Elasto-Plastic Stress Analysis in a Tube Made of Isotropic Material and Subjected to Pressure and Mechanical Load

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Abstract—This article deals with the study of elasto-plastic stress distribution in a tube made of isotropic material (*say* steel/copper) and subjected to uniform pressure and mechanical load. Seth's transition theory and generalized strain measure are used for finding the governing equation. Mathematical modeling is based on stress −strain relation and equilibrium equation. Analytical solutions are presented thick walled tube made of steel and copper materials. The effects of different pertinent parameters (*i.e.* load and pressure) are considered for tube made of steel/copper material. The behaviour of stress distribution, and pressure rise are investigated. From the obtained results, it is noticed copper material tube requires higher dimensionless pressure to yield at the internal surface in comparison to steel material. The value of pressure decreases with increasing mechanical loads. By applying mechanical loads, the values of hoop radial stresses are increasing at the external surface of the contraction/extension region of tube. The theoretical results are validated by comparing them with those obtained by Seth after performing some significant calculation examples.

1. INTRODUCTION

Elastic-plastic analyses in thick-walled tubes have attracted a lot of interest due to their important applications in engineering. The analytical solutions of stress distribution and displacement are given for idealized elastic-plastic by Timoshenko [1] and work hardening Chadwick [2] for homogeneous materials. Tube is a long hollow cylindrical and a hard hollow body which usually has a round cross-section, but can also be oval, square, rectangular or more complex in profile. It's used to convey liquid, gas, protect electrical, optical cables, wires, solid matter and wires and, on the other, as a construction element. The terms pipe and tube are almost interchangeable, although minor distinctions exist, a tube has tighter engineering requirements than a pipe. Both pipe and tube imply a level of rigidity and permanence. A tube and pipe may be specified by standard pipe size designations, e.g., nominal pipe size, or by nominal outside or inside diameter and/or wall thickness. A sufficiently long tube subjected to internal pressure is one of the classical problems in engineering mechanics. In the purely elastic domain, many researchers have explored the mechanical behaviors of the functionally graded material long tube such as pressure vessels and cylinders by assuming different kinds of functions of Young's modulus by You et al. [3] and Sburlati [4]. Bland [5] has investigated the problem of thick-walled tubes of work hardening material subjected to pressure and using thermal gradients condition. Gamer et al. [6] obtained the solution of elasto-plastic stress distribution in rotating tube by using Tresca's yield condition. Bree [7] investigated plastic stress deformation in a closed tube due to the interaction and thermal stresses. Mufit et al. [8] have investigated the stress distribution in a heat generating tube with yield stress and Yusuf et al. [9] studied plastic deformation in a tube with free ends subject to energy generation by using Tresca's yield condition and its associated flow rule. Figueiredo et al. [10] investigated stress analysis in a thick-walled functionally graded material pipes by using the von Mises yield criterion. El-Megharbel et al. [11] presented elastic−plastic bending of tubes and sections made of strain-hardening materials with different shape. Song et al. [12] have discussed plastic deformation in metal tubes with lateral blast loads. The deflection and the deformation angle of the tubes calculated numerically. Kozlovsky et al. [13] studied collapsible thin and thick-wall tubes by using general tube law. Xin et al. [14] investigated stress distribution in a functionally graded
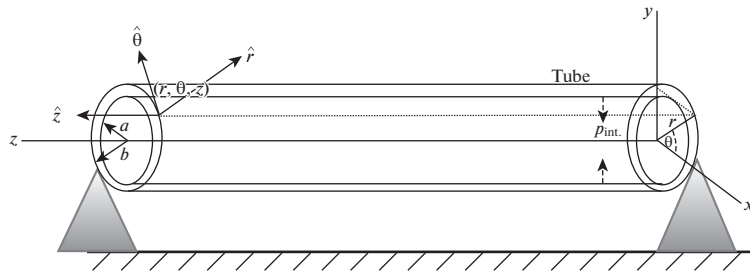
Fig. 1. Geometrical configuration of tube deformation.

thick-walled tube subjected to internal pressure by using the assumption of a uniform strain field within the representative volume element and the Tresca yield criterion. Matvienko et al. [15, 16] investigated elastoplastic deformation of dispersion-hardened aluminum tube under internal and external pressure. After that, Matvienko et al. [17] discussed plastic deformation in a tube from dispersion-hardened aluminum alloy in an inhomogeneous temperature field. This model is based on the principles of mechanics of deformable solids and equations of the physical plasticity theory. Qian et al. [18] studied mechanical properties of highly efficient heat exchange tubes. The objective of this article is to be investigating the solution of the elasto-plastic behavior in a mechanically loaded steel/ copper material tube under uniform pressure by using transition theory and generalized strain measure. In this present study, we discussed the effect of stress distribution and pressure in the contraction and the extension region of a thick walled tube under mechanical loads.

