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On the Changing Mechanisms of the Production of Large Irreversible Deformations in the Conditions of Rectilinear Motion in a Cylindrical Layer

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Abstract—In the framework of the theory of large deformations, a solution is obtained for the deformation of a material with nonlinear elastic, plastic and viscous properties located in the gap between two rigid coaxial cylindrical surfaces. The outer surface remains stationary, and the inner rectilinearly moves with variable speed. With uniformly accelerated surface motion, initially irreversible strains accumulate due to the viscous properties of the material as creep strains, and when the stressed state exits to the loading surface, the appearance and development of the viscoplastic flow region are considered. The further development of the flow at a constant speed and its deceleration at equally slow motion of the surface to a complete stop were investigated. The parameters of the stress-strain state of the medium are calculated, and stress relaxation after a complete stop of the cylinder is investigated.

Keywords: large deformations, creep, elasticity, viscosity, ductility, stress relaxation

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1. INTRODUCTION

The research is devoted to the study of the processes of intense deformation of materials when the irreversible deformations accumulated by them can be both creep and plastic deformations. An example of such a technological process is cold forming [1], when irreversible deformations accumulate due to the slow creep process, however, this does not exclude the occurrence of local areas of plastic flow, usually at the points of contact between the wrought material and the tooling. The presence of such regions leads to a significant redistribution of stress fields and, therefore, affects the creep process. Thus, when modeling such processes, it is necessary to use the creep theory while taking into account the possibilities of the emergence and development of plastic flow zones. Taking into account elastic deformations allows one to calculate the elastic response during unloading, including residual stresses and their relaxation after complete unloading.

Given that in most technological processes, the deformations acquired by the material are usually large, the corresponding model should be a model of large deformations of materials with elastic, plastic and viscous properties.

There are a lot of mathematical models of large elastoplastic deformations [2–13]. Here we will use a mathematical model in which reversible and irreversible strains are determined by differential equations of change [9, 12]. The model is generalized to the case when large irreversible strains sequentially accumulate first under creep and then plastic flow conditions recently [14, 15]. It is proposed to separate the irreversible deformations accumulated in the material into creep and plastic deformations by the mechanism of their production. Thus, the equation for changing irreversible strains is one and the same for creep and ductility strains, the difference is only in specifying the source of irreversible strains. For creep strains, the source is creep strain rates, and for plastic strains, plastic strain rates. At elastoplastic boundaries, the mechanism of accumulation of irreversible deformations changes from viscous to plastic and vice versa. Continuity in such an increase in irreversible deformations is ensured by the corresponding assignment of creep and ductility potentials.

The first solutions to boundary value problems that take into account the sequential accumulation of irreversible creep and ductility strains within the framework of the model of large elastic-viscoplastic strains were obtained in [15, 16], without taking viscosity into account in plastic flow, a similar approach was applied in the case of small strains in [17]. Here, using the model of large strains with a sequential change in the mechanism of accumulation of irreversible strains, we present a solution to the boundary-value problem of antiplane deformation of a cylindrical layer under creep conditions followed by plastic flow.

2. BASIC MODEL RELATIONSHIPS

In a rectangular system of spatial Cartesian Euler coordinates x_i, the kinematics of the medium is given by the relations [12]

