

# Calculation of Life Characteristics for Structural Alloys under the Mutual Influence of Fatigue and Long-Term Strength

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**Abstract**—The processes of degradation of the initial strength properties of polycrystalline structural alloys are considered under mechanisms that combine low-cycle fatigue and long-term strength of the material. From the point of view of mechanics of damaged medium (MDM) and fracture mechanics (FM), a mathematical model that describes the processes of cyclic viscoplastic deformation and damage accumulation in structural alloys under multiaxial disproportionate modes of combined thermo-mechanical loading has been developed. The model consists of three interrelated components: relations that determine the cyclic viscoplastic behavior of the material by taking into account the dependence on the fracture process; evolutionary equations describing the kinetics of damage accumulation; criterion of the strength of the damaged material. The viscoplasticity model is based on the idea of the existence of plasticity and creep surfaces in stress space and the principle of gradient vectors of plastic and creep strain rates to the corresponding surface at the loading point. This form of the equations of state reflects the main effects of cyclic viscoplastic deformation of the material for arbitrary complex loading trajectories. The form of kinetic equations for damage accumulation is based on the introduction of a scalar damage parameter and on energy principles. It also takes into account the main effects of the formation, growth and merging of microdefects under arbitrary complex modes of combined thermomechanical loading. A joint form of the evolutionary equation for damage accumulation in the areas of low-cycle fatigue and long-term strength of the material is proposed. As a criterion for the strength of a damaged material, the condition for reaching a critical value is used. The material parameters and scalar functions included in the constitutive relations of the MDM mathematical model are obtained. The results of numerical simulation of the processes of deformation and damage accumulation in structural alloys under the mutual influence of low-cycle fatigue and long-term strength of the material are presented. The results of comparison of calculated and experimental data show that the proposed MDM model qualitatively and with the accuracy necessary for practical calculations quantitatively describes the durability of materials under the mutual influence of low-cycle fatigue and long-term strength of the material.

**Keywords:** low-cycle fatigue, long-term strength, simulation, mechanics of a damaged medium, stress-strain state, damage, resource, material parameters, numerical and full-scale experiment

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## 1. INTRODUCTION

One of the main problems of modern mechanical engineering is the substantiation of the resource of equipment and systems of critical engineering objects (CEO) at the stage of their design, the assessment of the depleted resource and the forecast of the residual resource of structural units during operation, and the extension of the service life after the objects have worked out the standard operational life. These problems are especially relevant for facilities with a service life of several decades (nuclear power plants, petrochemical equipment, storage tanks for gaseous and liquefied chemical products, main gas and oil pipelines, etc.). Their working conditions are characterized by multi-parametric non-stationary thermomechanical effects, the effects of external fields of various physical nature, leading to the development of various mechanisms for the degradation of the initial strength properties of structural materials and the exhaustion of the initial resource of structural units of an engineering object [1, 2].

The processes of resource exhaustion are multi-stage, non-linear, interconnected and highly dependent on the specific conditions of manufacture and operation of an individual object.

There are many mechanisms that can determine the processes of exhaustion of the resource of a particular object depending on the conditions of its operation: high-cycle fatigue (HCF), low-cycle fatigue (LCF), long-term strength (LTS), etc. (taking into account their interaction). For these mechanisms, the formation of a macroscopic crack is the result of the successive action of a certain number of very complex (from a physical point of view) processes of transformation of the initial structure of a material, including the initiation, development and interaction of various defects in the crystal lattice in metals and the interaction of hierarchical structural components of various levels.

The actual loading conditions of many structural elements are characterized by cyclic changes in temperature and stresses. The process of destruction under the mutual influence of long-term strength and low-cycle fatigue occurs as a result of cyclic load application at elevated temperature, load holding and activation of time-dependent deformation mechanisms. Damage from creep (long-term strength) and non-stationary plastic deformation (low-cycle fatigue) of a material has a different physical character. From a microscopic point of view, it can be considered that creep damage has an intergranular character, while fatigue damage corresponds to the accumulation of microcracks along the grain body (transcrystalline character). When, in accordance with the loading conditions (cyclic deformation with holding at high temperature), both processes develop simultaneously, there is a possibility for their interaction, which manifests itself in the form of highly nonlinear effects that are unfavorable for the overall durability of the material [2–8]. At the microscopic level, this is explained by the fact that during creep, intergranular microcracks tend to form and develop at an early stage of the life of an elementary volume of a structural material. The presence of such defects due to creep contributes to the initiation of transcritical cracks during fatigue.

Currently, a system for classifying the criteria for low-cycle fracture during creep at high temperatures, depending on the accepted measure of the intensity of fatigue damage accumulation and the type of equations that determine the damage parameter, has been developed [9]. There are representative groups of criteria of the following types: deformation (including deformation or its components that characterize the deformation cycle as a whole); kinetic (determining the rate of damage and its critical value); energy (associating the accumulation of damage with the dissipation of the energy of inelastic deformation); criteria based on long-term strength characteristics. Several comprehensive reviews on this problem can be found in [10–13].

It should be noted that, while satisfactorily describing the patterns of low-cycle fracture under relatively simple loading programs, the above criteria lose this property when it is required to reflect the influence of the features of a particular cycle, for example, the nature of holding (creep, relaxation), the order of alternation of the stages of plastic and viscous deformation, as in a half-cycle of tension, and in the compression half-cycle, the law of temperature change, etc.

Neither the process of accumulation of damage as a result of creep, nor the process of accumulation of damage as a result of LCF has been properly studied yet; therefore, it is not surprising that the process in which both types of damage accumulate and interact simultaneously has also not been comprehensively studied. Nevertheless, in a number of practically important cases, these processes proceed simultaneously, interacting with each other and significantly affecting the resource of a structural element. Circumstances are complicated by the fact that during operation such elements are subjected to alternating stresses at constant temperatures, alternating temperatures at constant stresses, or when the stress-strain state and temperature change simultaneously with different periods. In addition, experiments indicate that the interaction of low-cycle fatigue and long-term strength processes is synergistic. At present, there is no general theory that would make it possible to predict long-term strength under cyclic temperature changes under constant stress, cyclic stress changes with exposures of various durations at a constant temperature and under conditions when both stress and temperature change according to different laws. Under real operating conditions of structural elements of power equipment, the duration of thermal force loading cycles can range from several seconds to tens and hundreds of hours, and the alternating viscoplastic deformation of the material in the area of structural or technological stress concentrators is accompanied by temperature-time effects and creep. Creep deformation and the damage process initiated by them can develop both in the process of load holding and during a monotonous loading or unloading process occurring at a rather low rate. Simultaneous development of damage processes in the load change cycle as a result of low-cycle fatigue and long-term strength has a noticeable effect on the mechanism of low-cycle failure and the durability of the material, and the important factors are the shape of the loading cycle, the asymmetry and sign of stresses, the type of stress state, against which creep deformations develop.