2. MATHEMATICAL MODEL

Consider a thick walled cylindrical tube made of isotropic materials (i.e. steel/copper), with an internal radius a and external radius $b(a < b)$, and subjected to uniform pressure p respectively. Further, if we assume that there are no body forces, body couples and couple stresses on the tube, and if only a steady deformation problem is considered as shown in Fig. 1.

3. BASIC GOVERNING EQUATION

Assuming that the tube is so long enough such that longitudinal extension can be neglected. Therefore the components of displacement in cylindrical polar coordinates (r, θ, z) are given [19]:

$$u = r(1 - \eta), \quad v = 0, \quad w = dz, \tag{3.1}$$

where η is a function of r. The Almansi strain components are given [20]:

$$\varepsilon_{rr} = \frac{1}{2}[1 - (r\eta' + \eta)^2], \quad \varepsilon_{\theta\theta} = \frac{1}{2}[1 - \eta^2],$$

$$\varepsilon_{zz} = \frac{1}{2}[1 - (1 - d)^2], \quad \varepsilon_{r\theta} = \varepsilon_{\theta z} = \varepsilon_{zr} = 0. \tag{3.2}$$

The stress −strain relations for isotropic material are given [1, 21]:

$$T_{ij} = \delta_{ij}I_1 + 2\mu\varepsilon_{ij}, \quad (i, j = 1, 2, 3), \tag{3.3}$$

where $I_1 = e_{kk}$ is the first strain invariant. Substituting (3.2) into (3.3), we get

$$\tau_{rr} = \lambda\left\{1 - \frac{1}{2}[(r\eta' + \eta)^2 + \eta^2]\right\} + \mu[1 - (r\eta' + \eta)^2],$$

$$\tau_{\theta\theta} = \lambda\left\{1 - \frac{1}{2}[(r\eta' + \eta)^2 + \eta^2]\right\} + \mu(1 - \eta^2), \tag{3.4}$$

$$\tau_{zz} = \lambda\left\{1 - \frac{1}{2}[(r\eta' + \eta)^2 + \eta^2]\right\} = 0, \quad \tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = 0.$$

The stress equation of equilibrium is given as:

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{(\tau_{rr} - \tau_{\theta\theta})}{r} = 0. \tag{3.5}$$

Boundary condition: The boundary conditions of tube are taken as:

$$\tau_{rr} = -p_{\text{int}} \quad \text{at} \quad r = a \quad \text{and} \quad \tau_{rr} = l_0 \quad \text{at} \quad r = b \quad \text{for the extension region,}$$
$$\tau_{rr} = -p_{\text{int}} \quad \text{at} \quad r = a \quad \text{and} \quad \tau_{rr} = l_0 \quad \text{at} \quad r = b \quad \text{for the contraction region,}$$

(3.6)

where l_0 is mechanical load applied at the external surface of tube made of isotropic materials.

Asymptotic solution at the transition points: Substituting (3.4) into (3.5) and after integration, we got the following nonlinear integro-differential equation:

$$(\lambda + 2\mu)[\eta^2 + (r\eta' + \eta^2)] + 2\mu \int r\eta'^2 \, dsr = A_0,$$

(3.7)

where A_0 is the constant of integration. Now, differentiating (3.7) with respect to r, we get:

$$\left[T^2 + \left(2 + \frac{1}{2}C\right)T + 2\right]\frac{d\eta}{dT} + \eta(1 + T) = 0$$

(3.8)

where $r\eta' = \eta T$ (T is the function of η and η is the function of r) and $C = 2\mu/(\lambda + 2\mu) = (1 - 2\nu)/(1 - \nu)$ be the compressibility factor and ν be the Poisson's ratio. Transition point T from (3.8) are $T \to -1$ and $T \to \pm\infty$. The transition point $T \to -1$ corresponds to extension region and $T \to \pm\infty$ corresponds to contractions region given by Seth (1970). In the general case, when Poisson's ratio varies in the range from 0 to 0.5, the parameter $0 < C < 1$, we have η and r in terms of T given as:

$$\ln \eta = CC_1 \tan^{-1}[C_1(4T + 4 + C)] - \frac{1}{2}\ln\left[T^2 + \left(2 + \frac{1}{2}C\right)T + 2\right] + A_1,$$

(3.9)

$$\ln r = \left[\frac{1}{4}\ln\left[T^2 + \left(2 + \frac{1}{2}C\right)T + 2\right] - \frac{1}{4}\ln T^2 \right.$$
$$\left. - \frac{1}{2}(4 - C)C_1 \tan^{-1}[C_1(4T + 4 + C)] + A_2\right],$$

(3.10)

where $C_1 = 1/\sqrt{16 - 8C - C^2}$ and A_1, A_2 are constants of integration. The resultant force normal to the plane $z = $ constant vanishes *i.e.* $\int_a^b r\tau_{zz}dr = 0$. The deformation of the tube walls is determined by the magnitude of the applied pressure. If mechanical load is increased, stresses in the wall of the tube increase.

