs in the channel with y polarization. Then, after measuring, the vector (2) reduces to:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle_y^a + |1\rangle_y^b). \quad (3)$$

or just to $|1\rangle_y^a$, and photon a occurs in the channel with y polarization after the polarization prism. If, on the contrary, after the measurement photon b occurs in the channel with x polarization, the opposite situation occurs.

There is no doubt that a real experiment will confirm this simple reasoning. Let us call the polarization measurement results from this experiment the baseline scenario of the conducted measurements.

Let us now attempt to start from the seemingly firmly established fact that the observed measured quantum value a priori does not exist [8]. In this case, depending on the existing interpretations of this statement, we have two options:

(a) Based on the interpretation that the quantum value a priori, i.e., before the measurement, *does not exist in reality* (in this case, the state vector acts as a mathematical method to describe its movement) the polarization of photon a does not exist prior to measuring it (in the best case, it would be natural).

(b) Based on the interpretation that the quantum value does not exist prior to measurements only in the "classical sense" (in other words, it *actually* exists, but is in a state of superposition of all its possible states) the photon a polarization prior to measuring will *actually* be in a superposition of two possible states (1). In this case, its state vector will begin to act as a component of *reality*, which will *actually reflect the real* parameters of photon a.

Let us study these two possibilities in detail.

In the first case, the $\lambda/2$ plate is not able to change the photon a polarization, because it does not exist and it is impossible to change something that does not exist. Therefore, this plate should have no effect on its polarization. Nevertheless, the experiment will certainly show that this is not so.

In fact, let us suppose that the photon b polarization is equal to x when the photon b polarization is measured at the time t_1 (at the moment when photon a is at the point T_1). In this case, the photon a polarization should immediately accept the y orthogonal value due to the action of the nonlocal relationship. Photon a will then be affected by a half-wave plate, which should turn its polarization to the x position. As a result, the polarization of both photons will be similar.

This scenario is in full compliance with "nonlocal" logic and the results of the experiment, which would undoubtedly confirm this in practice.

However, let us assume now that we conducted analogous measurement at the time t_2 (at the moment when photon a has already passed the $\lambda/2$ plate and is at point T_2). In this case, the obtained value of the photon b polarization is the same as in the first case, viz., equal to x. The photon a polarization would then seem to be equal to y due the fact that on the way from the point T_2 to the detector there is no device that could "turn it around." However, the conducted measurements will undoubtedly show that its polarization will still be equal to x.

In this case, the resulting value of the photon a polarization can be explained only by the fact that the $\lambda/2$ plate, while not "knowing" the photon polarization value (and whether it exists at all) still changed it to the opposite polarization. The plate changed something that, in accordance with this interpretation, does not and cannot exist. It is obvious that such treatment of the Copenhagen interpretation, which suggests that no quantum value really exists, leads to a logical paradox: a real plate "turns" something that does not exist and this can be determined empirically.

What will our experiment involve when we consider the second variant of the interpretation of the existence of the quantum particle? In this case, the $\lambda/2$ plate will affect photon "a," which will really be in a superposition of all its possible states. If we, as in the previous example, measure photon b at the time t_2 and record photon a in the state x (which will undoubtedly occur in the course of the experiment) this will mean that the $\lambda/2$ plate will change both of the possible states of photon a in a superposition state, i.e., all possible (both) polarization states will switch places, as occurred when passing from formula (1) to (2).

The adequacy of the last consideration is also confirmed by the experimental results via observation of three-beam interference [8], which prove the simultaneous presence of both one- and two-photon states in their superposition.

It is evident that the theoretical terms assume a third alternative explanation of these experiment results; in this case, we will accept that the measured quantities still have certain values in a classical sense prior to measurement. Such an interpretation would be contrary to the Copenhagen interpretation of quantum mechanics, but it remains theoretically possible. In fact, this is a nonlocal theory of hidden variables. It should be recalled that this interpretation, despite the assumption of hidden variables, contradicts neither Bell's theorem nor Aspect's experiments [9–11]. Such an explanation of reality, for example, is given in the interpretation by de Broglie–Bohm [12]. All this is possible due to the fact that it allows a higher degree of nonlocality in the behavior of quantum objects.

