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Problems of Experimental Verification of the Theory of General Relativity

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Abstract—On April 14, 2007, at a meeting of the American Physical Society in Jacksonville (Florida), the famous American gyroscopist Francis Everitt reported on the preliminary results of an experiment on a special artificial Earth satellite with cryogenic gyroscopes. In the experiment, which was started in August 2005 and completed in August 2006, a gyroscope was used to measure the curvature of space-time in the vicinity of Earth, which is determined by the distribution of matter due to the basic postulate of general relativity. In this note, we discuss the difficulties that had to be overcome in preparing this experiment.

Keywords: gyroscope, precession, general theory of relativity

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The idea of the experiment is to implement the effect of parallel transfer of a vector along a closed trajectory on the tested manifold. The role of the transferred vector is played by the kinetic moment of the gyroscope, the closed trajectory is the trajectory of the satellite, passed during the year (6227 revolutions). If the tested manifold were Euclidean, then as a result of such a transfer, the vector would coincide with itself. If the space is not Euclidean, then the vector will rotate by an angle, which is a measure of the curvature of this space. In general relativity, the curvature tensor is related to the momentum energy tensor by the well-known Hilbert formula, which makes it possible to calculate the expected effect for the experiment under consideration. It turns out to be equal to 7" per year of flight.

This experiment was proposed in 1960 by the American physicist L. Schiff. It was still being discussed when, in 1967, the French theoretical physicist R. Mathey showed in a paper published in the Reports of the Paris Academy of Sciences that such an experiment was not feasible in principle. The fact is that even a perfectly made gyroscope has a temperature different from absolute zero, and chaotic vibrations of the atoms of the crystal lattice lead to random precession of the gyroscope. Mathey calculated that for the conditions of the Schiff experiment, this drift is 3.5″ per year. Since such a drift is commensurate with what we need to determine, we cannot speak of any reliability of the experiment.

Such a pivotal motion of a parallel-transferable vector is called geodesic precession. On the surface of Earth, it is well known to navigation specialists. For example, if a torpedo tube for holding a target in a torpedo boat is stabilized relative to the vertical, then if it moves along a closed curve on the surface of the water area, the tube will turn around the vertical by an angle equal to the Gaussian spherical excess (Fig. 1). This is the same geodesic precession. In Fig.1, the gyroscope described an eighth of a sphere as an example. It must be emphasized that geodesic precession is a purely geometric effect. It does not depend on the magnitude of the gyroscope's kinetic momentum, nor on any kinematic parameters of motion along the trajectory, nor on any purely physical conditions.

Mathey's paper caused a great resonance. Although there are only a few people in the world who are professionally interested in general relativity, the question of the capabilities of a cryogenic gyroscope was of great interest to very many people. Here, we must remember that Russia's naval doctrine is based on missile-carrying nuclear submarines. Unlike the Americans, for whom aircraft carriers are the main striking force. In order for a submarine to be submerged for 2–3 months, ultra-high-precision gyroscopes are needed. Therefore, what was supposed to be used to test general relativity could find a very specific practical application.

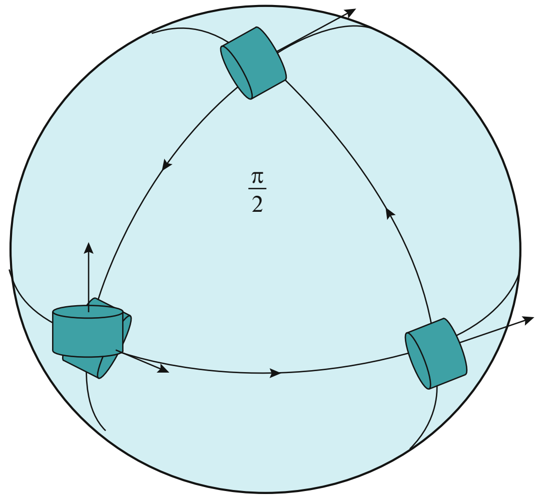

Fig. 1.