4. SOLUTION OF THE PROBLEM

(a) Contraction in the tube: The transition point $T \to \pm\infty$, correspond to contraction in the tube [22–33]. Putting $T = 1/\varepsilon$ where ε is small parameter in (3.9) and (3.10), we find the solution in the neighborhood of this point to be:

$$\ln(\eta/A_3) \approx \ln \varepsilon - \left(1 + \frac{C}{2}\right)\varepsilon,$$

(4.1)

$$\ln(r/A_4) \approx \varepsilon.$$

(4.2)

From (4.1) and (4.2), we get

$$\eta = A_3 e^{-\left(1 + \frac{C}{2}\right)} \ln(r/A_4),$$

(4.3)

where A_3, A_4 are constants of integration. By Substituting (4.3) into (3.4) and, after making estimates, one can obtain:

$$\tau_{rr} = \left(\frac{2\mu}{C}\right)[1 - A_5 e^{-C}].$$

(4.4)

The yielding stress in tension is given by Seth (1935): $Y = \mu(1 + \nu) = (3 - 2C)\mu/(2 - C)$. Now substituting the value of yielding stress condition in (4.4), we get

$$\tau_{rr} = \frac{(2 - C)^2 Y}{C(3 - 2C)}[1 - A_5 r^{-C}].$$

(4.5)

Using (3.6) into (4.5), we get $A_5 = b^C - \dfrac{l_0 C(3 - 2C)b^C}{Y(2 - C)^2}$ Further, (4.5) become:

$$\tau_{rr} = \frac{(2-C)^2 Y}{C(3-2C)}\left[1-\left(\frac{b}{r}\right)^C\right] + l_0\left(\frac{b}{r}\right)^C . \tag{4.6}$$

Now using boundary condition (3.6) into (4.6), we get

$$p_{\text{int.}} = \frac{(2-C)^2 Y}{C(3-2C)}[(a/b)^{-C} - 1] - l_0(a/b)^{-C}$$

$$\Rightarrow \frac{(2-C)^2 Y}{C(3-2C)} = \frac{p_{\text{int.}} + l_0(a/b)^{-C}}{\{(a/b)^{-C} - 1\}} . \tag{4.7}$$

Substituting (4.7) into (4.6) and using (3.5), we get stress on the contraction region:

$$\tau_{rr} = \frac{p_{\text{int.}} + l_0(a/b)^{-C}}{\left[\left(\frac{a}{b}\right)^{-C} - 1\right]}\left[1-\left(\frac{b}{r}\right)^C\right] + l_0\left(\frac{b}{r}\right)^C , \tag{4.8}$$

$$\tau_{\theta\theta} = \frac{p_{\text{int.}} + l_0(a/b)^{-C}}{\left[\left(\frac{a}{b}\right)^{-C} - 1\right]}\left[1-(1-C)\left(\frac{b}{r}\right)^C\right] + l_0(1-C)\left(\frac{b}{r}\right)^C . \tag{4.9}$$

From (4.8) and (4.9), we get:

$$\tau_{\theta\theta} - \tau_{rr} = \frac{(p_{\text{int.}} + l_0)C(b/r)^C}{(a/b)^{-C} - 1} . \tag{4.10}$$

Initial yielding stage: From (4.10), it has seen that $|\tau_{\theta\theta} - \tau_{rr}|$ is maximum at the inner surface (*i.e.* $r = a$), therefore yielding will take place at the outer surface of the tube and (4.10) becomes: $|\tau_{\theta\theta} - \tau_{rr}|_{r=a} = \left|\frac{(p_{\text{int.}} + l_0)C(b/a)^C}{\{(a/b)^{-C} - 1\}}\right| = Y$ (say); where Y is the yielding stress on the contraction region. The pressure required for the initial yielding is given by:

$$P_i = \frac{p_{\text{int.}}}{Y} = \left|\frac{(a/b)^{-C} - 1}{C(b/a)^C}\right| - \left|\frac{l_0}{Y}\right| \tag{4.11}$$

