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Structural Optimization of a Longitudinally Moving Layered Web Based on a Multi-Criteria Approach

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Abstract—The longitudinal motion at a constant speed of a thin continuous elastic web through a system of roller bearings under the action of a given constant tension is considered. One span between adjacent supports is considered. The web is modeled by a thin layered plate hinged on two opposite edges, the remaining two sides of the plate are free. It is assumed that the plate in the process of longitudinal movement can perform small transverse vibrations. The layers of the plate from a given set of materials are arranged symmetrically to the middle surface and fit tightly to each other. The total thickness of all layers is given and is small compared to the span length and plate width. Analytical expressions are derived for the effective characteristics of the plate, as a result of which the initial composite structure can be considered as an isotropic homogeneous plate, for which the known equations for calculating the critical velocity are applied. Within the framework of multi-criteria Pareto optimization, using a numerical method of non-local optimization, the order of the layers and their thickness are determined in order to satisfy a number of selected criteria: the maximum critical divergence rate, the maximum flexural stiffness, and the minimum unit weight of the layered web. An example of the found optimal structure of the plate and the constructed Pareto front for a given set of defining parameters of the problem are given.

Keywords: multicriteria optimization, layered web, moving materials, Pareto front

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1. INTRODUCTION

Moving elastic strings, beams, membranes and plates are the most popular models in the study of moving materials. The scientific literature presents a large number of works where these models are used, leading to the solution of differential equations with partial derivatives of the second and fourth orders. Detailed reviews of works on moving materials are contained, for example, in monographs [1–3]. A number of studies have been devoted to the issues of free oscillations, including the nature of wave propagation in a moving medium and the effects of axial motion of the frequency spectrum and eigenvalues. It was shown that the natural frequency of each mode decreases with increasing transport speed and that a moving string, beam, panel, and plate will experience divergence instability at a sufficiently high speed (see, for example, [4, 5]). The task of increasing the critical value of the transport speed, at which the phenomenon of static instability (divergence) of the web (panels, plates) occurs, is relevant for many technological processes associated with the longitudinal movement of materials (production of paper and films, rotation of disks and rods, movement of conveyor belts, drive belts, etc.). As a result of the theoretical studies carried out, the dependences of critical velocities on a number of determining parameters were analytically obtained earlier [1, 2, 6, 7]. One possible way to increase the critical speed is to change the internal structure of the web itself. Layered structures and their optimization can play an important role here [8].

In practice, it may be necessary to satisfy not one, but several quality criteria when designing structures. The issues of multiobjective and multiobjective optimization in mechanics based on the Pareto and Nash approaches are also the subject of a large number of studies. The first works devoted to the application of multiobjective optimization in mechanics were published by Stadler [10, 11], Eschenauer [12, 13] and others. Let us also note Stadler's review [14] on multiobjective optimization in mechanics, the collective monograph edited by Stadler [15], and Mittenen's monograph [16] devoted to nonlinear problems of multiobjective optimization. Multicriteria approaches to the problems of optimal design of structures were discussed in [17, 18]. The issues of multi-purpose optimization of structures were also considered in relation to the problems of contact interaction and high-speed penetration of bodies into deformable media in the monograph [19].

In this article, within the framework of multicriteria Pareto optimization, using the numerical method of nonlocal optimization (genetic algorithm [20–22]), the order of the layers of a moving web (layered plate) and their thickness are determined in order to satisfy a number of selected criteria: the maximum critical divergence rate, the maximum flexural stiffness and minimum linear weight of the layered web.