Since in real conditions long-term loading cycles can be quite complex, including areas of active loading and unloading at different speeds, holding at intermediate points of the cycle, studies were carried out to identify the most typical loading modes [6, 7, 14–16]. These experiments show that the imposition of tensile holding time of more than  $t = 1-2$  min at elevated temperatures can cause a sharp decrease in fatigue life and a change in the type of fracture from intragranular to intergranular. Compression holding time intervals or cycles that include equal tensile and compression holding time (symmetrical holding) results in less damage than tensile holding. Fracture in these cases is intragranular. In “slow-fast” cycles, when there is a low strain rate for the tensile part of the cycle, a significant decrease in durability (in terms of the number of cycles) is also observed compared to symmetric cycling with a transition to intergranular fracture. If each “slow-fast” cycle is followed by a “fast-slow” cycle with similar strain rates, then the fracture becomes intragranular, and the durability (in terms of the number of cycles) increases compared to the durability when cycling the “slow-fast” type. Damage generated during cycling with tensile holding is not reduced by subsequent continuous cycling.

The above results lead to the conclusion that the linear damage summation law (Palmgren-Miner rule) by the mechanism of joint degradation of low-cycle fatigue and long-term strength is not applicable, since these are completely different processes that exhibit a nonlinear interaction. To summarize these damages, several methods that are of an empirical or semi-empirical nature and valid only for particular loading laws have been proposed [4–7, 15]. Apparently, the only one of these methods that makes it possible to study complex loading processes is the method of separating the range of deformations in a cycle into plastic deformation and creep deformation. This method allows one to analyze the results of the combined action of creep and fatigue and interpret the influence of frequency, holding time and environmental conditions.

In this article, within the framework of MDM, fracture mechanics, a mathematical model that describes the processes of cyclic viscoplastic deformation and damage accumulation in structural materials (metals and their alloys) during the degradation of the initial strength properties of materials via the combined mechanisms of low-cycle fatigue and long-term strength of the material is developed. In order to qualitatively and quantitatively verify the model, a study of the influence of the laws of change in mechanical deformation and temperature (the shape of the loading cycle) on the durability of structural steels under various loading cycles has been made.

## 2. CONSTITUTIVE RELATIONS OF THE MATHEMATICAL MODEL OF THE MECHANICS OF A DAMAGED MEDIUM

The model of a damaged medium under degradation mechanisms that combine low-cycle fatigue and long-term strength of a material consists of three interrelated components [17–20]:

- constitutive relations describing the viscoplastic behavior of the material, taking into account the dependence on the fracture process;
- evolutionary equations describing the kinetics of damage accumulation;
- criterion of the strength of the damaged material.

### 2.1. Constitutive Relations of Thermoviscoplasticity

The constitutive relations of thermoviscoplasticity are based on the following basic provisions:

– the material of the medium is initially isotropic and there are no damages in it (only the anisotropy caused by the processes of plasticity and creep is taken into account; the anisotropy of the elastic properties caused by the processes of damage to the material is not taken into account);

– the tensors of strains and strain rates  $e_{ij}$ ,  $\dot{e}_{ij}$  represent the sum of the “instantaneous” and “temporal” components. The “instantaneous” component of the strain tensors and strain rates includes the elastic component  $e_{ij}^e$ ,  $\dot{e}_{ij}^e$  (independent of the loading history and determined by the final state of the process) and the plastic component  $e_{ij}^p$ ,  $\dot{e}_{ij}^p$  (depending on the history of the loading process). The time component of the strain tensors and strain rates  $e_{ij}^c$ ,  $\dot{e}_{ij}^c$ , is related to the evolution of the creep process;

– the initial yield surface is described by a surface in the form of a Mises hypersphere. The evolution of the change in the plastic loading surface is described by the change in its radius  $C_p$  and the displacement of its center  $\dot{p}_{ij}^p$ ;

- in the stress space, there is a family of equipotential creep surfaces of radius  $C_c$  and having a common center;
- the principle of gradiality is valid;
- the change in the volume of the elastic body  $e_{ii}^n = 0$ , where  $e_{ij}^n = e_{ij}^p + e_{ij}^c$  is the inelastic deformation, which includes the “instantaneous” and “time” components;
- the deformation processes characterized by small deformations are considered;
- the only structural parameter that characterizes the degree of material damage at the macrolevel is the scalar parameter  $\omega$ , namely, damage ( $\omega_0 \leq \omega \leq \omega_f$ ):
- the influence of the level of accumulated damage on the processes of deformation of the material is taken into account by introducing effective stresses.

It is assumed that in the elastic region the deviatoric  $\sigma'_{ij}$ ,  $e'_{ij}$  and spherical  $\sigma$ ,  $e$  components of the stress tensors  $\sigma_{ij}$  and strains  $e_{ij}$ , as well as their velocities are related by the generalized Hooke’s law:

$$\begin{aligned} \sigma &= 3K[e - \alpha(T - T_0)], \quad \sigma_{ij} = 2Ge_{ij}^{e'}, \quad \dot{\sigma} = 3K(\dot{e} - \dot{\alpha}T - \alpha\dot{T}) + \frac{\dot{K}}{K}\sigma, \\ \sigma'_{ij} &= 2Ge_{ij}^{e'} + \frac{\dot{G}}{G}\sigma'_{ij}, \end{aligned} \tag{2.1}$$

where  $\alpha(T)$  is the coefficient of linear thermal expansion of the material,  $K(T)$  is the bulk modulus,  $G(T)$  is the shear modulus,  $T_0$  is the initial temperature, and  $T$  is the current temperature.

To describe the effects of monotonic and cyclic deformation in the stress space, a yield surface, which is described by the von Mises hypersphere, is introduced [21, 22]:

$$F_p = S_{ij}S_{ij} - C_p^2 = 0, \quad S_{ij} = \sigma'_{ij}\rho_{ij}^p, \tag{2.2}$$

where  $C_p$  is the radius of the yield surface, and  $\rho_{ij}^p$  are the coordinates of its center.

To describe complex deformation modes in the deformation space, a cyclic “memory” surface is introduced [23]:

$$a_e = (e_{ij}^p - \xi_{ij})(e_{ij}^p - \xi_{ij}) - a_{\max}^2 = 0, \tag{2.3}$$

where  $a_{\max}$  is the maximum value of the intensity of the plastic strain tensor  $e_{ij}^p$ , and  $\xi_{ij}$  is the tensor of unilaterally accumulated plastic strain.