(4.8), (4.9) and (4.11), in non-dimensional form becomes:

$$\sigma_r = \frac{(P_i + L_0 R_0^{-C})(1 - R^{-C})}{(R_0^{-C} - 1)} + L_0 R^{-C},$$

$$\sigma_\theta = \frac{(P_i + L_0 R_0^{-C})}{(R_0^{-C} - 1)}[1 - (1-C)R^{-C}] + L_0(1-C)R^{-C}, \tag{4.12}$$

$$P_i = \left|\frac{R_0^{-C} - 1}{R_0^{-C}C}\right| - |L_0|,$$

where $R = r/b$, $\sigma_r = \tau_{rr}/Y$, $\sigma_\theta = \tau_{\theta\theta}/Y$, $L_0 = l_0/Y$, $R_0 = a/b$ and $P_i = p_{\text{int}}/Y$

Fully-plastic stage: (4.12) for fully-plastic stage when $P_{\text{int}} = P_f = p_f/Y$ and $C \to 0$ becomes:

$$\sigma_r = -(P_f + L_0)\frac{\ln R}{\ln R_0} + L_0, \quad \sigma_\theta = -(P_f + L_0)\left(\frac{\ln R + 1}{\ln R_0}\right) + L_0, \tag{4.13}$$

$$P_f = |\ln R_0| - |L_0|,$$

where $P_f = p_f/Y$ be the pressure required for fully-plastic stage.

(b) Extension in the tube: The transition point $T \to -1$ corresponds to extension on the tube [22−33]. Putting $1 + T + \varepsilon_1$ where ε_1 is small parameter in (3.9) and (3.10), we find the solution in neighborhood of this point to be:

$$\ln(\eta/A_6) \approx -\varepsilon_1^2/(2-C), \tag{4.14}$$

$$\ln(r/A_{67}) \approx -\varepsilon_1^2/(2 - C). \tag{4.15}$$

From (4.14) and (4.15), we get

$$\eta = A_6 A_7/r, \tag{4.16}$$

where A_6, A_7 are constants of integration. By Substituting (4.16) into (3.4) and, after making estimates, one can obtain:

$$\tau_{rr} = \frac{(2 - C)^2 Y}{C(3 - 2C)}[1 - A_8 e^{\frac{C}{1-C}}]. \tag{4.17}$$

Using (3.6) into (4.17), we get $A_8 = b^{\frac{-C}{1-C}} - \dfrac{l_0 C(3 - 2C)b^{\frac{-C}{1-C}}}{Y(2 - C)^2}$. Further, (4.17) become:

$$\tau_{rr} = \frac{(2 - C)^2 Y}{C(3 - 2C)}\left[1 - \left(\frac{r}{b}\right)^{\frac{C}{1-C}}\right] + l_0 \left(\frac{r}{b}\right)^{\frac{C}{1-C}}. \tag{4.18}$$

Now using boundary condition (3.6) into (4.18), we get

$$\begin{aligned} p_{\text{int.}} &= \frac{(2 - C)^2 Y}{C(3 - 2C)}[(a/b)^{C/(1-C)} - 1] - l_0(a/b)^{C/(1-C)} \\ &\Rightarrow \frac{(2 - C)^2 Y}{C(3 - 2C)} = \frac{p_{\text{int.}} + l_0(a/b)^{C/(1-C)}}{\{(a/b)^{C/(1-C)} - 1\}}. \end{aligned} \tag{4.19}$$

Substituting (4.19) into (4.18) and using (3.5), we get stress in the extension region:

$$\tau_{rr} = \frac{p_{\text{int.}} + l_0(a/b)^{C/(1-C)}}{\{(a/b)^{C/(1-C)} - 1\}}\left[1 - \left(\frac{r}{b}\right)^{\frac{C}{1-C}}\right] + l_0\left(\frac{r}{b}\right)^{\frac{C}{1-C}}, \tag{4.20}$$

$$\tau_{\theta\theta} = \frac{p_{\text{int.}} + l_0(a/b)^{C/(1-C)}}{\{(a/b)^{C/(1-C)} - 1\}}\left[1 - \left(\frac{1}{1-C}\right)\left(\frac{r}{b}\right)^{\frac{C}{1-C}}\right] + \frac{l_0}{(1-C)}\left(\frac{r}{b}\right)^{\frac{C}{1-C}}. \tag{4.21}$$

From (4.20) and (4.21), we get:

$$\tau_{\theta\theta} - \tau_{rr} = \frac{(p_{\text{int.}} + l_0)C(r/b)^{C/(1-C)}}{(1 - C)\{1 - (a/b)^{C/(1-C)}\}}. \tag{4.22}$$