2. BASIC RATIOS OF THE MECHANICAL MODEL

An elastic continuous layered web moving longitudinally at a constant transport velocity V_0, which performs transverse oscillations of small amplitude and is under the action of a given longitudinal tension, is considered. The web moves through a system of roller bearings (rolls), and in the Euler coordinate system one span is considered between two adjacent bearings. The canvas is modeled by an elastic layered plate moving at a constant longitudinal speed in the direction of the x axis, pivotally supported on rolls at the beginning and end of the span. Let us assume that the plate is effectively isotropic and homogeneous and occupies the region $\Omega = \{(x, y, z) : 0 < x < l, -b < y < b, -H/2 < z < H/2\}$ in the rectangular coordinate system x, y, z. In this case, the plate is hinged at $x = 0$ and $x = l$ and has free edges at $y = b$ and $y = -b$. The span length l, plate width $2b$, total plate thickness H and its velocity V_0 are given constants.

The dynamic behavior of a moving homogeneous isotropic web (plate) in the Euler coordinate system is described by the following partial differential equation and boundary conditions (Δ^2 is a biharmonic operator) [1, 2]:

$$m\left(\frac{\partial^2 w}{\partial t^2} + 2V_0\frac{\partial^2 w}{\partial x\partial t} + V_0^2\frac{\partial^2 w}{\partial x^2}\right) - T_0\frac{\partial^2 w}{\partial x^2} + D\Delta^2 w = 0, \tag{2.1}$$

$$(w)_{x=0,l} = \left(\frac{\partial^2 w}{\partial x^2}\right)_{x=0,l} = 0, \quad -b \le y \le b, \tag{2.2}$$

$$\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)_{y=\pm b} = \left(\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial x^2\partial y}\right)_{y=\pm b} = 0, \quad 0 \le x \le l. \tag{2.3}$$

Here, $w = w(x, y, t)$ are the lateral movements of the web, m is the mass per unit area, D is the flexural stiffness, ν is the Poisson's ratio, and T_0 is the constant mechanical tension applied to the ends of the web at $x = 0, l$. For the considered case of a layered effectively homogeneous and isotropic web, relations (2.1)–(2.3) can be used if we assume $m = m^{ef}$, $D = D^{ef}$, $\nu = \nu^{ef}$ in them, where m^{ef} is the effective mass of the plate per unit area, D^{ef} is the effective bending stiffness of the plate, ν^{ef} is the effective Poisson's ratio.

To determine the effective characteristics, we assume that the plate is symmetrically composed of an odd number $2n+1$ of elastic layers relative to the midplane, which are characterized by mass per unit area m_i, Young's modulus E_i, Poisson's ratio ν_i, and distance h_i from the midplane. In this case, the outer layers are numbered 1 and $2n+1$ (see Fig. 1).

Taking into account the symmetrical arrangement of the layers relative to the median plane ($z = 0$) and the fact that they are closely adjacent to each other, we obtain expressions for the effective bending stiffness of the plate D^{ef}, the effective Poisson ratio ν^{ef} and the effective mass of the plate m^{ef} per unit area. To do this, we apply the equations for stresses and strains and use the expression for the bending moment

$$\int_{-H/2}^{H/2}\sigma_x z\,dz = \left(\int_{-H/2}^{H/2}\frac{z^2E(z)\,dz}{1-(\nu(z))^2}\right)\frac{\partial^2 w}{\partial x^2} = D^{ef}\left(\frac{\partial^2 w}{\partial x^2}\right).$$

Taking into account the symmetry of the laying of the layers of the web structure, i.e.