When modeling the kinetics of the stress-strain state (SSS) during cyclic plastic deformation of a material, it is necessary to describe the processes of hardening and softening of structural materials as reliably as possible, since these mechanisms play a decisive role in the accuracy of estimates of the life characteristics of the material. Under fatigue loading, there is a competition between the processes of hardening and softening caused in the material by the mechanisms of changing the phase composition, microstructural state, changing the density of dislocations, as well as the effect of changing temperature [4, 5]. The physical mechanisms that occur in polycrystalline metals and alloys and affect the processes of hardening and softening depend on the operating temperature, the type of deformation trajectory, the length of the plastic deformation path, the effective amplitudes of the plastic deformation intensity, and the degree of stabilization of the deformation process.

We postulate that in the temperature range  $T$ , at which annealing effects can be neglected, the isotropic hardening (softening) of the material can be represented as follows:

$$\dot{C}_p(\chi, T) = q_\chi \dot{\chi}_p^{mon} + a_p(Q_p^p - C_p^{cyc(p)})\dot{\chi}_p^{cyc} + a_c(Q_p^c - C_p^{cyc(c)})\dot{\chi}_c^{cyc} + q_T \dot{T}, \tag{2.4}$$

$$C_p = C_p^0 + \int_0^t \dot{C}_p dt, \tag{2.5}$$

$$\begin{aligned} q_\chi &= q_2 A + (1 - A)q_1, \quad Q_p^p = Q_2^p A + (1 - A)Q_1^p, \quad Q_p^c = Q_2^c A + (1 - A)Q_1^c, \\ A &= 1 - \cos^2 \Theta, \quad \cos \Theta = n_{ij}^e n_{ij}^s, \quad n_{ij}^e = \frac{\dot{e}_{ij}}{(\dot{e}_{ij} \dot{e}_{ij})^{1/2}}, \quad n_{ij}^s = \frac{S_{ij}}{(S_{ij} S_{ij})^{1/2}}, \end{aligned} \tag{2.6}$$

$$\dot{\chi}_p = \left( \frac{2}{3} \dot{e}_{ij}^p \dot{e}_{ij}^p \right)^{1/2}, \quad \chi_p = \int_0^t \dot{\chi}_p dt, \quad (2.7)$$

$$\dot{\chi}_p^{mon} = \begin{cases} 0 & \text{at } a_e < 0 \vee e_{ij}^p \dot{e}_{ij}^p \leq 0, \\ \dot{\chi}_p & \text{at } a_e = 0 \wedge e_{ij}^p \dot{e}_{ij}^p \geq 0, \end{cases} \quad (2.8)$$

$$\dot{\chi}_p^{cyc} = \begin{cases} \dot{\chi}_p & \text{at } a_e < 0 \vee e_{ij}^p \dot{e}_{ij}^p \leq 0, \\ 0 & \text{at } a_e = 0 \wedge e_{ij}^p \dot{e}_{ij}^p \geq 0. \end{cases} \quad (2.9)$$

In formulas (2.4)–(2.9), the following notations are introduced:  $C_p^0$  is the value of the initial radius of the yield surface of the material;  $q_1, q_2, q_T$  are the modules of monotonous isotropic hardening (softening);  $Q_1^p, Q_2^p, Q_1^c, Q_2^c$  are the modules of cyclic isotropic hardening (softening);  $\dot{\chi}_p^{mon}, \dot{\chi}_p^{cyc}, \dot{\chi}_c^{cyc}$  is the length of the trajectory of plastic deformation of the material in monotonous sections, in sections of regular and irregular cyclic loading, and in sections of load holding, respectively.

The first term of Eq. (2.1) describes isotropic hardening as a result of monotonic plastic deformation of the material, the second one states for the areas of cyclic deformation, the third one states for the action of the creep process, and the fourth one states for a change in temperature  $T$ . Equations (2.1)–(2.9) describe the isotropic hardening of the material depending on the parameter of disproportionality of the loading process described by the parameter  $A$ .

We postulate that the rate of change of the microstress tensor  $\dot{\rho}_{ij}$  is taken as follows:

$$\dot{\rho}_{ij} = \dot{\rho}_{ij}^p + \dot{\rho}_{ij}^c, \quad \rho_{ij} = \int_0^t \dot{\rho}_{ij} dt, \quad (2.10)$$

$$\dot{\rho}_{ij}^p = f(\chi)(\dot{\rho}_{ij}^m + \dot{\rho}_{ij}^r), \quad (2.11)$$

$$\dot{\rho}_{ij}^m = g_1^p \dot{e}_{ij}^p - g_2^p \rho_{ij}^p \dot{\chi} + g_T^p \rho_{ij}^p \langle \dot{T} \rangle, \quad \dot{\chi} = \dot{\chi}_p + \dot{\chi}_c, \quad (2.12)$$

$$\dot{\rho}_{ij}^r = g_1^r \dot{e}_{ij}^p - g_2^r \left( \frac{\rho_{min} - \rho_u^r}{\rho_u^r} \right) \rho_{ij}^r \dot{\chi} \cos \gamma + g_T^r \rho_{ij}^r \langle \dot{T} \rangle, \quad (2.13)$$

$$\cos \gamma = \begin{cases} \left| \frac{\dot{\rho}_{ij} \rho_{ij}^r}{(\dot{\rho}_{ij} \dot{\rho}_{ij})^{1/2} (\rho_{ij}^r \rho_{ij}^r)^{1/2}} \right| & \text{at } \cos \gamma < 0, \quad \rho_u^r = (\rho_u^r \rho_u^r)^{1/2}, \\ 0 & \text{at } \cos \gamma > 0 \end{cases} \quad (2.14)$$

$$g_T^{p,r} = \frac{1}{g_1^{p,r}} \left[ \frac{\partial g_1^{p,r}}{\partial T} \right], \quad (2.15)$$

$$f(\chi) = f(\chi_p^{mon}) + f(\chi_p^{cyc}) + f(\chi_\xi), \quad f(\chi_{c,\xi}) = 1 + k_1^{c,\xi} (1 - e^{-k_2^{c,\xi} \chi_{c,\xi}}),$$

where  $g_1^{p,r}, g_2^{p,r}, k_1^{c,\xi}, k_2^{c,\xi}$  are the material parameters determined experimentally.