$$\begin{gathered}
d_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} - u_{k,i}u_{k,j}) = e_{ij} + p_{ij} - \frac{1}{2}e_{ik}e_{kj} - e_{ik}p_{kj} - p_{ik}e_{kj} + e_{ik}p_{km}e_{mj}, \\
\frac{De_{ij}}{Dt} = \varepsilon_{ij} - \gamma_{ij} - \frac{1}{2}((\varepsilon_{ik} - \gamma_{ik} + z_{ik})e_{kj} + e_{ik}(\varepsilon_{kj} - \gamma_{kj} - z_{kj})), \\
\frac{Dp_{ij}}{Dt} = \gamma_{ij} - p_{ik}\gamma_{kj} - \gamma_{ik}p_{kj}, \quad \frac{Dn_{ij}}{Dt} = \frac{dn_{ij}}{dt} - r_{ik}n_{kj} + n_{ik}r_{kj}, \\
\varepsilon_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}), \quad v_i = \frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + u_{i,j}v_j, \quad u_{i,j} = \frac{\partial u_i}{\partial x_j}, \\
r_{ij} = w_{ij} + z_{ij}(\varepsilon_{sk}, e_{sk}), \quad w_{ij} = \frac{1}{2}(v_{i,j} - v_{j,i}), \\
z_{ij} = A_1^{-1}((\varepsilon_{ik}e_{kj} - e_{ik}\varepsilon_{kj})A_2^2 + (\varepsilon_{ik}e_{ks}e_{sj} - e_{ik}e_{ks}\varepsilon_{sj})A_2 + e_{ik}\varepsilon_{ks}e_{st}e_{tj} - e_{ik}e_{ks}\varepsilon_{st}e_{tj}), \\
A_1 = 8 - 8E_1 + 3E_1^2 - E_2 - \frac{1}{3}E_1^3 + \frac{1}{3}E_3, \quad A_2 = 2 - E_1, \\
E_1 = e_{kk}, \quad E_2 = e_{ij}e_{ji}, \quad E_3 = e_{ij}e_{jk}e_{ki}.
\end{gathered} \tag{2.1}$$

Here u_i, v_i are the components of the displacement vectors and the velocity of the points of the medium; d_{ij} are the components of the Almansi strain tensor; e_{ij} and p_{ij} are their reversible and irreversible components; D/Dt is the operator of the used objective derivative of tensors with respect to time, which is written for an arbitrary tensor n_{ij}; r_{ij} are the components of the rotation tensor. Sources γ_{ij} and $\varepsilon_{ij}^e = \varepsilon_{ij} - \gamma_{ij}$ in the equations of change of reversible and irreversible deformations are the rates of their accumulation. For $\gamma_{ij} = 0$ the components of the tensor of irreversible deformations change in the same way as when the coordinate system is rotated or, as when the medium moves without deformation, i.e. $Dp_{ij}/Dt = 0$. The rotation tensor r_{ij} differs from the classical vortex tensor tensor w_{ij} by the presence of a nonlinear part z_{ij}. Note that if component z_{ij} of the rotation tensor r_{ij} is equal to zero, the derivative in (2.1) goes over to the Jauman derivative.

As in [9, 12], we assume that the thermodynamic potential used (free energy density distribution ψ) is an isotropic function of only reversible strains. Then, according to the law of conservation of energy, stresses in a medium are completely determined by reversible deformations and are connected with them by a formula similar to the Murnaghan formula in the nonlinear theory of elasticity [18, 19]. For an incompressible medium, this relation takes the form

$$\sigma_{ij} = -p\delta_{ij} + \frac{\partial W}{\partial e_{ik}}(\delta_{kj} - e_{kj}). \tag{2.2}$$

In relation (2.2), σ_{ij} are the components of the Euler–Cauchy stress tensor, p is the unknown function of additional hydrostatic pressure. For elastic potential $W = \rho_0\psi$ (ρ_0 is the density), we accept its expansion in the Maclaurin series with respect to the free state

$$\begin{gathered}
W = -2\mu I_1 - \mu I_2 + bI_1^2 + (b - \mu)I_1 I_2 - \chi I_1^3 + \ldots, \\
I_1 = e_{kk} - \frac{1}{2}e_{ks}e_{sk}, \quad I_2 = e_{ks}e_{sk} - e_{ks}e_{st}e_{tk} + \frac{1}{4}e_{ks}e_{st}e_{tn}e_{nk}.
\end{gathered} \tag{2.3}$$

Here μ is the shear modulus; b, χ are the constants of the material.