All of these three mentioned alternatives are undoubtedly interesting and do not leave room for simple local models. The first alternative is in contradiction with the real experiment and therefore leads to the conclusion that the quantum value probably still exists (albeit in a superposition state) prior to its measurement. In any case, it casts doubt on the converse statement related to the fact that the quantum value does not exist at all in this period of time.

The second alternative shows the reality of the quantum value being in a superposition state that characterizes its state vectors to some extent. The conclusions that follow from both the first and the second alternatives should advance the debate about the real nature of quantum–mechanical state vectors of quantum objects.

The third alternative takes an approach that is even closer to the reality that exists in the quantum and "ordinary" worlds, because it reveals the real nature of quantum values prior to measurement in the classical sense and is actually a nonlocal theory of hidden variables.

Let us study the last alternative more thoroughly, because the nonlocal theory of hidden variables can lead to completely unexpected and paradoxical conclusions. Let us consider, for example, a Mach–Zehnder interferometer, to whose inlet a single photon in a Fock state is released (Fig. 2). Initially, the second beam splitter positioned in front of the photo detectors is removed. In this case, the detectors will record single photocounts in one channel or another. They will never be recorded in both channels simultaneously, because only one photon is present in the system inlet.

What will happen after we return the second beam splitter to its place? The probability of photocounts in the detectors will be described by the harmonic function $1/2[1 \pm \cos(\Phi_1 - \Phi_2)]$, where Φ_1 and Φ_2 are phase delays in the interferometer arms [13]. The sign of this function depends on the type of detector (1 or 2) that makes the recording. This harmonic function cannot be expressed as the sum of the probabilities: $P(\Phi_1) + P(\Phi_2)$. Consequently, after the first beam splitter, the photon will be present as if it is in both

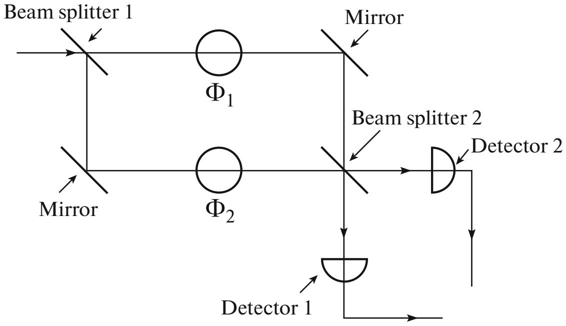

Fig. 2. A Mach–Zehnder interferometer.

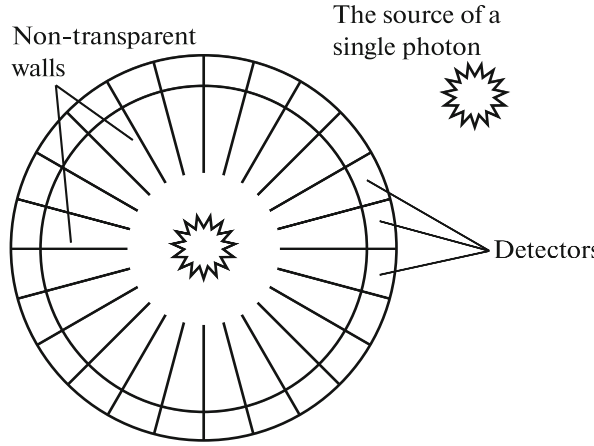

Fig. 3. A photon is generated in the center of the sphere totally covered by photodetectors; this photon runs to these detectors via sectors separated by non-transparent walls. The photon wave function occupies the entire surface of the sphere right before the photo detection, but after the detection it is directed only to one sector as if it passes "through" non-transparent walls.

interferometer arms simultaneously, although it was only in one arm in the first part of the experiment. This paradoxical situation is hard to explain from the standpoint of the spatial intuition of common sense that is common in the macroworld. Based on the conclusions drawn from the analysis of the first and second alternatives, we can assume that this photon was *really* situated in both interferometer arms in a superposition state. The cause of this behavior of quantum objects may be related to the fact that quantum state vectors belong to Hilbert space for which spatial locality is not mandatory.