It followed from Mathey's paper that the error of a cryogenic gyroscope falls like the square root of the absolute temperature. This means that when transiting from room temperature to the temperature of a cryogenic gyroscope, the accuracy increases by only one order of magnitude. To do this, it was not worth creating a whole branch of industry. In Russia, work has already been launched in this direction. Mathey's paper put an end to them.

In 1975, specialists interested in the problem turned to then director of the Institute for Problems in Mechanics, Russian Academy of Sciences, A.Yu. Ishlinsky with a request to clarify the situation. The point is that Mathey's paper did not contain the derivation of his famous formula, as is customary in the Reports of the Academy of Sciences. Therefore, checking how right he was required additional calculations.

An analysis of Mathey's result performed at that time at the Institute for Problems in Mechanics of the USSR Academy of Sciences of is presented below.

A dynamically symmetric rigid body, the main, central moments of inertia of which are $A = B, C$, rotates with angular velocity ω, which at the initial time coincides with the symmetry axis of the body. The direction vector of this axis is **e**. The body rotates in empty space, there are no forces and moments. Under these conditions, the center of mass of the body is motionless, its angular motion is considered in inertial reference frame X, Y, Z with the center at the center of mass and with the Z-axis directed at the initial time along vector **e**. The body has temperature TK and its angular motion, due to the thermal oscillations of its atoms, and is the subject of further analysis.

Let us calculate the kinetic momentum of the body (Fig. 2)

$$\mathbf{G} = m(\mathbf{R}_i + \mathbf{r}_i) \times (\dot{\mathbf{R}}_i + \dot{\mathbf{r}}_i) = m\left(\mathbf{R}_i \times \dot{\mathbf{R}}_i + \mathbf{R}_i \times \dot{\mathbf{r}}_i + \mathbf{r}_i \times \dot{\mathbf{R}}_i + \mathbf{r}_i \times \dot{\mathbf{r}}_i\right),$$

where $\mathbf{R}_i$ is the neutral position of the marked atom, $\mathbf{r}_i$ is the displacement vector of this atom during its vibrations, and m is the mass of the atom (summation over repeated indices).

Due to the absence of external moments $\dot{\mathbf{G}} \equiv 0$. The first term in the written sum is the kinetic momentum of the "cold" gyroscope, the derivative of it has form

$$\frac{d}{dt} m(\mathbf{R}_i \times \dot{\mathbf{R}}_i) = -m(\mathbf{R}_i \times \ddot{\mathbf{r}}_i + \mathbf{r}_i \times [\boldsymbol{\omega} \times [\boldsymbol{\omega} \times \mathbf{R}_i]]) = \mathbf{M}. \quad (1)$$

In formula (1), term $\mathbf{r}_i \times \ddot{\mathbf{r}}_i$ is omitted, the order of which in comparison with term $\mathbf{R}_i \times \ddot{\mathbf{r}}_i$ is equal to $r_i / R_i = 10^{-9}$.

The left-hand side of this equation can be written as [2]

$$\frac{d}{dt}(m\mathbf{R}_i \times \dot{\mathbf{R}}_i) = A\mathbf{e} \times \ddot{\mathbf{e}} + H\dot{\mathbf{e}} + \dot{H}\mathbf{e}, \quad (2)$$

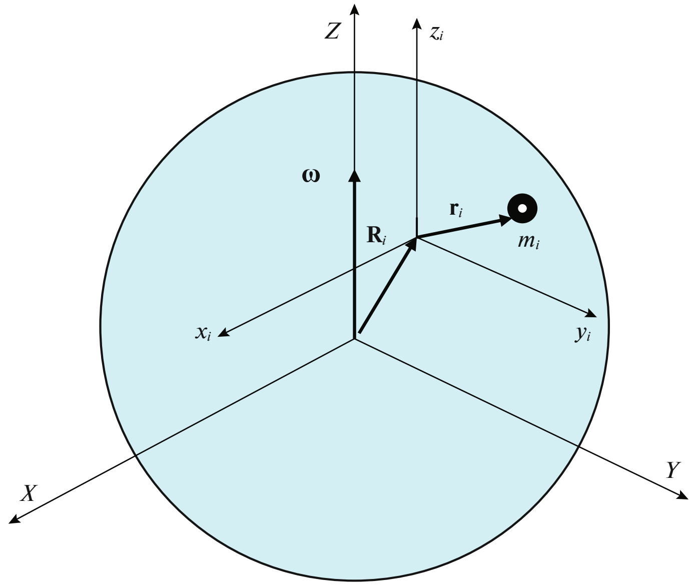

Fig. 2.

