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Contact between a Smooth Indenter and a Two-Layer Elastic Half-Space with Complicated Conditions on the Surface

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Abstract—The problem of loading a textured two-layer elastic foundation with a rigid smooth indenter is considered. At the interface between the textured layer and the half-space, the conditions for complete adhesion are specified. Relief elements are elastic cylinders, characterized by height and radius, located on the surface of the base with a given period. The one-dimensional Winkler model is used to describe the mechanical properties of relief elements. The contact problem is solved using the boundary element method. Pressure, displacements, and also the shape of the indenter are approximated by piecewise constant functions. The influence coefficients are constructed using a method based on double integral Fourier transforms. The analysis of the influence of the texture elements arrangement density and their compliance on the distribution of the contact pressure is carried out. It was found that in most cases contact is made only with the elements of the texture. For a relatively stiff textured layer, the combined effect of layer curvature and additional pliability imparted by texture elements is important. For relatively stiff and pliable textured layers, load-penetration curves were obtained depending on the texture period. Within the framework of the proposed setting, the limiting case of contact of an indenter with a Winkler layer completely covering the surface of a two-layer elastic half-space is also considered. Load-penetration curves are obtained and analyzed for an indenter in the form of a Berkovich pyramid.

Keywords: contact problem, coating, textured surface

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1. INTRODUCTION

Formation of a certain relief (texture) on a surface is a common technological technique that pursues different goals, such as creating volumes for retaining lubricant and removing wear products under conditions of frictional contact, controlling the coefficient of friction and contact stiffness by varying the geometry and relative position of texture elements, etc. When solving contact problems for textured surfaces, it is customary to use the methods and approaches developed for the mechanics of discrete contact. For homogeneous elastic bodies, periodic problems have been considered in a number of works, such as [1–5]. Contact problems for coatings of variable thickness arising from the presence of texture can be considered using approximate methods, as, for example, in [6], where the corresponding plane problem was considered. The spatial problem of loading a textured layer coupled to a rigid half-space was considered in [7]. An approximate method for solving the spatial problem for a coating, the scale of irregularities on the surface of which is significantly inferior to the thickness of the coating, was developed in [8].

In this article, we propose a formulation and a method for solving the problem of contact between a smooth indenter and a two-layer elastic half-space, on the surface of which there is a periodic system of identical texture elements.

Also, thin layers can exist on the surface of coatings, that are, as a rule, rather pliable, arising during operation. A typical example is a layer that occurs during friction (the so-called tribolayer). Determination of the properties of this layer provides important information about the processes occurring during frictional interaction. The mechanical properties of the layer can be determined using nanoindentation. There are experimental works on the indentation of friction surfaces [10–12], but for correct identification of the mechanical properties of surface films when interpreting the results, it is necessary to take into

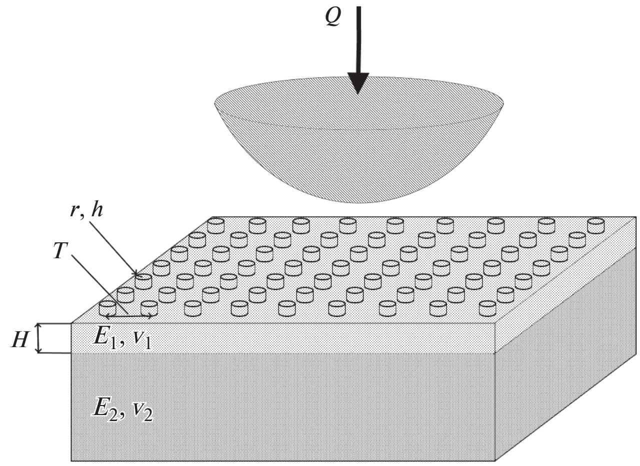

Fig. 1. Contact scheme.

account the deformation of the coating and substrate, especially in the case of relatively hard coatings. In [12], the results of determining the mechanical properties of a tribofilm on the surface of a carbon coating during indentation with a ball (axisymmetric problem) are presented. In this paper, it is shown that within the framework of the proposed model for solving the problem of the contact of a textured surface, it is possible to consider as a limiting case the spatial problem of the contact of a smooth indenter of arbitrary shape and a two-layer elastic half-space with a relatively compliant layer on the surface.