Initial yielding stage: From (4.22), it has seen that $|\tau_{\theta\theta} - \tau_{rr}|$ is maximum at the outer surface (*i.e.* $r = b$), therefore yielding will take place at the outer surface of the tube in the extension region and (4.22) becomes: $|\tau_{\theta\theta} - \tau_{rr}|_{r=b} = \left|\dfrac{(p_{\text{int.}} + l_0)C}{(1 - C)\{1 - (a/b)^{-C}\}}\right| = Y^*$ (say); where Y^* is the yielding stress in the extension region. The pressure required for the initial yielding is given by:

$$P_i = \frac{p_{\text{int.}}}{Y^*} = \left|\frac{\{1 - (a/b)^{C/(1-C)}\}(1 - C)}{C}\right| - \left|\frac{l_0}{Y^*}\right| \tag{4.23}$$

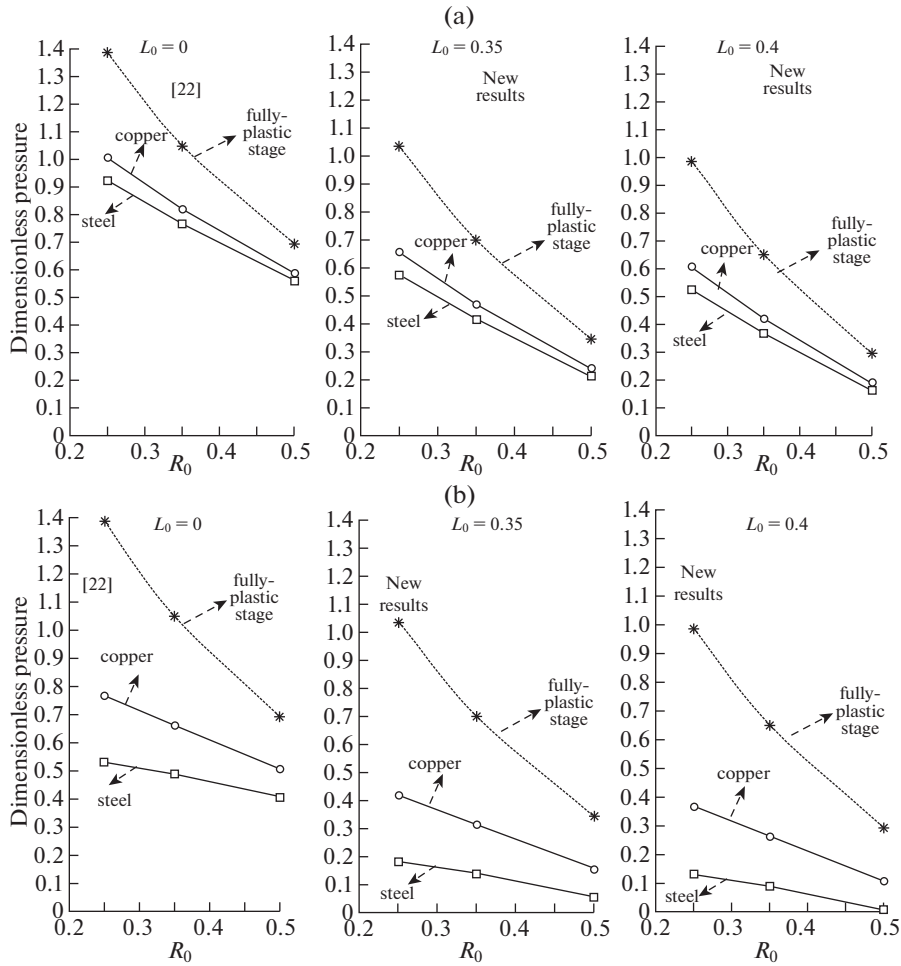(4.20), (4.21) and (4.23), in non-dimensional form becomes:

Fig. 2. Graphical comparisons between new and published results [22], dimensionless pressure required for yielding/fully-plastic stage versus radii ratio $R_0 = a/b$ in the (a) contraction (b) extension region.

$$\sigma_r^* = \frac{P_i + L_0 R_0^{C/(1-C)}}{\{R_0^{C/(1-C)} - 1\}}\left[1 - R^{\frac{C}{1-C}}\right] + L_0 R^{\frac{C}{1-C}},$$

$$\sigma_\theta^* = \frac{P_i + L_0 R_0^{C/(1-C)}}{\{R_0^{C/(1-C)} - 1\}}\left[1 - \left(\frac{1}{1-C}\right)R^{\frac{C}{1-C}}\right] + \frac{L_0 R^{\frac{C}{1-C}}}{(1-C)},$$

and

$$P_i^* = \left|\frac{\{1 - R_0^{C/(1-C)}\}(1-C)}{C}\right| - |L_0|. \tag{4.24}$$

Fully-plastic stage: (4.24), for the fully-plastic stage when $C \to 0$, becomes:

$$\sigma_r^* = -(P_f + L_0)\left(\frac{\ln R}{\ln R_0}\right) + L_0, \quad \sigma_\theta^* = -(P_f + L_0)\left(\frac{\ln R + 1}{\ln R_0}\right) + L_0,$$

$$P_f^* = |\ln R_0| - |L_0| \tag{4.25}$$

where $P_f^* = p_f/Y$ be the pressure required for fully-plastic stage.