where $H = C\omega$ is the intrinsic momentum of the gyroscope, and $\omega = \boldsymbol{\omega} \cdot \mathbf{e}$ is the projection onto the axis of symmetry, coinciding at the initial moment of time with the modulus of angular velocity $\boldsymbol{\omega}$.

From (1) and (2), $\dot{H} = \mathbf{M} \cdot \mathbf{e}$ follows. The change in H is not of interest, so this equation is not considered further. Assuming that H is large, and also keeping in mind the exceptionally small level of the perturbation on the right-hand side of Eq. (1), we restrict ourselves further to the precessional part of equation

$$H\dot{\mathbf{e}} = \mathbf{M} - (\mathbf{M} \cdot \mathbf{e})\mathbf{e}. \quad (3)$$

Since $\dot{\mathbf{e}} = (\dot{\beta}, -\dot{\alpha}, 0)$, where α and β are small angles of rotation of vector $\mathbf{e}$ around the x- and y-axes, respectively, then Eq. (3) takes form

$$H\dot{\beta} = -m(Y_i\ddot{z}_i - Z_i\ddot{y}_i) - m\omega^2 Y_i z_i,$$
$$H\dot{\alpha} = m(Z_i\ddot{x}_i - X_i\ddot{z}_i) - m\omega^2 X_i z_i.$$

We will consider the components of vector $\mathbf{r}_i$, which determines the vibrations of the atom, to be random functions of time [4, 5] with correlation function

$$K[x_i] = K[y_i] = K[z_i] = \tau^2 V^2 \exp(-|t - t'|/\tau),$$

where V^2 is the average value of the square of the speed of random oscillations of an atom, τ is the correlation time constant and calculate the dispersion of angle α: (similar calculations can be done for angle β). To do this, we first find the correlation function of corresponding equation

$$K[H\dot{\alpha}] = m^2(Z_i^2 + X_i^2)K[\ddot{x}_i] + m^2\omega^4 X_i^2 K[x_i]. \quad (4)$$

Taking into account that $\sum m(Z_i^2 + X_i^2) = A$, $\sum mX_i^2 = C/2$, and $mV^2 = \langle E \rangle/3$, where $\langle E \rangle$ is the average value of the total energy of atomic vibrations, we obtain

$$K[H\dot{\alpha}] = \frac{\langle E \rangle}{3\tau^2}(A + C\tau^4\omega^4/2)e^{-|t-t'|/\tau}.$$

Since $\tau = 10^{-6}$, $\omega = 2\pi \times 10^3$, then, neglecting $\tau^4\omega^4$ in comparison with unity, we write

$$K[H\alpha] = \frac{A\langle E\rangle}{3\tau^2}\int_0^t dt\int_0^{t'} e^{-|t-t'|/\tau}dt'$$
$$= \frac{A\langle E\rangle}{3\tau}(\tau(e^{-|t|/\tau} + e^{-|t'|/\tau} - e^{-|t-t'|/\tau} - 1) + |t| + |t'| - |t-t'|),$$

which allows us to calculate dispersion

$$D[H\alpha] = K[H\alpha]_{t=t'} = 2A\langle E\rangle t/3\tau,$$

(linear terms in t are left, which are predominant in comparison with those discarded).