2. STATEMENT OF THE PROBLEM

Loading by a rigid indenter of a two-layer elastic foundation, on the surface of which a periodic relief is applied (Fig. 1), is considered. The lower part of the base is an elastic half-space with elastic modulus E_2 and Poisson's ratio ν_2. The top layer is also elastic and is described by the finite thickness H, as well as the modulus of elasticity E_1 and Poisson's ratio ν_1. The conditions at the boundary between the half-space and the upper layer correspond to complete adhesion:

$$\begin{aligned} &w^{(1)} = w^{(2)}, \quad u_x^{(1)} = u_x^{(2)}, \quad u_y^{(1)} = u_y^{(2)}, \\ &\sigma_z^{(1)} = \sigma_z^{(2)}, \quad \tau_{xz}^{(1)} = \tau_{xz}^{(2)}, \quad \tau_{yz}^{(1)} = \tau_{yz}^{(2)}. \end{aligned} \tag{2.1}$$

Here, σ_z, τ_{xz}, τ_{yz} are normal and tangential stresses, w, u_x, u_y are normal and tangential displacements in the layer material (1) and half-space material (2).

The relief elements are elastic cylinders of height h and radius r, located on the surface of the base with a period T. To describe the mechanical properties of relief elements, a one-dimensional model of an elastic material is used, characterized by compliance $\eta = h/E_a$. The right-hand Cartesian coordinate system XYZ is located in such a way that its center is in a plane passing through the tops of relief elements in an undeformed state, and the applicate axis is directed along the normal to the specified plane in the direction opposite to the base. The shape of the indenter is described by a smooth function $f(x, y)$. The indenter is loaded with a vertical force Q. Boundary conditions in the plane $z = 0$:

$$\begin{aligned} &f^*(x, y) + w^{(1)}(x, y) + w^*(x, y) = f(x, y) - D, \quad (x, y) \in \Omega, \\ &\sigma_z = 0, \quad (x, y) \notin \Omega, \\ &\tau_{xz} = 0, \quad \tau_{yz} = 0. \end{aligned} \tag{2.2}$$

Here, Ω is the nominal contact area, $f^*(x, y)$ is the shape of the surface in an undeformed state, $w^{(1)}(x, y)$ are the vertical displacements of the upper boundary of the two-layer base on which the texture elements are located, $w^*(x, y)$ are the vertical displacements of the top points of the texture elements relative to the surface of the two-layer base (outside of the texture elements $w^*(x, y) = 0$), D is the penetration of the indenter. In this case, the contact pressure $p(x,y) = -\sigma_z(x,y)$, the nominal contact area Ω, and the penetration D are unknown.

The equilibrium condition is also satisfied:

$$Q = \iint_\Omega p(x,y)dxdy. \tag{2.3}$$

3. SOLUTION METHOD

The contact problem is solved using the boundary element method. The rectangular area Ω^*, which includes the unknown contact area, is divided into N square elements Ω_i, $i = 1..N$. Pressure, displacements, and also the shape of the indenter inside this area are approximated by piecewise constant functions (p_i, $\tilde{w}_i$, w_i^*, f_i, f_i^*, $i = 1..N$). To determine the dependence of the vertical displacements of the boundary of a two-layer foundation on the applied pressure, we use the solution to the problem of the action of a load q uniformly distributed inside a square with side $2a$ on the surface of a two-layer elastic foundation [9]:

$$w'(x',y',0) = -\frac{1+\nu_1}{E_1}\int_0^{\pi/2}\int_0^{\infty}\Delta(\gamma,\varphi,\lambda,\chi)\cos(x'\gamma\cos\varphi)\cos(y'\gamma\sin\varphi)d\gamma d\varphi, \tag{3.1}$$
$$\text{где } \chi = E_1/E_2, \quad (x',y',w') = (x,y,w)/a, \quad \lambda = H/a.$$