Hereinafter, for any value  $B$  contained in  $\langle \rangle$ , the following condition is satisfied:

$$\langle B \rangle = \begin{cases} B, & B > 0, \\ 0, & B \leq 0. \end{cases}$$

In formula (2.11), the first term, indicated in brackets, describes the evolution of  $\rho_{ij}^p$  that is associated with the formation of macroscopic plastic deformations, and the second one describes unilaterally accumulated plastic deformations  $\xi_{ij}$ .

Dependence (2.11) makes it possible to describe the main effects of anisotropy caused by inelastic deformation of a material under alternating loading, as well as the effects arising from the implementation of “hard” (“fitting” of the plastic hysteresis loop) and “soft” (“stepping” of the loop) loading modes.

The function  $f(\chi)$  takes into account the change in  $\rho_{ij}^p$  for cyclic block asymmetrical modes of cyclic plastic loading of the material.

The components of the plastic strain rate tensor  $\dot{e}_{ij}^p$  are determined based on the associated flow law:

$$\dot{e}_{ij}^p = \lambda_p S_{ij}, \quad S_{ij} = \sigma'_{ij} - \rho_{ij}. \quad (2.16)$$

If the magnitude of stresses, temperature, and loading rate are such that the creep effects are significant, the parameters of the material deformation process should be determined at the loading stage, taking into account the creep process.

To describe creep processes in the stress space, a set of equipotential creep surfaces  $F_c$  with a common center  $\rho_{ij}^c$  and different radii  $C_c$  determined by the current stress state is introduced [22]:

$$F_c^{(i)} = S_{ij}^c S_{ij}^c - C_c^2 = 0, \quad i = 0, 1, 2, \dots \quad (2.17)$$

By virtue of associated law, we have

$$\dot{e}_{ij}^c = \lambda_c \frac{\partial F_c^{(i)}}{\partial S_{ij}^c} = \lambda_c S_{ij}^c, \quad (2.18)$$

where  $\lambda_c$  corresponds to the current surface  $F_c^{(i)}$ , which determines the current stress state  $S_{ij}^c$ .

Among these equipotential surfaces, one can single out a surface with a radius  $\bar{C}_c$  corresponding to zero creep velocity:

$$F_c^{(0)} = \bar{S}_{ij}^c \bar{S}_{ij}^c - \bar{C}_c^2 = 0, \quad \bar{S}_{ij}^c = \bar{\sigma}'_{ij} - \rho_{ij},$$

where  $\bar{S}_{ij}^c$  and  $\bar{\sigma}'_{ij}$  is the set of stress states corresponding (with a certain tolerance) to zero creep velocity.

We postulate that, taking into account the mutual influence of the processes of plasticity and creep, the expression for the radius of the creep surface of the zero level has the form

$$\begin{aligned} \bar{C}_c &= \bar{C}_c(\chi_c, T) + C_c^{cyc}(\chi_c^{cyc}, T), \\ \dot{C}_c^{cyc} &= a_c(Q_c^p - C_c^{cyc})\dot{\chi}_c^{cyc}, \\ Q_c^p &= Q_2^{cr} A + (1 - A)Q_1^c, \\ \dot{\chi}_c &= \left(\frac{2}{3} \dot{e}_{ij}^c \dot{e}_{ij}^c\right)^{1/2}, \quad \chi_c = \int_0^t \dot{\chi}_c dt, \\ \dot{\chi}_c^{mon} &= \begin{cases} 0 & \text{at } a_e < 0 \vee e_{ij}^c \dot{e}_{ij}^c \leq 0 \\ \dot{\chi}_c & \text{at } a_e = 0 \wedge e_{ij}^c \dot{e}_{ij}^c \geq 0 \end{cases}, \quad \dot{\chi}_c^{cyc} = \begin{cases} \dot{\chi} & \text{at } a_e < 0 \vee e_{ij}^c \dot{e}_{ij}^c \leq 0, \\ 0 & \text{at } a_e = 0 \wedge e_{ij}^c \dot{e}_{ij}^c \geq 0, \end{cases} \\ \lambda_c &= \lambda_c(\psi_c, T) = \lambda_c \psi_c, \\ \psi_c &= \left[ \frac{(S_{ij}^c S_{ij}^c)^{1/2} - \bar{C}_c}{C_c} \right], \quad \lambda_c = \begin{cases} 0, & \psi_c \leq 0, \\ \lambda_c, & \psi_c > 0, \end{cases} \end{aligned} \quad (2.19)$$

where  $\bar{C}_c$  and  $\lambda_c$  are experimentally determined functions of temperature  $T$ .

The evolutionary equation for changing the coordinates of the center of the creep surface has the form [29]:

$$\dot{\rho}_{ij}^c = g_1^c \dot{e}_{ij}^c - g_2^c \rho_{ij}^c \dot{\chi} + g_T^c \rho_{ij}^c \langle \dot{T} \rangle,$$

where  $g_1^c$  and  $g_2^c > 0$  are experimentally determined material parameters.

By refining the relation (2.18), the gradient law can be represented as follows:

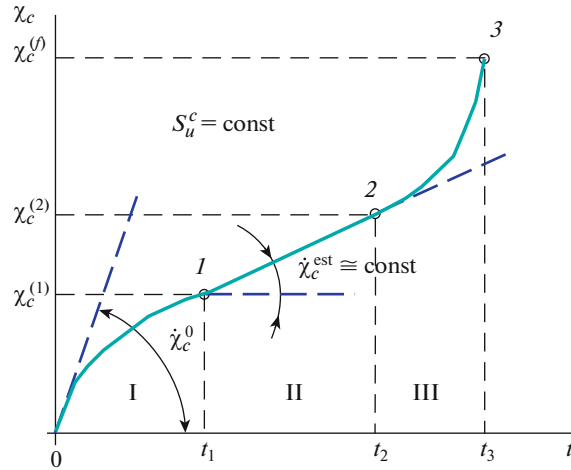


Fig. 1. Creep curve under multiaxial deformation.

$$\dot{e}_{ij}^c = \lambda_c(\psi_c, T) S_{ij}^c = \lambda_c \psi_c S_{ij}^c = \lambda_c \left( \frac{\sqrt{S_{ij}^c S_{ij}^c} - \bar{C}_c}{C_c} \right) S_{ij}^c.$$

The intensity of the creep strain rate tensor has the form:

$$\dot{e}_u^c = \lambda_c \left( \frac{\sqrt{S_{ij}^c S_{ij}^c} - \bar{C}_c}{C_c} \right) S_u^c. \quad (2.20)$$

Taking into account (2.20), the expression for the length of the trajectory of creep deformations takes the form:

$$\dot{\chi}_c = \sqrt{\frac{2}{3}} \dot{e}_c^u = \sqrt{\frac{2}{3}} \lambda_c (\sqrt{S_{ij}^c S_{ij}^c} - \bar{C}_c),$$

where:

$$\lambda_c = \frac{\dot{\chi}_c}{\frac{2}{3} (\sqrt{S_{ij}^c S_{ij}^c} - \bar{C}_c)}. \quad (2.21)$$

The dependence  $\chi_c$  on time  $t$  of the process at  $S_u^c = \text{const}$  for multiaxial deformation along a ray trajectory has the form shown in Fig. 1.