Hence the root mean square deviation of angle α is

$$\sqrt{\alpha^2} = \frac{1}{H}\sqrt{\frac{2A\langle E\rangle t}{3\tau}}. \quad (5)$$

The relationship between the average energy of atomic oscillations and the absolute temperature of the body follows from Debye heat capacity theory [5]

$$\langle E\rangle = \frac{9kT^4}{\theta^3}\int_0^{\theta/T}\frac{\varphi^3 d\varphi}{e^\varphi - 1}. \quad (6)$$

Formula (5), in which $\langle E\rangle$ is determined by formula (6) and represents the desired result, relating the drift of a warm gyroscope to its temperature without any restrictions on the latter.

Two consequences from Debye formula (6) are commonly used at high and low temperatures. At room temperatures, $T \to \infty$, and formula (6) gives Dulong and Petit law $\langle E\rangle = 3kT$. Substituting it into (5) we get

$$\sqrt{\alpha^2} = \sqrt{\frac{2AkTt}{C^2\omega^2\tau}}, \quad (7)$$

which coincides with the result given in [1] without derivation for the case of a ball $A = B = C$. In [1], the following data were used to calculate the formula

$$A = C = 3 \text{ g cm}^2, \quad \omega = 2\pi\times 10^3 \text{ s}^{-1}, \quad k = 1.38\times 10^{-16} \text{ g cm}^2/\text{s}^2\text{deg},$$
$$\theta = 252°\text{K}, \quad t = 1 \text{ year} = 3.15\times 10^7 \text{ s}, \quad \tau = 10^{-6} \text{ s}$$

for two temperatures. For $T = 293$ K, using formula (7), we find $\sqrt{\alpha^2} = 30''$ for the year of flight. In the case of a cryogenic gyroscope, $T = 4$ K, and by the same formula, following [1], we obtain $\sqrt{\alpha^2} = 3.5''$. For a year of flight, the geodesic precession of the gyroscope, according to [2, 3], is 7″. These calculations made the author doubt the possibilities of the experiment described in [6].

Meanwhile, at temperatures close to absolute zero, from (6) for $T \to 0$ we have

$$\int_0^\infty \varphi^3/(e^\varphi - 1)d\varphi = \pi^4/15,$$

and formula (6) takes form

$$\langle E\rangle = 3\pi^4 kT^4/5\theta^3.$$

Substituting this expression into Eq. (5) allows us to obtain the root-mean-square deviation of angle α for low temperatures in form

$$\sqrt{\alpha^2} = \frac{\pi^2 T^2}{H\theta}\sqrt{\frac{2Akt}{5\tau\theta}}. \quad (8)$$

For temperature $T = 4$ K, using this formula, we obtain $\sqrt{\alpha^2} = 0.03''$, which is two orders of magnitude less than that obtained in [1].

Note that in this calculation, the characteristic Debye temperature was assumed to be the value for niobium 252°K. Meanwhile, for quartz, the characteristic temperature is higher and the calculated drift for a real cryogenic gyroscope will be even smaller.

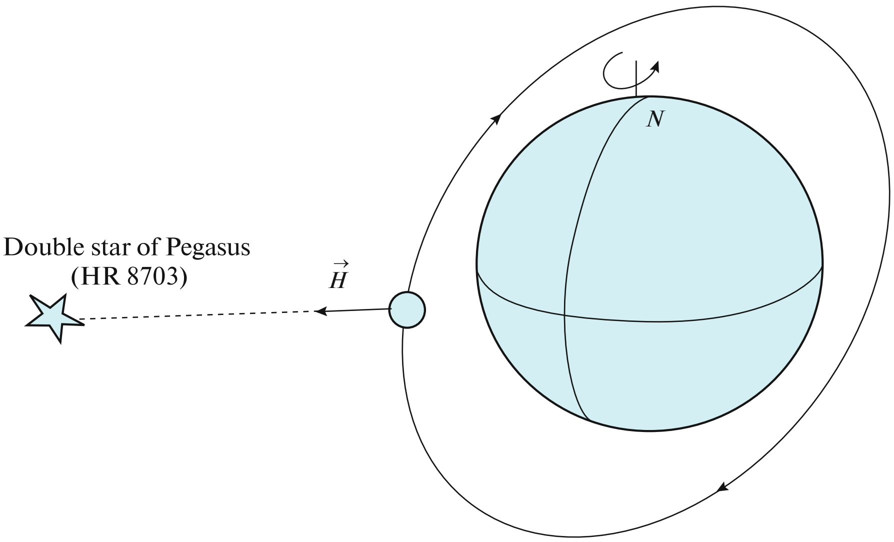

Fig. 3.