The function $\Delta(\gamma,\varphi,\lambda,\chi)$ is determined from the solution of a system of linear functional equations obtained from the boundary conditions as a result of using biharmonic functions to determine stresses and displacements, as well as a double integral Fourier transform applied to a constant load. The function $\Delta(\gamma,\varphi,\lambda,\chi)$ is linearly dependent on the result of applying the double Fourier transform to constant pressure:

$$\overline{q} = q\frac{4}{\pi^2}\frac{\sin(\gamma\cos\varphi)\sin(\gamma\sin\varphi)}{\gamma^2\sin\varphi\cos\varphi}. \tag{3.2}$$

Due to the fact that the vertical displacements of the boundary linearly depend on the applied pressure inside each element, the vertical displacements of the boundary of a two-layer base, as well as displacements of relief elements, can be expressed in terms of pressure as follows:

$$\begin{pmatrix} \mathrm{k}_1^1 & \cdots & \mathrm{k}_N^1 \\ \vdots & \ddots & \vdots \\ \mathrm{k}_1^N & \cdots & \mathrm{k}_N^N \end{pmatrix}\cdot\begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix} = \begin{pmatrix} \tilde{w}_1 \\ \vdots \\ \tilde{w}_N \end{pmatrix}, \quad \begin{pmatrix} \eta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \eta_N \end{pmatrix}\cdot\begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix} = \begin{pmatrix} w_1^* \\ \vdots \\ w_N^* \end{pmatrix}, \tag{3.3}$$

where k_i^j is the vertical displacement of the surface in the center of i element as a result of the action of unit pressure inside element j, η_i is the compliance:

$$\mathrm{k}_i^j = \frac{1+\nu_1}{E_1}\int_0^{\pi/2}\int_0^{\infty}\Delta'(\gamma,\chi,\varphi,\lambda)\cos(y_{ij}\gamma\sin\varphi)\cos(x_{ij}\gamma\cos\varphi)d\gamma d\varphi. \tag{3.4}$$

Here, $(x_{ij}^2 + y_{ij}^2)^{1/2}$ is the distance between the centers of the square elements.

Boundary conditions (2.2) and equilibrium condition (2.3) can be expressed using the introduced piecewise constant functions:

$$\begin{pmatrix} 4a^2 & \cdots & 4a^2 & 0 \\ \mathrm{k}_1^1 + \eta_1 & \cdots & \mathrm{k}_N^1 & 1 \\ \vdots & \ddots & \vdots & \vdots \\ \mathrm{k}_1^N & \cdots & \mathrm{k}_N^N + \eta_N & 1 \end{pmatrix} \times \begin{pmatrix} p_1 \\ \vdots \\ p_N \\ D \end{pmatrix} = \begin{pmatrix} Q \\ f_1 - f_1^* \\ \vdots \\ f_N - f_N^* \end{pmatrix}, \tag{3.5}$$

Since the area Ω^* is certainly larger than the contact area, when solving system (3.5), the pressures p_i, $i = 1..N$ can take positive, negative, and zero values, which contradicts the boundary conditions. Since the elements with negative pressure are not part of the contact area, they are assigned a zero value, the rank