$$E(z) = E(-z), \quad \nu(z) = \nu(-z)$$

we find an expression for the effective bending stiffness in the form

$$D^{ef} = 2\int_0^{H/2}\frac{z^2E(z)}{1-(\nu(z))^2}dz.$$

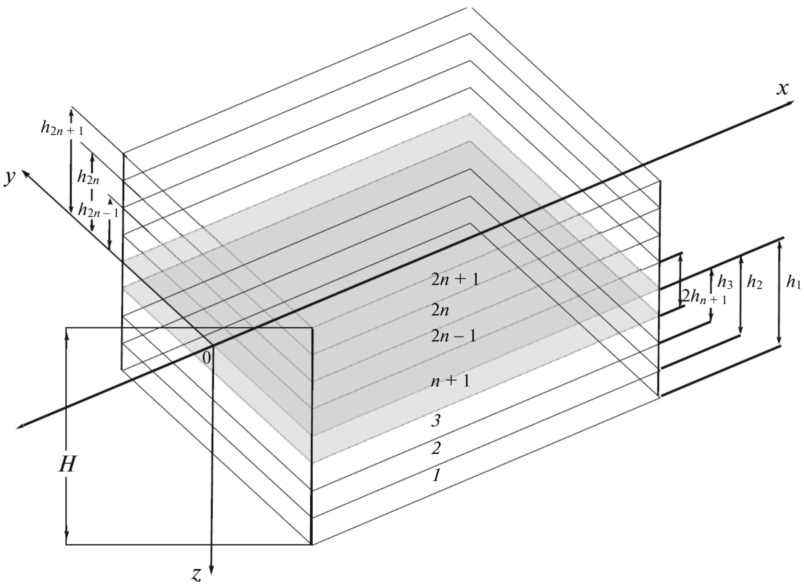

Fig. 1. Layered plate.

Using the mechanical and geometric characteristics of the web layers E_i, ν_i, h_i, we obtain the following expression:

$$D^{ef} = \frac{2}{3}\left[\frac{E_{n+1}h_{n+1}^3}{1-\nu_{n+1}^2} + \sum_{i=1}^{n}\frac{E_i}{1-\nu_i^2}(h_i^3 - h_{i+1}^3)\right]. \tag{2.4}$$

The equation for effective Poisson ratio of an inhomogeneous isotropic layered web is derived similarly. We have

$$\nu^{ef} = \frac{2}{D^{ef}}\int_0^{H/2}\frac{z^2\nu(z)E(z)}{1-(\nu(z))^2}dz = \frac{2}{D^{ef}}\left[\frac{\nu_{n+1}E_{n+1}h_{n+1}^3}{1-\nu_{n+1}^2} + \sum_{i=1}^{n}\frac{E_i\nu_i}{1-\nu_i^2}(h_i^3 - h_{i+1}^3)\right]. \tag{2.5}$$

The equation for the effective mass m^{ef} per unit area of the laminated web is obtained by direct summation of the respective masses $2n+1$ of the layers. We get

$$m^{ef} = m_{n+1} + 2\sum_{i=1}^{n} m_i. \tag{2.6}$$

One of the most important parameters characterizing the mechanical behavior of the considered moving system is the critical speed of the static form of plate buckling (critical divergence rate). In the stationary case, when all time derivatives vanish, we arrive at an eigenvalue boundary value problem for a homogeneous and isotropic plate with the equation

$$(mV_0^2 - T_0)\frac{\partial^2 w}{\partial x^2} + D\Delta^2 w = 0 \tag{2.7}$$

and boundary conditions (2.2), (2.3). In the case of a homogeneous isotropic moving canvas, an explicit expression for the critical divergence rate V_0^{div} (static instability) was obtained [1, 2]. This expression can also be written for the considered case of a layered, effectively homogeneous and isotropic web. We have

$$(V_0^{div})^2 = \frac{T_0}{m^{ef}} + \frac{\gamma_*^2\pi^2 D^{ef}}{m^{ef}l^2}, \tag{2.8}$$

where γ_* is the root of the following transcendental equation [1]:

$$\Phi(\gamma,\mu)-\Psi\left(\gamma,\nu^{ef}\right)=0,$$
$$\Phi\equiv\tanh\left(\frac{\sqrt{1-\gamma}}{\mu}\right)\coth\left(\frac{\sqrt{1+\gamma}}{\mu}\right),$$
$$\Psi\equiv\frac{\sqrt{1+\gamma}\left(\gamma+\nu^{ef}-1\right)^2}{\sqrt{1-\gamma}\left(\gamma-\nu^{ef}+1\right)^2},\quad \mu=\frac{l}{\pi b}.$$

Thus, for any layered web with a symmetrical internal structure and given parameters m_i, E_i, ν_i $(i=1,2,\ldots,n+1)$, we can determine the values m^{ef}, D^{ef}, ν^{ef} and, therefore, determine the value of the critical divergence rate V_0^{div} using expression (2.8).