Validation of results: Initial Yielding stage: By taking $L_0 \to 0$ into (4.12) and (4.24), we get:
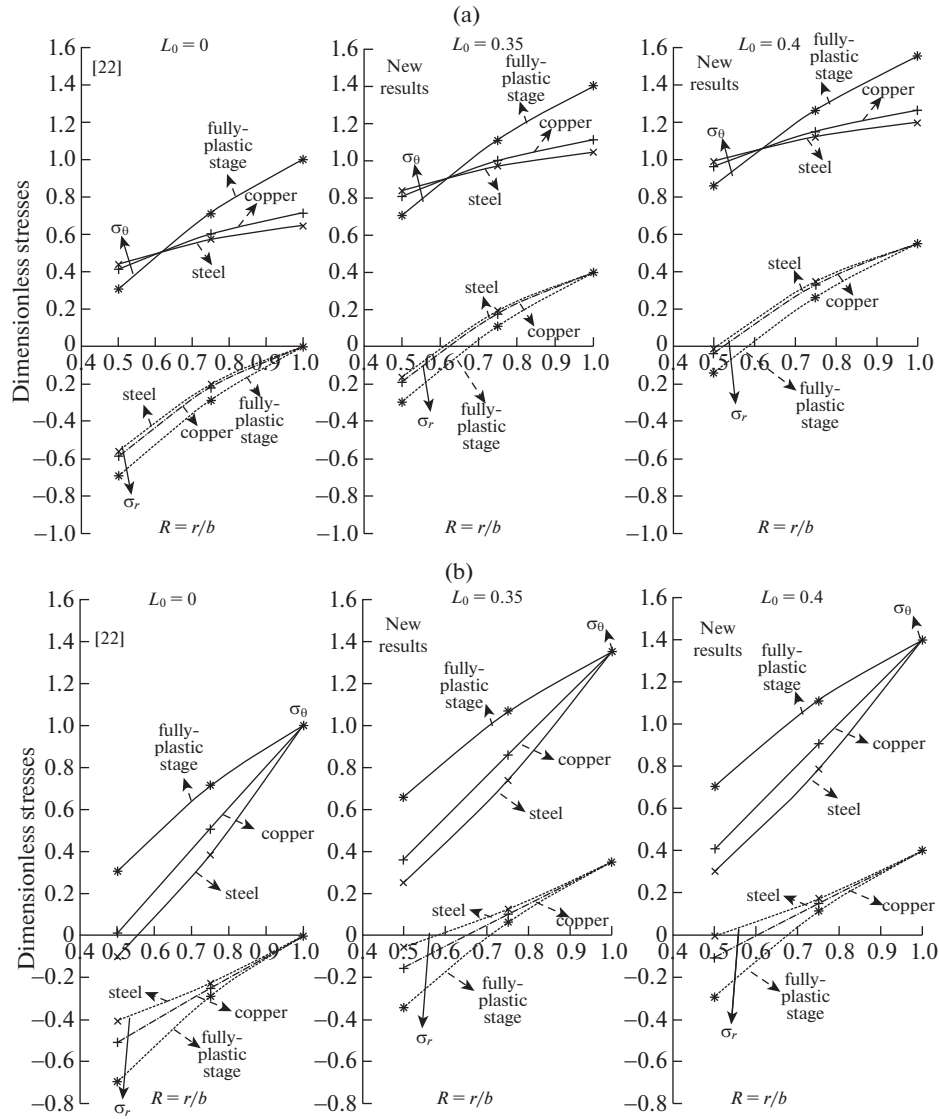
Fig. 3. Graphical comparisons between new and published results [22], dimensionless stress distribution versus radii ratio $R = r/b$ for initial yielding/fully-plastic stage in the with (a) contraction, (b) extension region.

$$\sigma_r = \frac{P_i(1 - R^{-C})}{(R_0^{-C} - 1)}, \quad \sigma_\theta = \frac{P_i}{(R_0^{-C} - 1)}[1 - (1 - C)R^{-C}], \quad P_i = \left|\frac{R_0^{-C} - 1}{R_0^{-C}C}\right| - |L_0| \quad (4.26)$$

in the contraction region.

$$\sigma_r^* = \frac{P_i^*}{\{R_0^{C/(1-C)} - 1\}}\left[1 - R^{\frac{C}{1-C}}\right], \quad \sigma_{\theta r}^* = \frac{P_i^*}{\{R_0^{C/(1-C)} - 1\}}\left[1 - \left(\frac{1}{1-C}\right)R^{\frac{C}{1-C}}\right],$$

$$P_i^* = \left|\frac{\{1 - R_0^{C/(1-C)}\}(1 - C)}{C}\right| \quad (4.27)$$

in the extension region.