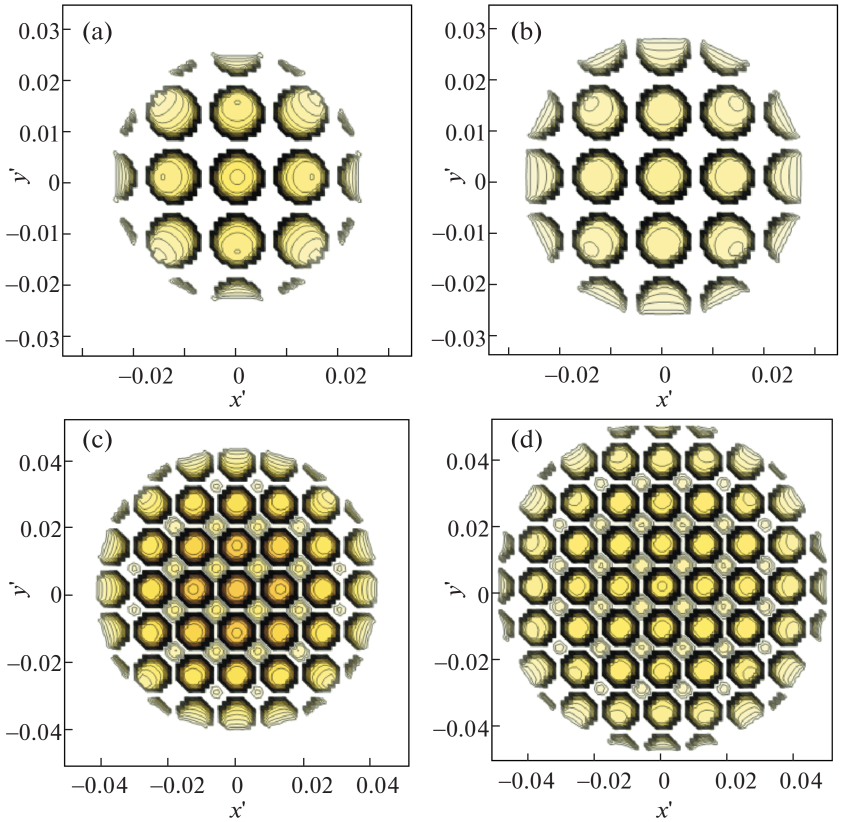

Fig. 2. Contact pressure distribution for relatively rigid (b, d) and relatively compliant textured coatings (a, c) at different values of force: $Q' = 0.3571\times10^{-4}$ (a), $Q' = 0.3571\times10^{-5}$ (b), $Q' = 2.1426\times10^{-4}$ (c), $Q' = 2.1426\times10^{-5}$ (d); $E_1/E_2=10$, $\nu_1 = 0.3$, $\nu_2 = 0.4$ (b, d); $E_1/E_2 = 0.1$, $\nu_1 = 0.4$, $\nu_2 = 0.3$ (a, c); $r_a' = 5\times10^{-3}$; $H_a' = H' = 10^{-4}$, $E_a = E_1$; $T' = 1/80$.

of the matrix of the system of equations (3.5) is reduced, then the system is solved again. The process continues until there are elements with negative pressure in the solution. As a result, the contact area, contact pressure, and penetration of the indenter are approximately determined. It should be noted that the condition of contact only over the surface of the texture elements is not set in advance; moreover, for some combinations of input parameters, the indenter can contact with the deformed surface of the elastic layer outside the texture elements.

4. CALCULATION RESULTS (TEXTURED LAYER)

The calculations were carried out for an indenter in the form of a paraboloid $f(x,y) = (x^2 + y^2)/2R$, where R is the radius. In the calculations, relief elements were considered, having the shape of a cylinder with a radius r_a and a height H_a and located periodically with a period T, while the modulus of elasticity of the relief elements is equal to the modulus of elasticity of the layer ($E_a = E_1$). The results are obtained for a half-space covered with a relatively hard layer and for a half-space covered with a relatively soft layer. The results are presented in dimensionless form, with the dimensionless parameters obtained as follows: $(x', y', z') = (x, y, z)/R$, $p' = p/E_1$, $Q' = Q/E_1R^2$.

Figure 2 shows the distribution of contact pressure when a spherical indenter is introduced into various types of bases under the action of various forces. In Figs. 2a and 2b, incomplete contact of the base and the indenter is observed: only the surfaces of the relief elements come into contact with the indenter. In the cases shown in Figs. 2 c and 2d we have full contact near the center of the indenter and incomplete at the edges. Pressure peaks are concentrated at the edges of the irregularities. In all the cases considered, the maximum contact pressure is at the edge of the roughness element, the center of which coincides with the axis of symmetry of the indenter.