On the curve  $\chi_c \sim t$  (Fig. 1), with a certain degree of conditionality, three sections can be distinguished:

I. Section of unsteady creep ( $0 - \chi_c^{(1)}$ ): the rate of creep deformation  $\dot{\chi}_c$  decreases;

II. Section of steady creep ( $\chi_c^{(1)} - \chi_c^{(2)}$ ): creep strain rate  $\dot{\chi}_c$  is approximately constant  $\dot{\chi}_c^{est} \cong \text{const}$ ,  $\dot{\chi}_c \cong \text{const}$ ;

III. Section of unsteady creep ( $\chi_c > \chi_c^{(2)}$ ): creep strains grow rapidly (precedes destruction) and increases sharply.

The lengths of the sections essentially depend on the value of  $S_u^c = \text{const}$ .

Equations (2.19)–(2.23) describe the unsteady and steady sections of the creep curve at different stress levels and the main effects of the creep process under alternating stress. Creep equations (2.19)–(2.23) are related with Eqs. (2.1)–(2.18), which describe “instantaneous” plastic deformations, at the loading stage via the stress deviator  $\sigma'_{ij}$  and the corresponding algorithm for determining  $\dot{e}_{ij}^c$  and  $\dot{e}_{ij}^p$  at the loading stage by means of certain relations between “time” and “instantaneous” scalar and tensor quantities.

At the stage of development of damages scattered over the volume, the influence of damage on the physical and mechanical characteristics of the material is observed. This effect can be taken into account by introducing effective stresses [17, 18]:

$$\tilde{\sigma}'_{ij} = F_1(\omega)\sigma'_{ij} = \frac{G}{\tilde{G}}\sigma'_{ij} = \frac{\sigma'_{ij}}{(1-\omega)\left[1 - \frac{(6K+12G)\omega}{(9K+8G)}\right]}, \quad (2.22)$$

$$\tilde{\sigma} = F_2(\omega)\sigma = \frac{K}{\tilde{K}}\sigma = \frac{\sigma}{4G(1-\omega)/(4G+3K\omega)}, \quad (2.23)$$

where  $\tilde{G}$ ,  $\tilde{K}$  are the effective moduli of elasticity determined by the Mackenzie formulas [24].

The effective variable  $\tilde{\rho}_{ij}$  is defined similarly:

$$\tilde{\rho}_{ij} = F_1(\omega)\rho_{ij}$$

## 2.2. Evolutionary Equations of Damage Accumulation

Damage accumulation equations are based on the relation between the damage value and internal macroscopic parameters, the critical value of which determines the moment of destruction (formation of a macroscopic crack). The dependence of the dissipated energy spent on the formation of defects under fatigue loading is determined via the work of the microstress tensor  $\tilde{\rho}_{ij}$  on irreversible deformations  $\dot{e}_{ij}^n$ :

$$W_0 = \tilde{\rho}_{ij}\dot{e}_{ij}^n. \quad (2.24)$$

In the problems of assessing the life characteristics, it is necessary to take into account the influence of loading multiaxiality, the presence of which significantly reduces the resource due to both an increase in the influence of the acting components of the strain and stress tensors under proportional loading, and due to the rotation of the main areas of the stress and strain tensors under nonproportional loading.

Numerous studies of the influence of multiaxial loading under various types of stress states, such as biaxial tension-compression, triaxial tension, etc., allow us to conclude that the resource of the material of dangerous zones of structural elements is significantly affected by the “volume” of the stress state, characterized by the parameter  $\beta = \sigma/\sigma_u$ , where  $\sigma$  is the hydrostatic component of the stress tensor,  $\sigma_u$  is the intensity of the stress tensor.

Accounting for the influence of the “bulkness” of the stress state on the damage growth rate is carried out by introducing the function  $f_1(\beta)$  into the damage accumulation rate equation, that increases the damage accumulation rate  $\dot{\omega}$  under loading for  $\beta \rightarrow +\infty$  and decreases the rate for  $\beta \rightarrow -\infty$ . Under loading with in some polycrystalline metals and alloys, a partial decrease in the accumulated damage is possible (the “recovering” effect).

Under conditions of disproportionate loading, in which the stress and strain tensor guides are not coaxial, the realized strain trajectory significantly affects the kinetics of the stress-strain state and the resource characteristics of the structural material as a whole.

Taking into account the considered effects that affect the life characteristics, the equation for the rate of accumulation of fatigue damage under conditions of low-cycle and high-cycle loading can be represented as follows:

$$\dot{\omega} = f_1(\beta)f_2(\omega)f_3(W_0)f_4(\Theta)\dot{W}_0. \quad (2.25)$$

In (2.25), the following notations are introduced for functions that take into account the influence of the factors described above:  $f_1(\beta)$  is “bulk” stress state;  $f_2(\omega)$  is the accumulated level of damage;  $f_3(W_0)$  is the relative level of dissipated energy used for the formation of microdefects  $f_4(\Theta)$  is the function of the parameters for the deformation trajectory.

$$f_1(\beta) = \exp(k\beta), \quad (2.26)$$



$$f_2(\omega) = \begin{cases} 0, & \text{at } W_0 \leq W_a, \\ \omega^{1/3}(1-\omega)^{2/3}, & \text{at } W_0 > W_a \wedge \omega \leq 1/3, \\ \frac{\sqrt[3]{16}}{9}\omega^{-1/3}(1-\omega)^{-2/3}, & \text{at } W_0 > W_a \wedge \omega > 1/3, \end{cases} \quad (2.27)$$

$$f_3(W_p) = \frac{W_0 - W_a}{W_f - W_a}, \quad (2.28)$$

$$f_4(\Theta) = (1 - \cos^2 \Theta)\beta + \cos^2 \Theta, \quad (2.29)$$

$$W_f = W_{pf}^{cyc} \frac{\dot{\chi}_p^{cyc}}{\dot{\chi}} + W_{cf}^{cyc} \frac{\dot{\chi}_c^{cyc}}{\dot{\chi}} + W_{pf}^{mon} \frac{\dot{\chi}_p^{mon}}{\dot{\chi}} + W_{cf}^{cyc} \frac{\dot{\chi}_p^{mon}}{\dot{\chi}}, \quad (2.30)$$

where  $W_{pf}^{cyc,mon}(T)$  and  $W_{cf}^{cyc,mon}(T)$  are the parameters of the material.