3. MULTIOBJECTIVE OPTIMIZATION PROBLEM

Having the ability to form the internal structure of the web, that is, to change the filling and arrangement of layers, we can influence the effective characteristics and basic properties of the system, satisfying the selected optimality criteria. For this, it is convenient to use natural parametrization. We assume that the number r of acceptable materials for the manufacture of web layers is given. Considering that each of these materials can be numbered using one parameter, we apply natural parameterization using the scalar variable k, which can take the values k_1, k_2, ..., k_s, ..., k_r, i.e. $k\in\{k_1,k_2,\ldots,k_s,\ldots,k_r\}$. Since the layers are characterized by Young's modulus E, Poisson's ratio ν, mass per unit area m, then $E_s=E(k_s)$, $\nu_s=\nu(k_s), m_s=m(k_s)$. Thus, the layered web under consideration consists of a discrete set of layers (materials) distributed along the z axis and is characterized by a set of parameters $\{E_s,\nu_s,m_s\}$, $s=1,2,\ldots,r$. The distributions of parameters $(E(z),\ \nu(z),\ m(z))$ over the web thickness are given by piecewise constant functions defined on the interval $0\le z\le H/2$. For each point $z\in[0,H/2]$, these functions take values from a given finite set, i.e. $E(z)\in\{E_s\}$, $\nu(z)\in\{\nu_s\}$, $m(z)\in\{m_s\}$, $s=1,2,\ldots,r$.

Let us apply the parametrization described above, using a piecewise constant function $k=k(z)$ $(z\in[0,H/2])$ that takes values $k=k_s=s$ from a given set, i.e. $k\in\{k_s=s\}$. The following relations are valid:

$$E_s=E(k(z))_{k=k_s=s},\quad \nu_s=\nu(k(z))_{k=k_s=s},\quad m_s=m(k(z))_{k=k_s=s}. \tag{3.1}$$

We will consider the function $k=k(z)$ as the desired design variable, which determines the structure of the layered web under consideration. As optimization criteria (objective functionals), we choose the critical divergence rate of the web, the reciprocal of the total effective mass, and the effective bending stiffness. Let us define the vector functional $J=J(k(z))$ in the form

$$J=J(k(z))=\begin{Bmatrix}J_1\\J_2\\J_3\end{Bmatrix},$$
$$J_1=V_0^{div}(k(z)),\quad J_2=\frac{1}{m^{ef}(k(z))},\quad J_3=D^{ef}(k(z)) \tag{3.2}$$

and formulate the following vector (multi-objective) optimization problem:

$$(J)_*=\max_k J(k(z)),$$
$$k(z)\in\{k_s\}:\ z_{i-1}<z<z_i, \tag{3.3}$$
$$i=1,2,\ldots,n+1,\quad s=1,2,\ldots,r,\quad z_1=H/2,\quad z_{n+1}=0.$$

The maximum in (3.3) is understood in the Pareto sense [17, 18]. This means that the expression

$$k_*=\arg\max_k J(k) \tag{3.4}$$

is the optimal solution of the formulated problem with a vector functional if there is no other design variable k_{**} satisfying the condition $k_{**}\in\{k_s=s\}$ such that

$$J_i(k_{**}) \geq J_i(k_*), \quad i = 1,2,3 \tag{3.5}$$

and at least one of the components (j-th) of the vector functional satisfies the strict inequality

$$J_j(k_{**}) > J_j(k_*). \tag{3.6}$$

To solve the multiobjective optimization problem (3.3), we use the target weighting method. Let us construct the weighting functional J_C (preference functional) as a sum