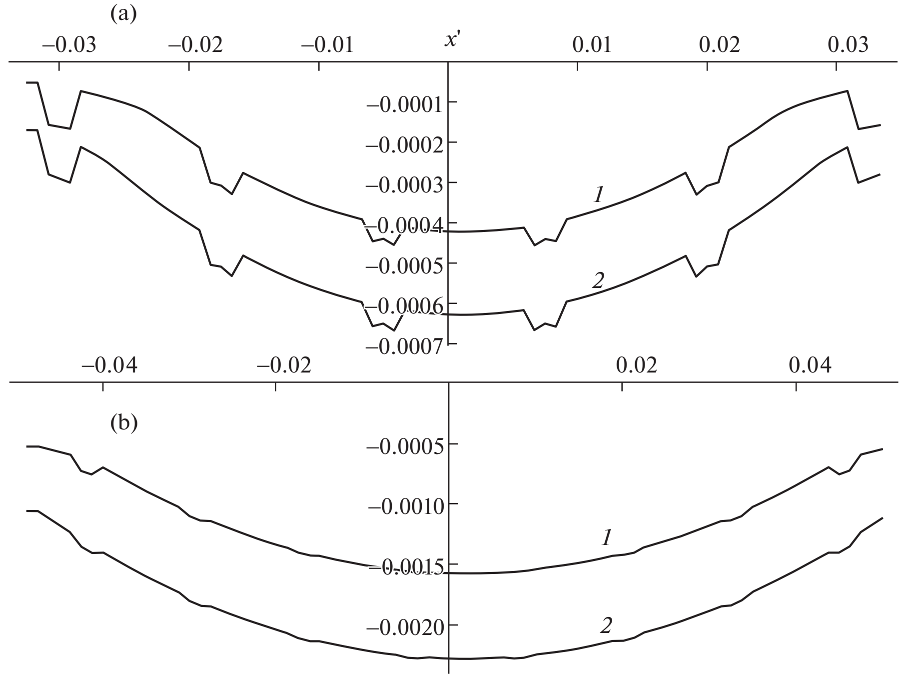

Fig. 3. The shape of the layer surface under the texture elements for a relatively pliable (a) and relatively rigid (b) layer: $E_1/E_2 = 0.1$, $\nu_1 = 0.4$, $\nu_2 = 0.3$ (a); $E_1/E_2 = 10$, $\nu_1 = 0.3$, $\nu_2 = 0.4$ (b); $Q' = 0.3571 \times 10^{-5}$ (a, curve *1*), $Q' = 0.3571 \times 10^{-4}$ (a, curve *2*), $Q' = 2.1426 \times 10^{-5}$ (b, curve *1*), $Q' = 2.1426 \times 10^{-4}$ (b, curve *2*); $r_a' = 5 \times 10^{-3}$, $H_a' = H' = 10^{-4}$, $E_a = E_1$; $T' = 1/80$.

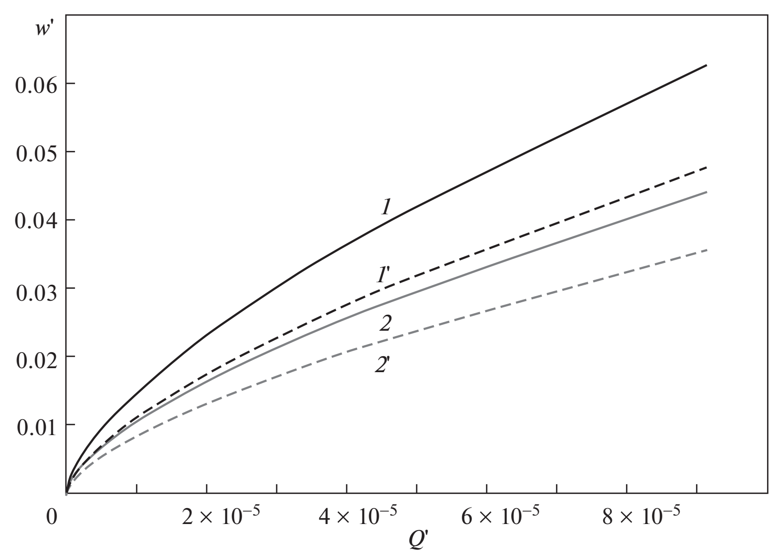

Fig. 4. Load-penetration curves for relatively rigid (curves *1* and *1'*) and relatively compliant (curves *2* and *2'*) layers at periods $T' = 1/60$ (curves *1* and *2*) и $T' = 1/80$ (curves *1'* и *2'*); $E_1/E_2 = 10$, $\nu_1 = 0.3$, $\nu_2 = 0.4$ (curves *1* and *1'*); $E_1/E_2 = 0.1$, $\nu_1 = 0.4$, $\nu_2 = 0.3$ (curves *2* and *2'*); $r_a' = 5 \times 10^{-3}$; $H_a' = H' = 10^{-4}$; $E_a = E_1$.