Does this mean that experiments with the Mach–Zehnder interferometer can prove the mandatory nonlocality of the quantum theory by themselves? In other words, can any local model describe the interference of a single photon in a Mach–Zehnder, Michelson, or Jung interferometer? It turns out that it can. This problem is readily solved, for example, by the "pilot wave" interpretation. In fact, let us direct a single quantum particle through a particular path by vacuum frequency fluctuations that correspond to the de Broglie frequency of this particle. It is obvious that these quasi-monochromatic vacuum modes will interfere and direct this particle to the interference maxima. This model can explain the interference of single quantum particles, however, it actually is a local theory of hidden variables, which is known to be firmly refuted by Bell's tests for inequalities [9, 10] (see also [12–14]).

So, how can this experiment be interpreted in terms of the nonlocal theory of hidden variables? In this case, it turns out that the photon "nonlocally" knows in advance what will happen to it in the future, viz., whether it should interfere and split into two channels or it should entirely go to one channel and then be recorded with one of the detectors (in this case, the detectors should be installed in all possible channels; they should cover all alternative photon paths that are necessary for its subsequent interference).

In assuming this interpretation, we should clearly understand that this paradoxical situation, which seemingly has no physical sense and violates the causality principle, is not that bad. After all, we can install high-speed (for example, electrooptic) valves in the channels, which will "deceive" the photon that already passed the first beam splitter. In this situation, when it is passing the beam splitter and "hoping" to interfere, the photon will face a barrier in the form of a valve that impedes the interference, which will send it to the photodetector. However, it is evident that we will fail to deceive the photon in such a way: it will readily adapt to the changed situation as if it already "knew" about the future actions of the valve. Hence, either the photon is outside of space and time [15], or it continuously and instantaneously "jumps" from one interferometer arm to another to always be ready for both interference and detection. Now let us suggest an absolutely fantastic hypothesis: all material objects should be related to each other by an unknown nonlocal communication, which will, depending on the surrounding events (in particular, the valve state), change the following behavior of a photon.

All of the explanations of single photon interference that are given in the Mach–Zehnder scheme appear to be very exotic. Leaving the first assumption outside the scope of this discussion, we will comment on the second one. Let us imagine that we change our experiment in Fig. 2 by adding a non-transparent wall, which will securely separate both interferometer beams from each other. It is obvious that the photon behavior will remain unchanged. Hence, we come to a new, hitherto unknown type of nonlocal interaction. When discussing the variant of the interaction that results in continuous "jumps" of the photon, it should be mentioned that in case of its existence we simultaneously solve the problem of instantaneous reduction in the state vector of the quantum system that is under measurement: in fact, local information on measurements can instantly pass through non-transparent walls through an unknown carrier.

This has the following form: let us assume that a single photon is generated in the center of a sphere that is fully covered by photodetectors separated from each other by non-transparent sectors directed to the center (Fig. 3). It is clear that the measurement will result in the activation of only one detector and the wave function will collapse as if "through" non-transparent walls.

CONCLUSIONS

Returning to the initial experience in Fig. 1 and the corresponding question, we make the following conclusions: quantum values *really* exist in the superposition of all their states and these states are *real*, because we can see not only a strict correlation between such states of related entangled particles, but also can change them using real macroobjects; quantum objects still have hidden variables, and their behavior should be explained on the basis of a hitherto unknown nonlocal theory using an unknown interaction. What conclusion can be drawn from these results? First, both discussed alternatives are indisputably indicative of the nonlocal nature of quantum objects. The second conclusion and perhaps the most important is that the state vector exists in reality; it is not an artificial mathematical approach in the area of virtual reality that makes it possible to perform calculations that are consistent with the experimental results, for example, the negative absolute temperature that formally describes the population inversion of active laser environments or the fictitious non-Hermitian Hamiltonian function, which is also suitable for successful analytical calculation of three-level quantum systems (for example, [1], p. 47 and references therein). The actual existence of the state vector of quantum systems and the alternative hypothesis of the nonlocal theory of hidden variables, in their turn, confirm the reality of quantum objects not only at the time of recording, but also prior to measurement (a priori).

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