$$J_C = \sum_{i=1}^{3} C_i \hat{J}_i, \tag{3.7}$$

with weight coefficients C_i, satisfying the following conditions:

$$C_i \geq 0, \quad i = 1,2,3, \quad \sum_{i=1}^{3} C_i = 1. \tag{3.8}$$

By $\hat{J}_i$ in (3.7) we mean the dimensionless variables $\hat{J}_i = J_i/J_i^0$, $i = 1,2,3$, where J_i^0 are some characteristic quantities of velocity, reciprocal mass, and flexural rigidity ("the cap" for the considered variables in relation (3.7) is omitted below).

In accordance with the target weighting method used, the solution of the multi-objective optimization problem (3.3) and the search for the set of Pareto-optimal layered web structures reduces to solving the following problem of determining the maximum of the scalar functional

$$J_C^* = J_C\left(k_*\right) = \max_{k \in \{k_s = s\}} J_C\left(k\right) \tag{3.9}$$

under restrictions (3.8) imposed on the weight coefficients. Thus, for any given set of weight coefficients that satisfy condition (3.8), the optimal structure of a layered plate (web) will be determined, which has the maximum critical divergence rate and, at the same time, the maximum bending stiffness and minimum mass per unit length.

4. NUMERICAL SOLUTION BASED ON THE GENETIC ALGORITHM

The problem of maximizing the functional J_C (3.9), (3.8) for various values of the problem parameters l, b, H, n, r, T_0, and the characteristics of the materials used was solved numerically using a genetic algorithm [20, 8, 19]. This global optimization method was chosen due to the large number of parameters in order to avoid the difficulties caused by the possible appearance of local extrema in the problem under consideration.

According to the terminology adopted for this method, each allowable distribution of materials over layers is taken as an "individual" belonging to a certain population (generation), and is characterized by a set of values $k\left(j,i\right)$, where j is the number of the "individual" in the generation, and i is the layer number. The number of "individuals" N in the population is given even and is unchanged in the process of further updating of generations. The initialization of the algorithm consists in generating an initial generation from randomly generated admissible distributions $k\left(j,i\right)$. Each "individual" corresponds to some value of the functional J_C being maximized. This is followed by an iterative process of successive formation of new improved generations of "individuals". This process is based on the formation of $N/2$ parental pairs of "individuals" for each generation and obtaining $N/2$ pairs of offspring from them using the crossover operation, which form the next generation. In this case, both probabilistic and deterministic selection mechanisms are involved: each parent is selected from a randomly generated subgroup N^T of "individuals" so that the value of J_C corresponding to was at maximum. The crossing operation for each generated pair is performed with a given probability p_{co}. As a result, the offspring either completely copy the parents, or a partial exchange of their properties occurs as a result of crossing at an arbitrarily chosen point (i). For the formed new generation of "individuals" the mutation operation is applied. With a very small given value of probability, a change of the "individual" is carried out at an arbitrarily chosen point (i). The mutation operation is important for overcoming a possible hit in a local maximum. For the finally formed generation, the best "individual" is established, that is, the distribution of $k_*\left(j,i\right)$ materials over the layers of the web, in the sense of fulfilling conditions (3.9), (3.8). This best solution is remembered, and the algorithm proceeds to the next iteration to build a new improved generation. The constructed new optimal solution is compared with the solution fixed at the previous iteration, and the best of them is saved. The

Table 1. Material characteristics

Material, s	E, kg/cm^2	ν	m, g/m^2
1	14.0×10^6	0.460	450
2	1.7×10^6	0.038	400
3	9.36×10^6	0.017	410

algorithm is executed with a given number of iterations or until a certain convergence condition is met. Typically, multiple reinitializations are performed on the same source data.

Let us give an example of solving the problem of optimizing the web structure for the following values of the parameters of the problem: $l = 1.2$ m, $b = 0.47$ m, $T_0 = 16$ n/m, $H = 10^{-3}$ m, $n = 10$, $r = 3$. Characteristics of materials considered acceptable for the optimal design of an inhomogeneous isotropic layered web are presented in Table 1.