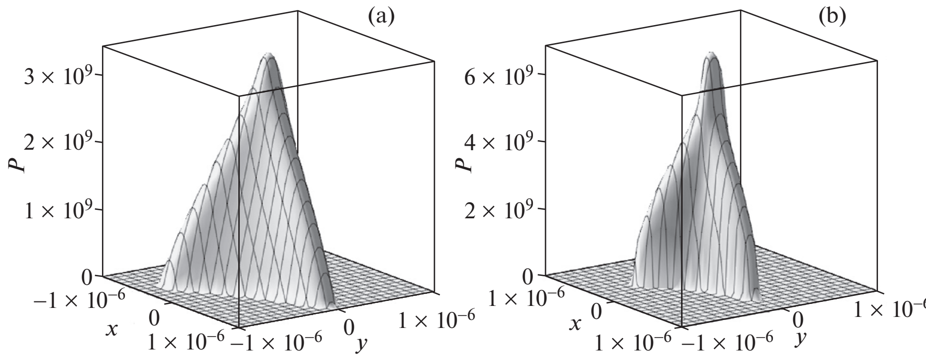

Fig. 5. Distribution of contact pressure under the pyramid with film (a) and without film (b). $E_1 = 70$ GPa, $E_2 = 7$ GPa, $E_w = 0.14$ GPa, $\nu_1 = 0.3$, $\nu_2 = 0.4$, $H_w = 5$ nm, $H = 10$ nm, $R_b = 400$ nm, $Q = 2.3$ mN.

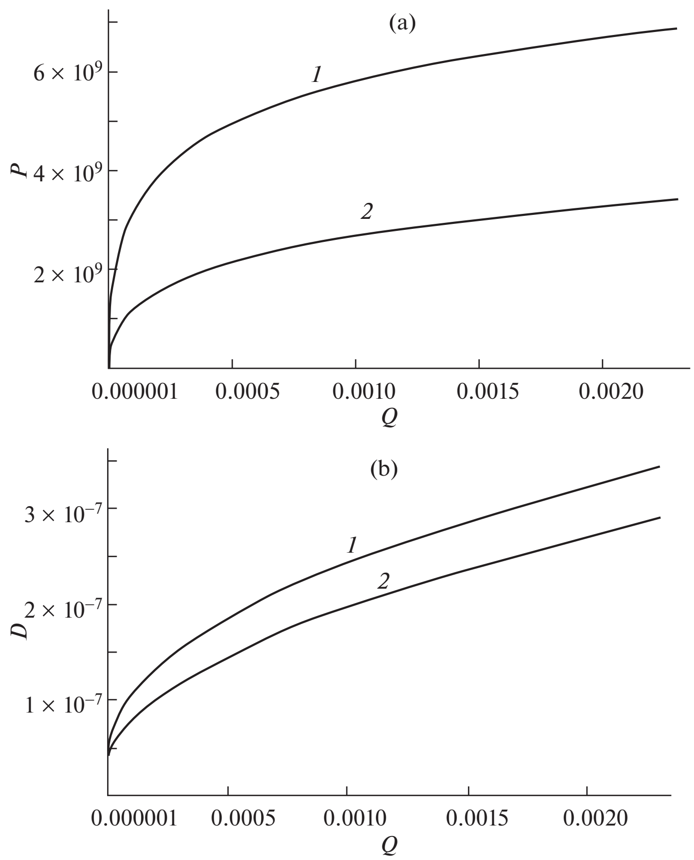

Fig. 6. Dependence of the maximum contact pressure (a) and penetration (b) on the load in the presence of a film on the coating surface (curve *2*) and without it (curve *1*). $E_1 = 70$ GPa, $E_2 = 7$ GPa, $E_w = 0.14$ GPa, $\nu_1 = 0.3$, $\nu_2 = 0.4$, $H_w = 5$ nm, $H = 10$ nm, $R_b = 400$ nm.