The dissipative mechanism of irreversible deformation is associated with the rheological and plastic properties of the material. We assume further that irreversible deformations accumulate from the beginning of the deformation process and are initially associated with the creep of the material.

In areas where the stress state has not yet reached the yield surface, or where the plastic flow has but stopped, the corresponding dissipative deformation mechanism is specified in the form of the Norton power law of creep [20], in which we assume the rates of irreversible strains γ_{ij} to be equal to the creep rates ε_{ij}^{v}

$$V(\sigma_{ij}) = B\Sigma^{n}(\sigma_1,\sigma_2,\sigma_3), \quad \Sigma = \max\left|\sigma_i - \sigma_j\right|, \quad \gamma_{ij} = \varepsilon_{ij}^{v} = \frac{\partial V(\Sigma)}{\partial \sigma_{ij}}. \tag{2.4}$$

In dependences (2.4), $V(\sigma_{ij})$ is the thermodynamic potential; σ_1, σ_2, σ_3 are the main values of the stress tensor; B, n are the creep parameters of the material.

Over time, the stress state reaches the yield surface, the dissipative mechanism of deformation changes, and plastic flow begins in the material. In this case, in the plastic flow suppose $\gamma_{ij} = \varepsilon_{ij}^{p}$. Without dividing the irreversible deformations into components, we assume that the irreversible creep deformations (2.4) accumulated by the time the plastic flow started (2.4) are the initial values for the plastic deformations that accumulate further in the flow region. If the viscous properties of the medium during plastic flow are taken into account, the rates of irreversible deformations must also coincide when the deformation mechanism changes from viscous to plastic.

According to the Mises maximum principle, the relation between the plastic strain rates ε_{ij}^{p} and stresses is established by the associated law of plastic flow

$$\alpha_{ij} = \lambda\frac{\partial F}{\partial \sigma_{ij}}, \quad F(\sigma_{ij},\alpha_{ij}) = k, \quad \lambda > 0, \quad \alpha_{ij} = \varepsilon_{ij}^{p} - \varepsilon_{ij}^{v_0},$$

where $\varepsilon_{ij}^{v_0}$ are the components of the creep strain rate tensor at the beginning of the plastic flow.

As a plastic potential, we will use the Tresca yield condition, generalized to the case of taking into account the viscous properties of the material

$$\max\left|\sigma_i - \sigma_j\right| = 2k + 2\eta\max\left|\alpha_n\right| \tag{2.5}$$

in which α_n are the principal values of the tensor α_{ij}, k is the yield strength, η is the viscosity coefficient.

3. STATEMENT AND SOLUTION OF THE PROBLEM BEFORE VISCOPLASTIC FLOW

Let the layer of incompressible material be located in the gap between two rigid coaxial cylindrical surfaces of radii r_0 and R $(r_0 < R)$. Consider the process of deformation of the layer during the rectilinear motion of the inner surface and the immobility of the outer surface. We believe that the adhesion conditions are satisfied on the cylindrical walls. Then, in the cylindrical coordinate system r, φ, z the boundary conditions of the problem take the form

$$v\big|_{r=R} = 0, \quad u\big|_{r=R} = 0, \quad v\big|_{r=r_0} = v_0, \quad u\big|_{r=r_0} = u_0 = \int_0^t v_0 dt, \quad \sigma_{rr}\big|_{r=R} = a_0. \tag{3.1}$$

Here $v = v_z(r,t)$, $u = u_z(r,t)$ are the only non-zero components of the velocity and displacement vectors, respectively, $v_0 = v_0(t)$, a_0 are the specified function and constant. From relations (2.1) in this case, we establish that the kinematics of the medium is described by the dependences

$$d_{rr} = -\frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2, \quad d_{rz} = \frac{1}{2}\frac{\partial u}{\partial r}, \quad \varepsilon_{rz} = -w_{rz} = \frac{1}{2}\frac{\partial v}{\partial r}, \quad r_{rz} = \frac{2\varepsilon_{rz}(1 - e_{zz})}{e_{rr} + e_{zz} - 2}. \tag{3.2}$$