### 2.3. Strength Criterion for Damaged Material.

As a criterion for destruction, the condition for achieving a damage value of a critical value is taken:

$$\omega = \omega_f \leq 1. \quad (2.31)$$

## 3. NUMERICAL RESULTS

As the first example of assessing the reliability of the MDM mathematical model under the mutual influence of the processes of creep and fatigue of the material, as well as the accuracy of determining the material parameters and scalar functions of the MDM model, the problem of calculating the kinetics of stress-strain state and damage accumulation in laboratory samples of P92 steel under “hard” loading- uniaxial tension-compression with and without time delays at maximum strain for two strain amplitudes has been solved [8]. The temperature in the studies is 600°C. Figure 2 shows the loading law implemented in the experiments.

The main physical and mechanical characteristics of R92 steel [20] and the material parameters of the MDM model at a temperature of 650°C are given in Table 1–3.

Figures 3 and 4 show the cyclic hysteresis loops for the first loading cycle and for a loading cycle equal to half the durability ( $0.5N_f = 335$ ), respectively (the dots mark the experimental data, the solid line shows the numerical results).

Figure 5 shows the numerical results of the dependence of the maximum and minimum stresses in a cycle on the number of cycles, where the experimental data are marked with dots

Figure 6 shows a comparison of the stress relaxation curves for the first loading cycle and for a loading cycle equal to half the durability ( $0.5N_f = 335$ ) obtained numerically (solid line) and experimentally (points).

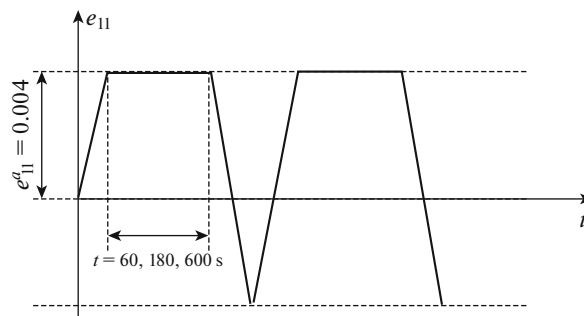


Fig. 2. The law of loading of laboratory samples of steel R92.

**Table 1.** Physical and mechanical characteristics and material parameters of the MDM model of steel R92

$K$ (MPa)	141 667
$G$ (MPa)	65 385
$C_p^0$ (MPa)	249
$\bar{C}_{c0}$ (MPa)	125
$g_1^p$ (MPa)	38 900
$g_2^p$ (MPa)	660
$g_1^c$ (MPa)	5100
$g_2^c$ (MPa)	260
$a_p$	4
$a_c$	0
$W_{pf}^{cyc}$ (MJ/m <sup>3</sup> )	4130
$W_{cf}^{cyc}$ (MJ/m <sup>3</sup> )	33

**Table 2.** Modulus of cyclic hardening  $Q_1^p(a_{e\max})$  and  $Q_1^c(a_{e\max})$  (MPa) of steel R92

$a_{e\max}$	0	0.001	0.003	0.004	0.006	0.01
$Q_1^p(a_{e\max})$	0	-92	-92	-92	-92	-92
$Q_1^c(a_{e\max})$	0	-50	-50	-50	-50	-50

Table 4 compares the calculated and experimental values of the durability of laboratory specimens under cyclic loading with time delays at maximum strain.

As a second example of the practical implementation of the MDM mathematical model under the mutual influence of the processes of creep and fatigue of the material, as well as the accuracy of determining the material parameters and scalar functions of the MDM model, the problem of determining the kinetics of stress-strain state and damage accumulation in laboratory samples of steel 316 under rigid loading – tension – compression with time delays at maximum strain and without them for two strain amplitudes [7]. The temperature in the studies is 650°C. Figure 7 shows the loading laws used in experiments [7].

The main physical and mechanical characteristics of steel 316 [7] and the material parameters at a temperature of 650°C of the damaged medium model [7–23] are given in Table. 5–8.

Figure 8 shows cyclic hysteresis loops for the loading laws shown in Fig. 7.

Figure 9 compares the results of the experiment (markers) and numerical simulation of experimental processes (solid line) by the values of stress ranges in cycles depending on the number of cycles.

Figure 10 compares the deviations of the results of numerical simulation of experimental processes and experimental data from the law of linear damage summation (light dots are experimental data, black dots are numerical results). Where  $N_h$  is the number of cycles to failure in tests with a holding duration in each

**Table 3.** Dependence of proportionality coefficients  $\lambda_c^0$  and  $\lambda_c^{II}$  on stress intensity for steel R92

$\sigma_u$ , MPa	0	50	100	150	170	250
$\lambda_c^0$ , 1/MPa h	0	0.00014	0.00014	0.00014	0.00035	0.0015
$\lambda_c^{II}$ , 1/MPa h	0	$1.3 \times 10^{-6}$	$1.3 \times 10^{-6}$	$1.3 \times 10^{-6}$	$3.2 \times 10^{-6}$	$1.75 \times 10^{-5}$

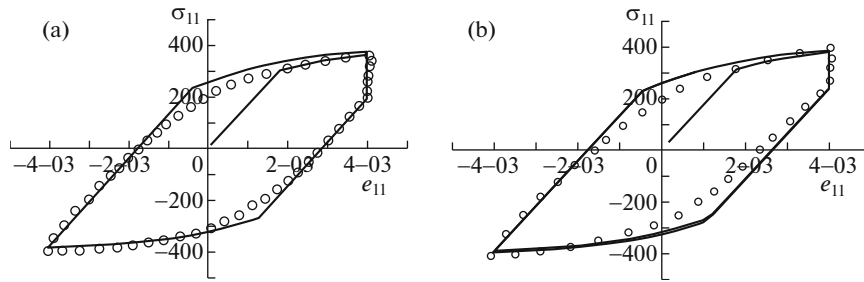


Fig. 3. Cyclic hysteresis loop for the first loading cycle: (a) – holding time 600 s, (b) – holding time 60 s.