Thus, there is no thermodynamic obstacle to test the general theory of relativity with a gyroscope. If such an experiment is not feasible, then it is for some other reason.

In the same year (1975), this result was handed over to customers, and in 1995, after it ceased to have a non-public status, the refutation of R. Mathey's result was reported in INRIA (France). It was published in our press [9].

In the last years of the second millennium, the work on the experiment in the United States entered the final phase and on April 20, 2004, with four cryogenic (temperature 1.8°K) gyroscopes, the satellite was launched into a circular, near-polar orbit with an altitude of 642 km (Fig. 3). The initial direction of the kinetic moments of the gyroscopes was oriented to the reference star IM Pegasus (binary star HR 8703). The scientific phase lasted 353 days. The information volume collected was 1 terabyte. In August 2006, the experiment was completed. Another year was spent processing the received data, and the final results were summed up in December 2007. With an accuracy better than 1%, the theoretical value of 6.606" per year was confirmed. The cost of the experiment was $760 billion.

Thus, for the first time in the history of science, the fact of the curvature of our space-time near Earth is confirmed by a direct laboratory experiment.

There is an opinion that until now there were two facts confirming the theory: the redshift and the precession of the perihelion of Mercury.

Actually it is not. The red shift can also be explained within the framework of the special theory of relativity, and to explain the shift of the perihelion of Mercury, it is enough to take into account the dipole moment of the Sun's gravitational field, abandoning the assumption that the Sun is a uniform ball. Thus, the experiment completed in 2006 is so far the only reliable confirmation of general relativity.

The role of this experiment is not limited by the fact of confirmation of general relativity. Other reasons are also known that could lead to the precession of the gyroscope. The Lense–Thirring precession is determined by taking into account the rotation of the Earth. In addition, the Sun and the Moon also lead to the curvature of space near Earth. These effects are known, they appear only in the third decimal place after the decimal point.

There are two other controversial sources of gyroscope precession: Thomas precession [7] and Logunov precession [8].

Thomas precession is an indisputable fact of relativistic kinematics, when the composition of three motions with constant velocities, the vectors of which form a flat triangle, is not an identical transformation, as it would be in the case of classical kinematics, but a uniform rotation in the plane of this triangle. In the case of a relativistically translational motion of some non-inertial trihedron along a circle, a continuous accumulation of the angle of rotation of this trihedron also occurs.

These facts are purely mathematical, they follow from the non-commutativity of the Lorentz group, and they are indisputable. Disputable is the identification of this kinematic rotation with the rotation of any physical object located in this trihedron (the spin of an electron moving around the nucleus in an atom, or the kinetic moment of a gyroscope onboard a satellite).

The Thomas precession for such physical objects has not yet been experimentally confirmed, and the experiment we are discussing simply refutes the fact of its existence.

The situation is similar with precession according to Logunov. An alternative theory of gravity according to Logunov starts from a flat space-time with an indefinite metric, followed by the calculation of gravitational forces using the Hilbert formula.

In flat space-time, there is no geodesic precession, and the effect of gyroscope precession, according to Logunov, is not kinematic, but dynamic and is expressed by formula

$$\Omega = \frac{3}{2}\left[\mathbf{v}, \nabla\left(\frac{GM}{c^2 r}\right)\right] = 0.8'' \text{ in year.}$$

In fact, there is no dynamics here, since the resulting expression does not depend on the magnitude of the gyroscope's angular momentum. It is just another derivation of the formula for the Thomas kinematic precession.

And this result is also refuted by the experiment.

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