In problems of this class, the diagonal components of the strain tensors are small of a higher order in comparison with off-diagonal components [21–24]; therefore, in the future, we restrict ourselves to terms of the first order with respect to diagonal components and the second with respect to off-diagonal components. The accepted restriction is not fundamental for solving the problem and is introduced with the aim of significantly simplifying the calculations. From relations (2.2) and (2.3) in the case under consideration, we find the stresses in the medium

$$\begin{gathered}\sigma_{rr} = -(p+2\mu)+2(b+\mu)e_{rr}+2be_{zz}+\mu e_{rz}^2 = -P+2\mu e_{rr},\\ \sigma_{\varphi\varphi} = -(p+2\mu)+2b(e_{rr}+e_{zz})-2\mu e_{rz}^2 = -P-3\mu e_{rz}^2,\\ \sigma_{zz} = -(p+2\mu)+2(b+\mu)e_{zz}+2be_{rr}+\mu e_{rz}^2 = -P+2\mu e_{zz},\\ \sigma_{rz} = 2\mu e_{rz},\quad \frac{\sigma_{rr}-\sigma_{zz}}{\sigma_{rz}} = \frac{e_{rr}-e_{zz}}{e_{rz}}.\end{gathered} \tag{3.3}$$

In the framework of the quasistatic approximation, we write the equilibrium equations for the case under consideration

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{rz}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\varphi\varphi}}{r} = 0,\quad \frac{\partial\sigma_{rz}}{\partial r}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r} = 0. \tag{3.4}$$

We integrate equations (3.4) under the assumption that the stresses are finite, i.e. $\partial P/\partial z = 0$:

$$\sigma_{rz} = \frac{c(t)}{r},\quad e_{rz} = \frac{c(t)}{2\mu r},\quad P = f(r,t). \tag{3.5}$$

For the components of the strain tensors and strain rates from (2.1), the following relations are valid:

$$\begin{gathered}d_{rz} = e_{rz}+p_{rz},\quad \varepsilon_{rz} = \frac{\partial e_{rz}}{\partial t}+\frac{\partial p_{rz}}{\partial t} = \varepsilon_{rz}^e+\gamma_{rz},\\ \frac{\partial p_{zz}}{\partial t} = -\gamma_{rz}\frac{p_{zz}-e_{rz}^2}{e_{rz}}+\frac{4\varepsilon_{rz}p_{rz}}{2+e_{rz}^2}\left(1+p_{zz}-\frac{1}{2}e_{rz}^2-2e_{rz}p_{rz}\right),\\ e_{rr} = p_{zz}-\frac{3}{2}e_{rz}^2-2e_{rz}p_{rz},\quad p_{rr}+p_{zz} = -2p_{rz}^2,\quad e_{rr}+e_{zz} = -e_{rz}^2.\end{gathered} \tag{3.6}$$

We consider that with an increase in the speed of the inner cylinder, irreversible strains initially accumulate due to the slow creep process.

The potential $V(\sigma_{ij})$ in relation (2.4) in a cylindrical coordinate system takes the form

$$V(\sigma_{ij}) = B(4\sigma_{rz}^2+(\sigma_{rr}-\sigma_{zz})^2)^{n/2}.$$

In this potential, we restrict ourselves to terms of the order of stresses due to the remark described before the dependences (3.3). Then for the creep strain rates and the component p_{rz} from formulas (2.4), (3.3), (3.5) and (3.6) we obtain the relations

$$\begin{gathered}\varepsilon_{rz}^v = f_1(c^{n-1},r),\quad \varepsilon_{rr}^v = -\varepsilon_{zz}^v = \frac{\varepsilon_{rz}^v}{2}\frac{e_{rr}-e_{zz}}{e_{rz}},\quad f_1(c^{n-1},r) = (-1)^n 2^{n-1}Bn\frac{c^{n-1}}{r^{n-1}},\\ p_{rz} = f_1(c_1,r),\quad c_1(t) = \int_0^t c^{n-1}dt,\quad c = (\dot{c}_1)^{\frac{1}{n-1}}.\end{gathered} \tag{3.7}$$