Fully-plastic stage: By taking $L_0 \to 0$ into (4.13) and (4.25), we get:

$$\sigma_r = \sigma_r^* = -P_f\frac{\ln R}{\ln R_0}, \quad \sigma_\theta = \sigma_\theta^* = -P_f\left(\frac{\ln R + 1}{\ln R_0}\right), \quad P_f = P_f^* = |\ln R_0| \quad (4.28)$$

Table 1. Percentage in pressure required for initial /fully plastic stage

Materials	Yielding starts at $r = a$				Percentage increase in pressure from Initial yielding to fully-plastic stage	
					P %	
	Mechanical Load L_o	Pressure P_i (Pressure required for initial yielding stage)		Pressure P_f (Pressure required for fully-plastic stage)	$P = \left(\dfrac{P_f - P_i}{P_i} \right) \times 100$	
		contraction	extension		contraction	extension
Steel ($i.e.\ v = 0.27$)	0	0.5616	0.406	0.693	23.4%	70.7%
	0.35	0.2116	0.056	0.343	61.8%	512.5%
	0.4	0.1616	0.006	0.293	80.8%	4783.3%
Copper ($i.e.\ v = 0.333$)	0	0.5879	0.506	0.693	17.8%	36.9%
	0.35	0.2371	0.156	0.343	44.7%	119.8%
	0.4	0.1881	0.106	0.293	55.8%	176.4%

where $P = \left(\dfrac{P_f - P_i}{P_i} \right) \times 100$ is the percentage (%) increase in pressure from initial yielding stage to become fully plastic stage and having mechanical load $L_0 = 0.\ 0.35, 0.4$.

in the contraction/extension region. The present results obtained from (4.26)−(4.28) are same as [22] in the contraction/extension region.

5. NUMERICAL RESULTS AND DISCUSSION

To see the combined effect of stress distribution and pressure in a tube made of steel (say $C = 0.63$ or $v = 0.27$) and copper material (say $C = 0.488$ or $v = 0.333$) [21], for the initial/fully plastic stage based upon the following numerical values has been taken: $L_0 = 0, 0.35, 0.4$, $a = 1$ and $b = 2$ respectively.

Figures 2 and 3, show the graphical comparison between the new and previously published results, dimensionless pressure versus radii ratio $R_0 = a/b$ for the initial/fully-plastic stage, and dimensionless
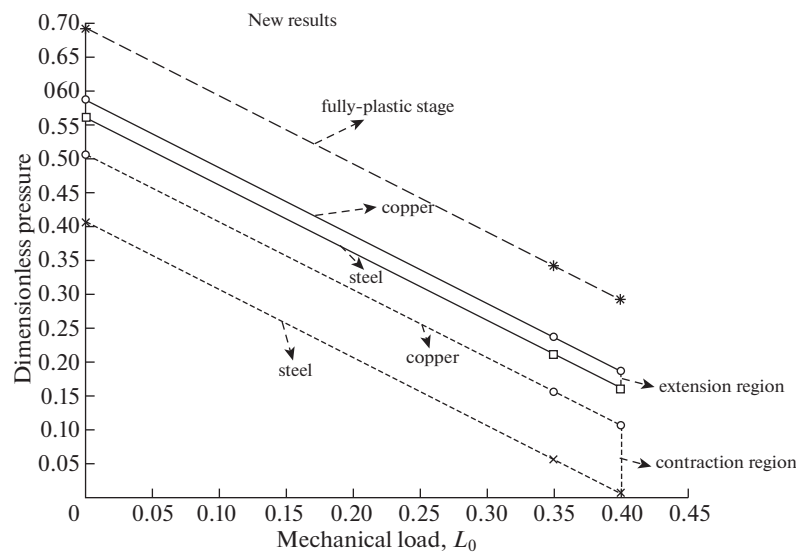

Fig. 4. Graphical comparisons between dimensionless pressure versus mechanical load at $R_0 = 0.5$ for initial yielding/fully-plastic stage along the contraction/extension region.

stress distribution versus radii ratio $R = r/b$ in the contraction/extension regions. In (4.26) and (4.28), the present result reduces to the previously published outcomes done [22]. From this comparison, it is found that the present results are correct and make sure the validity of the present solutions. The shortcomings of this study [22] are that, the numerical and graphical representations are not discussed, but in the present study we discussed the numerical and graphical representation of isotropic materials (i.e. steel/ copper), with the new addition of mechanical load condition on the outer surface of the tube.