The deformed surface of the layer and texture elements is shown in Fig. 3 for the same parameters of the problem as considered in Fig. 2. In the case of incomplete contact, the surface between the indenters has a curvature, while the maximum vertical displacements are near the boundaries of the irregularities, which is due to the concentration of contact pressure at the edges of the irregularities.

The dependence of the penetration of the indenter on the load was investigated for two different cases of the relative stiffness of the layer and two periods characterizing the mutual arrangement of irregularities

(Fig. 4). Herewith, the dimensionless load $Q' = Q/E_1R^2$ in both cases was obtained with respect to the elastic modulus E_1 of the rigid coating in order to be able to compare the results for the same absolute one. In all cases, there is a non-linear relationship between penetration and load. The largest increase in implementation with increasing load is observed near zero. As the load increases, the $\partial D/\partial Q$ value gradually decreases to some constant value. As expected, a smaller value of the period of relative position of irregularities corresponds to a smaller penetration of the indenter. The nature of the dependence of penetration on the load is similar for the cases of relatively hard and relatively soft textured coatings.

5. CALCULATION RESULTS (HOMOGENEOUS LAYER)

The developed solution method for the case $w^*(x, y) \neq 0$ at all interior points of the domain Ω was used to study the contact of a two-layer elastic half-space, on the surface of which there is a layer of constant thickness H_w, described by the Winkler model with compliance $\eta_w = H_w/E_w$, and a smooth indenter of arbitrary shape. From a practical point of view, the most popular problem is the indentation of the Berkovich pyramid, since it can be used to interpret the results of indentation of surfaces of coatings with thin films. This type of head is a triangular pyramid with a rounded end (according to GOST R. 8.904-2015, the curvature radius R_b can range from 20 to 50 nm, increasing during exploitation). In the study of thin films, indentation occurs more and more often in the elastic mode, which does not allow determining the hardness of the materials under study, but can provide information on their elastic properties.

To demonstrate the capabilities of the method, the penetration of an indenter, the geometry of which is determined by GOST R. 8.904-2015, into a relatively rigid coating with a Young's modulus of 70 GPa, applied to a substrate with a modulus of 7 GPa was studied; there is a pliable film (0.14 GPa) on the coating surface. It was shown in [13] that, in the case of relatively rigid coatings, substrate deformation has a significant effect on the load-penetration curve even at low load values.

The results obtained are shown in Figs. 5 and 6 in dimensional terms (coordinates in meters, load and pressure in Newtons and Pascals, respectively). Figure 5 shows the distribution of the contact pressure under the indenter in the presence or absence of a surface film. In addition to the expected result - an increase in the contact area and a decrease in the maximum pressure, it should be noted that the shape of the distribution in the presence of a film is closer to pyramidal.

An important aspect is the effect of the film on embedding and the maximum contact pressure during indentation. Figure 6 shows the corresponding results for the selected load range, the lower limit of which is due to the sensitivity of the numerical solution method. The curves illustrating the results obtained for the cases of coating with and without a surface film have significantly different gradients in the region close to zero (at the early stages of loading), then the effect of the film decreases. The question of the legitimacy of using the Winkler model to interpret the results of indentation of pliable thin films on a rigid foundation was considered in [14], where experimental verification of the model is also presented.

CONCLUSIONS

Within the scope of this study, an effective method was proposed for modeling the contact of a rigid indenter with an elastic body covered with a textured layer. The calculations were performed for a parabolic indenter, as well as a relatively soft and relatively rigid coating with cylindrical irregularities on its surface. The results show that the pliability of the coating significantly depends on the period of the irregularities. The selected frequently used shape of the irregularities (cylindrical) causes the concentration of contact pressure at the edges of the irregularities, which can potentially cause uneven wear of such a coating.

The limiting case of the proposed formulation of the problem is the contact of a smooth indenter with a two-layer elastic half-space, on the surface of which there is a thin film (one-dimensional model of an elastic material). The results obtained show that when the Berkovich pyramid is indented with low loads, the effect of the film on the load-penetration curve is significant. It follows from this that the proposed model can be used to interpret the results of indentation, diagnose the presence of surface films of various nature, and assess their mechanical properties.

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