The dependence of γ_* on the effective Poisson's ratio ν^{ef} for the considered parameters of the problem is shown in Fig. 2.

The computational process was performed for 500 generations (populations formed sequentially) with the following algorithm parameters $n = 10$, $N = 10$, $N^T = 4$, $p_{co} = 0.5$, $p_m = 0.05$ for 10 initializations.

For simplicity and clarity, let us give an example when the vector functional J has only two components J_1 and J_2. In expressions (3.7), (3.8) we should assume $C_3 = 0$. Figure 3 shows the Pareto front in the axes J_1, J_2. Each point of the front corresponds to some value of the weight coefficient C_1 (it follows from condition (3.8) that $C_2 = 1 - C_1$) and the corresponding maximum value of J_C^*.

Points numbered 1–8 correspond to the values of the coefficient $C_1 = 1$; 0.3, 0.2; 0.15; 0.12; 0.1; 0.008 and $C_1 = 0$, respectively.

Figure 4 shows the dependence of the maximum value J_C^* of the optimized functional on the coefficient C_1.

For the case $C_1 = 1$, Fig. 5 shows the optimal distribution of materials over the layers of the web: materials $s = 1, 2, 3$ are marked in white, pink and blue, respectively. It can be seen that in order to fulfill criterion (3.9), it is efficient to place a harder material on the upper and lower layers of the web, and soft materials closer to the middle surface.

The convergence of the genetic algorithm for this case is shown in Fig. 6.

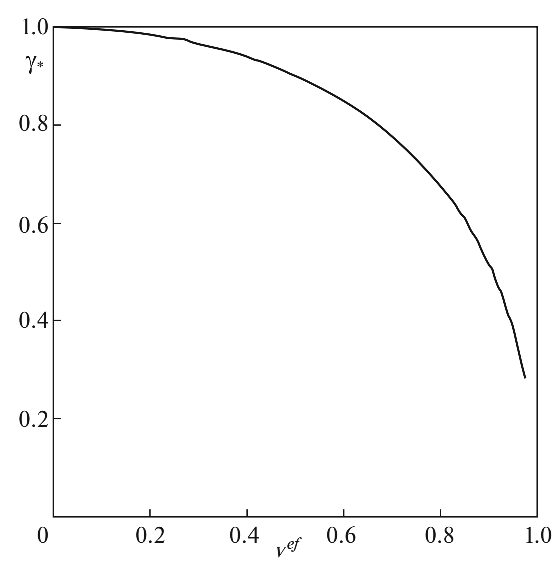

Fig. 2. Dependence of the parameter γ_* on the effective Poisson's ratio ν^{ef}.

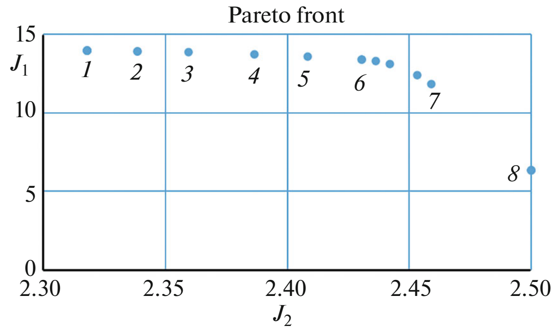

Fig. 3. Pareto front.

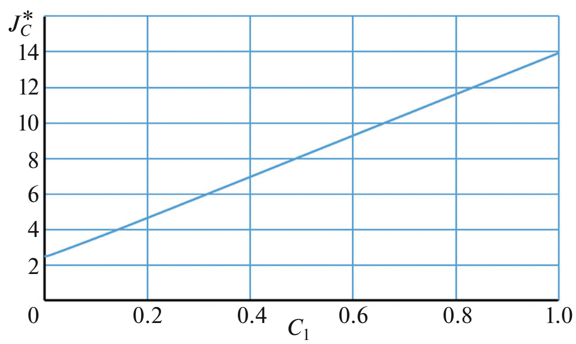

Fig. 4. Dependence of the maximum value J_C^* of the optimized functional on the weight coefficient C_1.