From dependences (3.2), (3.5)–(3.7) and the sticking conditions (3.1) on the surface $r = R$ the relations for speed and displacement follow

$$\begin{gathered}v = f_2(\dot{c},r,R)+f_3(c^{n-1},r,R),\quad u = f_2(c,r,R)+f_3(c_1,r,R),\\ f_2(c,r,R) = \frac{c}{\mu}\ln\frac{r}{R},\quad f_3(c_1,r,R) = \frac{(-1)^n 2^n Bnc_1}{2-n}\left(\frac{1}{r^{n-2}}-\frac{1}{R^{n-2}}\right).\end{gathered} \tag{3.8}$$

Hereinafter, the point on top means the time derivative.

Taking into account the boundary conditions (3.1) on the inner wall $r = r_0$ from (3.8) we obtain the ordinary differential equation for the unknown function $c_1(t)$

$$\dot{c}_1 = f_2^{1-n}(1,r_0,R)(u_0+f_3(c_1,r_0,R))^{n-1},\quad c_1(0) = 0. \tag{3.9}$$

Differential equation (3.9) and system (3.6) are solved numerically in the Wolfram Mathematica package. The stress component σ_{rr} is found from the first equilibrium equation (3.4) using the boundary condition from (3.1). Further, from the relations (3.3), the additional hydrostatic pressure P and the stress tensor components $\sigma_{\varphi\varphi}$ and σ_{zz} are determined.

With an increasing speed of the inner cylinder v_0, the obtained solution of the problem will be valid up to a certain point in time $t = t_0$, at which, in the vicinity of the inner wall $r = r_0$, the plasticity condition (2.5) is satisfied, which in our case takes the form $\sigma_{rz}\big|_{r=r_0} = -k$. From the first relation (3.5) we obtain an equation for determining the moment of time $t = t_0$ at which the viscoplastic flow begins: $c(t_0) = -kr_0$.

4. VISCOPLASTIC FLOW

The region of viscoplastic flow $r_0 \le r \le m(t)$ developing from moment of time $t = t_0$ is bounded by surfaces $r = r_0$ and $r = m(t)$. Viscoelastic deformation still occurs in the region $m(t) \le r \le R$, i.e., the surface $r = m(t)$ is the moving boundary of the viscoplastic flow region.

In the region $m(t) \le r \le R$, by integrating the equilibrium equations (3.4) under the sticking conditions (3.1) on the surface $r = R$, we establish that the dependences (3.5) continue to hold with the unknown integration function $c(t)$. Also in this area, relations (3.7) and (3.8) remain true.

Integrating the equilibrium equations in the viscoplastic flow region $r_0 \le r \le m(t)$ and using the condition for the continuity of the stress components at the elastoplastic boundary $r = m(t)$, we find that dependences (3.5) also hold in this region.

The plastic potential (2.5) in our case is written in the following form

$$\sigma_{rz} = -k + \eta(\varepsilon_{rz}^{p} - \varepsilon_{rz}^{v_0}), \quad \varepsilon_{rz}^{v_0} = -2^{n-1}k^{n-1}Bn. \tag{4.1}$$

From relation (4.1) we determine the rate of plastic deformation

$$\varepsilon_{rz}^{p} = \frac{1}{\eta}\left(\frac{c(t)}{r} + k\right) - 2^{n-1}k^{n-1}Bn. \tag{4.2}$$