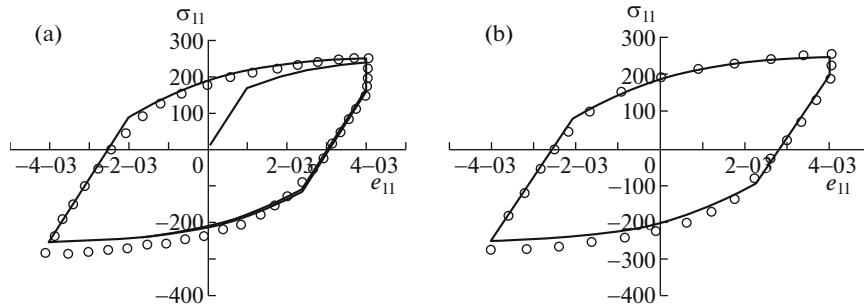


Fig. 4. Cyclic hysteresis loop for a cycle equal to half the life of the material (a) holding time 600 s, (b) holding time 60 s.

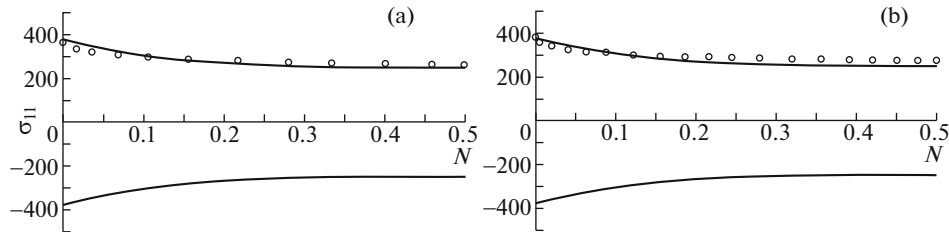


Fig. 5. Dependence of the amplitude values of stresses (MPa) on the number of loading cycles: (a) – holding time 600 s, (b) – holding time 60 s.

cycle;  $N_f$  is the number of cycles to failure in similar tests without holding;  $t_f$  is the time to failure that is determined from the curve of long-term strength at stress and holding temperature.

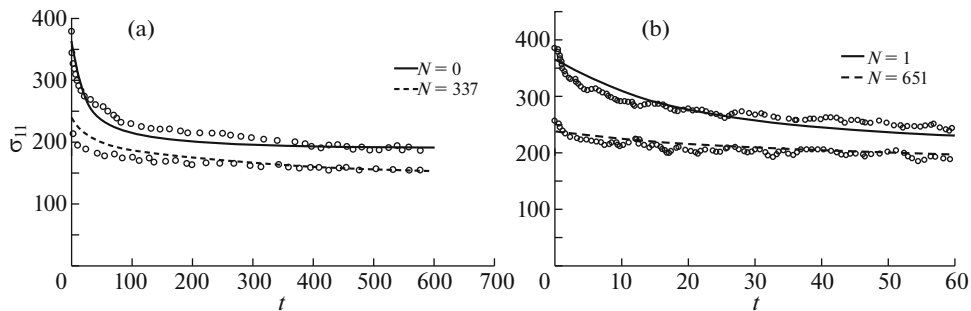


Fig. 6. Stress relaxation curves (MPa) for cyclic loading with holding at the maximum strain value: (a) 600 s, (b) 60 s.

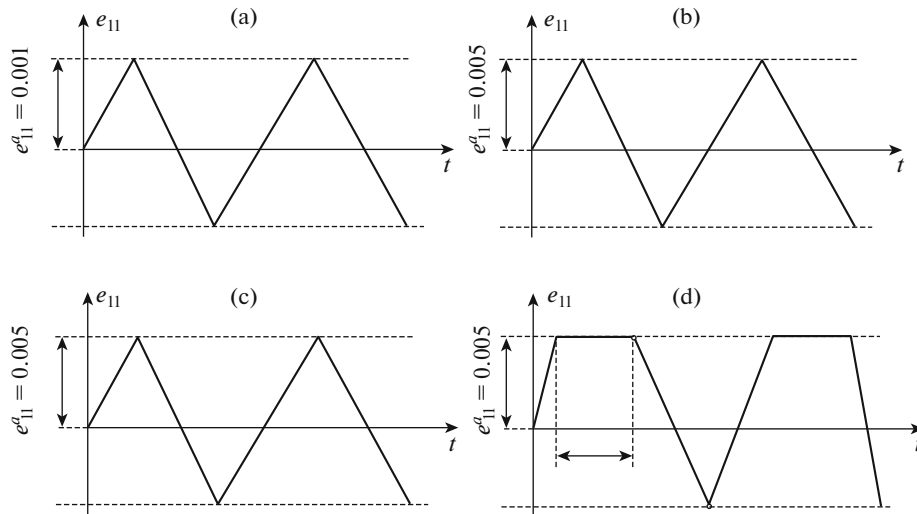


Fig. 7. The law of loading of laboratory samples from steel 316 (min).

Table 4

Holding time, s	Experimental number of cycles to failure	Estimated number of cycles before failure
60	1303	1302
180	1190	984
600	675	801

Table 9 compares the number of cycles obtained numerically and experimentally for the loading laws shown in Fig. 7.

The obtained life characteristics of structural steels given in this article demonstrate the qualitative and quantitative agreement between the numerical and experimental data and indicate the reliability of the constitutive relations of the MDM and the developed method for determining the material parameters.

Table 5. Physical and mechanical characteristics and material parameters of the MDM model of steel 316 at  $T = 650^\circ\text{C}$

$K$ (MPa)	153450
$G$ (MPa)	58450
$C_p^0$ (MPa)	74
$g_1^p$ (MPa)	6858
$g_2^p$ (MPa)	290.4
$g_1^c$ (MPa)	5100
$g_2^c$ (MPa)	260
$a_p$	3.2
$A_c$	75
$W_{pf}^{cyc}$ (MJ/m <sup>3</sup> )	820
$W_{cf}^{cyc}$ (MJ/m <sup>3</sup> )	10

**Table 6.** Loading surface radius corresponding to zero creep rate  $\bar{C}_c$  (MPa) of steel 316 at  $T = 650^\circ\text{C}$ 

$\chi_c$	0	0.01	0.05	0.08	0.1	0.12	0.14	0.16	0.17
$\bar{C}_c$	40	40	40	35	27	14	6	3	2

**Table 7.** Modulus of cyclic hardening  $Q_1^p(a_{e\max})$  and  $Q_1^c(a_{e\max})$  (MPa) of steel 316 at  $T = 650^\circ\text{C}$ 

$a_{e\max}$	0	0.001	0.003	0.004	0.006	0.01
$Q_1^p(a_{e\max})$	0	50	80	90	95	110
$Q_1^c(a_{e\max})$	0	-30	-33	-33	-33	-33

**Table 8.** Dependence of proportionality coefficients  $\lambda_c^0$  and  $\lambda_c^{II}$  on stress intensity  $\sigma_u$  for steel 316 at  $T = 650^\circ\text{C}$ 

$\sigma_u$ , MPa	0	50	100	150	170	200
$\lambda_c^0$ , 1/MPa · h	0	$9.0 \times 10^{-6}$	$9.0 \times 10^{-6}$	$9.0 \times 10^{-6}$	$2.1 \times 10^{-6}$	$7.2 \times 10^{-5}$
$\lambda_c^{II}$ , 1/MPa · h	0	$1.455 \times 10^{-6}$	$1.455 \times 10^{-6}$	$1.455 \times 10^{-6}$	$3.6 \times 10^{-6}$	$1.854 \times 10^{-5}$

The differences obtained can also be explained by the fact that the experimental data have been obtained on one laboratory sample for each loading mode.