5. SOME CONCLUSIONS AND REMARKS

The article considers a multicriteria statement of the problem of optimizing the layered structure of a moving elastic web, modeled by a thin layered plate. Using an approach based on the construction of Pareto-optimal solutions for a problem with several objective functionals and the use of a numerical method of non-local optimization (genetic algorithm), the optimal distributions of materials from a given set over the layers of the canvas are found. Technologically important parameters of the moving canvas were chosen as optimization criteria: critical divergence rate (static form of buckling), linear mass (or its

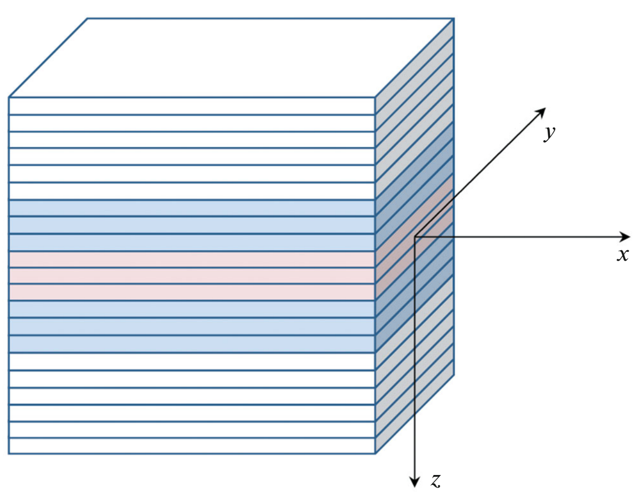

Fig. 5. Optimum distribution of materials over the layers of the web at $C_1 = 1$.

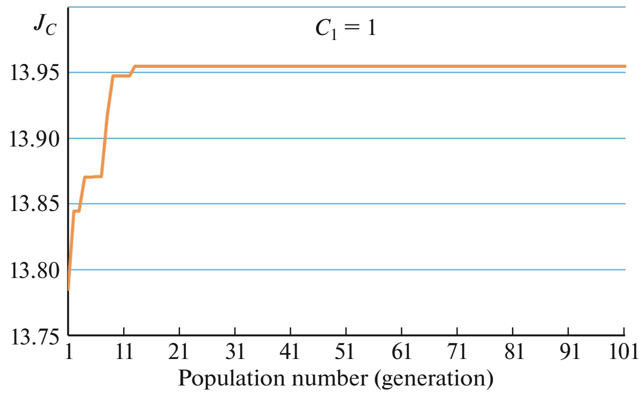

Fig. 6. Convergence of the genetic algorithm for $C_1 = 1$.

reciprocal) and bending stiffness. Multi-criteria Pareto optimization allows taking into account the selected criteria with different weighting coefficients depending on their significance in certain conditions of production or operation of the web. To perform the optimization algorithm, the effective characteristics of the layered material were analytically determined, which made it possible to consider it as effectively isotropic and homogeneous and apply the previously obtained analytical expressions for the critical divergence rate of a moving elastic plate to it. An example of constructing an optimal solution is given, the graphs show the results of calculations and data on the convergence of the genetic algorithm. It should be noted that the found optimal distribution of materials over the layers of the canvas can serve as an answer to several questions of the designer at once: how many layers should be used to make the optimal design, what material to take for each layer, what thickness each layer will be and what will be the order of laying these layers. Note that the described multicriteria approach to solving the problem of optimizing the layered structure of a moving web can also be applied in the case of thermal effects on it. In this case, the thermoelastic properties of the selected materials will also be taken into account in the optimization process.

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