From the condition that the rates of irreversible strains coincide on the elastoplastic boundary $r = m(t)$ from (4.2) we find

$$c(t) = -km(t).$$

From dependences (3.2), (3.5), (3.7), (4.2) and the boundary condition for the velocity on the surface $r = r_0$ (3.1), we find the velocity in the viscoplastic flow region

$$\begin{gathered} v = f_2(\dot{c}, r, r_0) + f_4(c, r, r_0) + v_0, \\ f_4(c, r, r_0) = \frac{2}{\eta}\left(c(t)\ln\frac{r}{r_0} + k(r - r_0)\right) - 2^{n}k^{n-1}Bn(r - r_0). \end{gathered} \tag{4.3}$$

The position of the elastoplastic boundary $r = m(t)$ at each moment of time is determined by the ordinary differential equation following from (3.8), (4.3) and the continuity condition for the velocities of the points of the medium at this boundary

$$f_2(-k\dot{m}, r_0, R) + f_3((-km)^{n-1}, m, R) - f_4(-km, m, r_0) = v_0. \tag{4.4}$$

In the region of viscoelastic deformation $m(t) \le r \le R$ equations (3.6) remain valid for irreversible and reversible deformations.

Integrating the plastic strain rate ε_{rz}^{p} (4.2) over time with allowance for (3.5) and (3.7), we find the component of irreversible strains p_{rz} in the flow region $r_0 \le r \le m(t)$

$$p_{rz} = f_5(t, r) + g(r), \quad f_5(t, r) = \frac{1}{\eta}\left(\frac{c_2(t)}{r} + kt\right) - (2k)^{n-1}Bnt, \quad c_2(t) = \int_{t_0}^{t} c(t)dt. \tag{4.5}$$

Here $g(r)$ is the unknown integration function, which we find from the continuity condition for irreversible deformations (3.7) and (4.5) on a moving elastoplastic boundary:

$$g(r) = f_1(c_1(\zeta), r) - f_5(\zeta, r)$$

in which the function $\zeta = \zeta(r)$ is determined from the ordinary differential equation

$$\zeta'(f_3((-kr)^{n-1}, r, R) - f_4(-kr, r, r_0) - v_0(\zeta)) = f_2(k, r_0, R), \quad \zeta(r_0) = t_0. \tag{4.6}$$

For components of reversible and irreversible deformations in the viscoplastic flow region $r_0 \le r \le m(t)$, system (3.6) is valid.

Taking into account relations (3.2), (3.5), (3.6), (4.5) and the condition for continuity of displacements at the elastoplastic boundary $r = m(t)$, we find the expression for displacements in the viscoplastic flow region

$$u = f_2(c, r, R) + f_3(c_1, m, R) + tf_4(t^{-1}c_2, r, m) + 2\int_m^r g(r)dr.$$

When the inner cylinder moves at a constant speed from moment of time $t = t_1 > t_0$, the viscoplastic flow region continues to develop according to the law (4.4). The solution obtained above remains valid both in the region $r_0 \le r \le m(t)$ and in the region $m(t) \le r \le R$. A change in the boundary condition will only lead to a change in the function $\zeta(r)$: in the region $m(t_1) \le r \le m(t)$, it is now determined from the differential equation

$$\zeta'(f_3((-kr)^{n-1}, r, R) - f_4(-kr, r, r_0) - v_0(t_1)) = f_2(k, r_0, R), \quad \zeta(m(t_1)) = t_1, \tag{4.7}$$

while in the region $r_0 \le r \le m(t_1)$ it is still a solution of equation (4.6).

5. INHIBITION OF VISCOPLASTIC FLOW AND UNLOADING OF THE MEDIUM

The initial decrease in the speed of movement of the inner surface, starting from time $t = t_2 > t_1$, does not lead to significant changes in the deformation process. Unlike the case when the cylinder moves at a constant speed, the function $\zeta(r)$ in the region $r_0 \le r \le m(t_1)$ is found from equation (4.6), in the region $m(t_1) \le r \le m(t_2)$ it is found from (4.7), and in the region $m(t_2) \le r \le m(t)$ it is again from (4.6) with the boundary condition $\zeta(m(t_2)) = t_2$.