From Table 1, it shows that percentage increase in pressure required for the initial yielding to become fully-plastic stage has been discussed. It can also see from Table 1, that tube made of steel material requires higher percentage values in pressure (i.e. $P = 70.7\%$, 512.5%, 4783.3%), to become fully plastic as compared to the copper material (i.e. $P = 36.9\%\%$, 119.8%, 176.4%) in the expansion region with mechanical load $L_0 = 0$. 0.4 and 0.55 respectively. Further increasing mechanical loads, the percentage ratio (i.e. $P\%$) is also increased, for the initial yielding stage to become a fully plastic stage. Moreover, the contraction region of the tube requires lower percentage values in pressure to become fully plastic in comparison to extension region of the tube.

In Fig. 2, curves have been drawn between dimensionless pressure required for initial yielding/fully-plastic stage versus radii ratio $R_0 = a/b$ for the contraction/extension region and having mechanical loads $L_0 = 0$, 0.35, and 4, respectively. It has been observed that tube made of copper material requires higher dimensionless pressure to yield at the internal surface as compared to the tube made of steel material for the initial yielding stage. Further, the value of pressure decreases with increasing mechanical loads (*say* $L_0 = 0.35$ and 4) at the internal surface of a tube made of copper material and also in a steel material for the initial as well as fully-plastic stage. Moreover, the tube made of copper/steel material requires higher pressure in the contraction region as compared to extension region.

Figures 3a and 3b are portrayed in order to demonstrate the behaviour of dimensionless stress distribution versus radii ratio $R = r/b$ in the contraction/extension region and having mechanical loads $L_0 = 0$, 0.35 and 4 respectively. It is observed that tube made of copper material requires maximum hoop stress at the external surface in comparison to tube made of steel material. Further, the values of the hoop/radial stress also increase with increasing mechanical load in the contraction/extension region of the tube. Moreover, the extension region in the tube requires maximum hoop/radial stress at the external surface of the initial yielding stage as compared to contraction region.

Figures 4a and 4b is prepared to illustrate the behaviour of dimensionless pressure versus mechanical load for the initial yielding/fully-plastic stage at $R_0 = 0.5$. It is shown that the extension/contraction region, the value of pressure, decreasing with increased mechanical load (i.e. $L_0 = 0$, 0.35, 0.4) for initial yielding as well as fully-plastic stage.

6. CONCLUSIONS

In this work, behaviour of stress distribution and pressure in a tube made of isotropic material (*say* steel/copper) under uniform pressure and mechanical load by using the transition theory is investigated. The problem is designed under uniform pressure and mechanical loads. Solutions of the stress distribution are obtained, for initial and fully-plastic stages, whereas the expressions for pressure versus loads are evaluated graphically. The main findings can be concluded as follows:

• The copper material tube requires higher dimensionless pressure to yield at the internal surface in comparison to tube made of steel material.

• The value of pressure decreases with increasing mechanical loads (*say* $L_0 = 0.35$ and 4).

• The tube made of copper/steel material requires higher pressure in the contraction region as compared to extension region.

• The values of the hoop / radial stress also increase with increasing mechanical load in the contraction as well as extension region of tube.

• Moreover, the extension region in the tube requires maximum hoop/radial stress at the external surface of the initial yielding stage as compared to contraction region.

• The tube made of copper material is convenient than that of steel material.

• The results for [22] can be obtained by taking $L_0 \to 0$ in the resulting equations.

ABBREVIATIONS

λ, μ	— Lame's constants
e_{kk}	— First strain invariant
C	— Compressibility factor
a, b	—Internal and external radii
u, v, w	—Displacement components
ν	— Poisson's ratio
$\tau_{ij}, \varepsilon_{ij}$	—Stress and strain components
Y, Y^*	—Stress
δ_{ij}	— Kronecker's delta
p_{int}	— Internal surface pressure
P_i	— Pressure required for initial yielding stage
P_f	— Pressure required for fully-plastic stage
l_0	—Load at the external surface
η	— Function of r
r	— Function of x and y
T	— Function of η

NON-DIMENSIONAL QUANTITIES

$R_0 = a/b$, $R = r/b$ (Radii ratio), $\sigma_r = \tau_{rr}/Y$ (Radial stress component), $\sigma_\theta = \tau_{\theta\theta}/Y$ (Circumferential stress component), $L_0 = l_0/Y$ (Mechanical load), $P = p/Y$ (Pressure)

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