#### 4. CONCLUSIONS

A mathematical model of the MDM that describes the processes of deformation and damage accumulation, based on the energy approach and a unified form of representation of the damage accumulation process under mechanisms that combine fatigue and material creep has been developed.

The material parameters and scalar functions included in the constitutive relations of the MDM mathematical model for a number of structural steels are obtained.

Numerical studies have been carried out to determine the kinetics of stress-strain state and damage accumulation in laboratory specimens of steels 316 and P92 under “hard” loading (controlled strains) – uniaxial tension-compression with and without time delays at maximum strain for two strain amplitudes.

**Table 9.** Comparison of calculated and experimental values of cycles to failure

Loading law	Experimental number of cycles to failure	Estimated number of cycles before failure
$e_{11}^a = 0.01$ without holding in time (Fig. 7a)	863	964
$e_{11}^a = 0.005$ without holding in time (Fig. 7b)	4000	3639
$e_{11}^a = 0.01$ with holding in time (Fig. 7c)	338	385
$e_{11}^a = 0.005$ with holding in time (Fig. 7d)	985	950

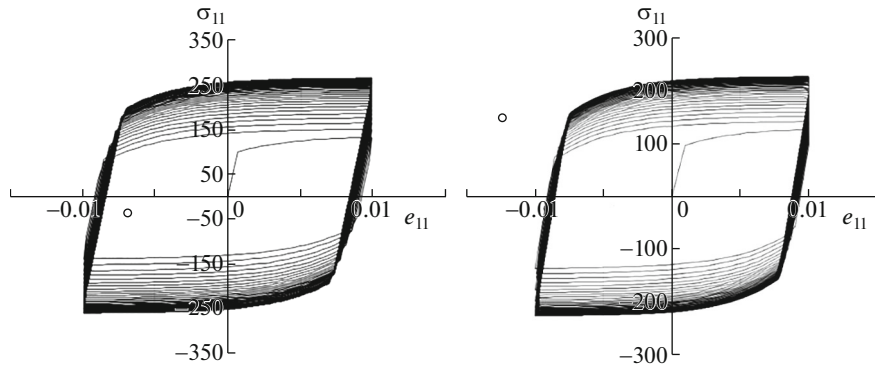


Fig. 8. Cyclic hysteresis loops for loading laws, presented in Fig. 7a and 7b.

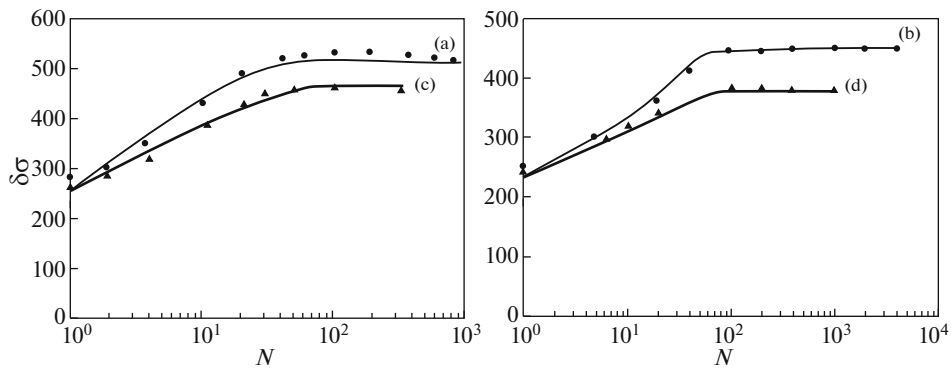


Fig. 9. Stress range (MPa) in loading cycles depending on the number of cycles for loading modes presented in Fig. 7. Fig. 9a: • – amplitude  $\epsilon_{11} = 0.01$ , ▲ – amplitude  $\epsilon_{11} = 0.01$  with holding of 30 min. Figure 9b: • – amplitude  $\epsilon_{11} = 0.005$ , ▲ – amplitude  $\epsilon_{11} = 0.005$  with holding of 30 min.

The reliability assessment results show that the developed model describes loading processes with degradation mechanisms that combine fatigue and long-term strength of the material with sufficient accuracy for engineering calculations.

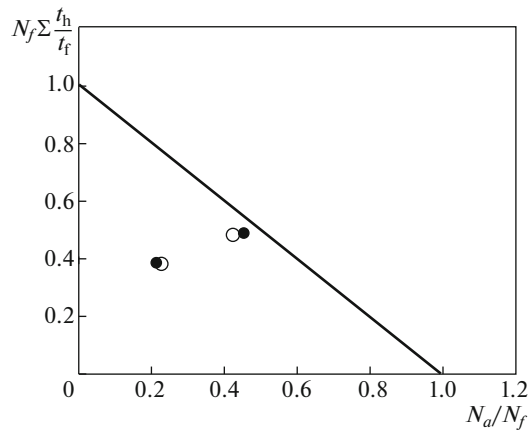


Fig. 10. Dependence of the relative operating time of the material under the mutual influence of fatigue and material strength.

The analysis of the equations of mechanics of a damaged medium presented in this article and the results of assessing their reliability allow, according to the authors, to recommend them at this stage for calculating the durability of structural units of machine-building objects operating under conditions of non-stationary thermomechanical loading under degradation mechanisms that combine fatigue and long-term strength material. At the same time, further experimental and theoretical verification of their reliability is necessary for the cases of non-stationary multiaxial stress-strain states and non-isothermal processes accompanied by a significant rotation of the main areas of stress and strain tensors (for various complex loading trajectories) [26]. It is also necessary to further analyze experimental information on the joint mechanisms of degradation of structural materials during the complex development of degradation processes caused by fatigue, unsteady creep, corrosion processes, radiation exposure, etc., and formulate on this basis mathematical models of joint processes of degradation of the initial strength properties of structural materials .

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