At the calculated moment of time $t = t_3 > t_2$, the growth of the viscoplastic flow region ceases. A new boundary $r = m(t)$ appears, moving from the stationary surface $r = m(t_3)$ to the inner surface $r = r_0$ and separating the decreasing flow region $r_0 \le r \le m(t)$ from the unloading region $m(t) \le r \le m(t_3)$, in which, like in the region $m(t_3) \le r \le R$, irreversible creep deformations now accumulate.

In the viscoelastic strain region $m(t_3) \le r \le R$ and in the flow region , the relations valid in the time interval $t_2 \le t \le t_3$ remain valid.

In the region $m(t) \le r \le m(t_3)$, relations (3.5) and (3.6) are satisfied for the stress components and the rates of irreversible strains.

From (3.2), (3.5), (3.7) and the condition for the continuity of velocity at the elastoplastic boundary $r = m(t_3)$, we establish that in the region $m(t) \le r \le m(t_3)$ the velocity is calculated from the first dependence (3.8).

From the condition of continuity of velocities at the elastoplastic boundary $r = m(t)$, we obtain the equation for its change of the form (4.4).

Integrating the dependence (3.7) over time in the unloading region $m(t) \le r \le m(t_3)$, we obtain the relation for the component of irreversible deformations in this region

$$p_{rz} = f_1(c_1, r) + g_1(r). \tag{5.1}$$

In expression (5.1), we find the unknown integration function $g_1(r)$ from the condition for the continuity of irreversible strains on the elastoplastic boundary $r = m(t)$

$$g_1(r) = f_1(c_1(\zeta), r) + f_5(\zeta, r) + g(r). \tag{5.2}$$

Here, the function $\zeta = \zeta(r)$ satisfies the ordinary differential equation (4.6) with the initial condition $\zeta(m(t_3)) = t_3$.

Integrating the second equation (3.2) in the region $m(t) \le r \le m(t_3)$, taking into account relations (3.5), (3.6), (5.1) and the continuity condition for the displacements on the boundary $r = m(t_3)$, we find the displacement component

$$u = f_2(c, r, R) + f_3(c_1, r, R) + 2\int_{m(t_3)}^{r} g_1(r)dr. \quad (5.3)$$

In the region $r_0 \le r \le m(t)$, from the second equation (3.2), taking into account (3.5), (3.6), (4.5) and the continuity condition for the displacements on the boundary $r = m(t)$, we find

$$u = f_2(c, r, R) + f_3(c_1, m, R) + tf_4(t^{-1}c_2, r, m) + 2\int_{m}^{r} g(r)dr + 2\int_{m(t_3)}^{m} g_1(r)dr. \quad (5.4)$$

At moment of time $t = t_4 > t_3$, the inner cylinder will stop ($v_0(t_4) = 0$). The velocity in the viscoplastic flow region $r_0 \le r \le m(t)$ will be calculated as

$$v = f_2(\dot{c}, r, r_0) + f_4(c, r, r_0).$$

The equation for calculating the position of the elastoplastic boundary $r = m(t)$ takes the form

$$f_2(-k\dot{m}, r_0, R) + f_3((-km)^{n-1}, m, R) - f_4(-km, m, r_0) = 0.$$

In the region $m(t_3) \le r \le R$, the component of irreversible strains is calculated from (4.5), in the region $m(t) \le r \le m(t_3)$, from (5.1). The function $g_1(r)$ has the form (5.2), in which $\zeta = \zeta(r)$ in the region $m(t_k) \le r \le m(t_3)$ is found from equation (4.6) with the initial condition $\zeta(m(t_2)) = t_2$, and in the region $m(t) \le r \le m(t_k)$ it is calculated from the equation

$$\zeta'(f_2((-kr)^{n-1}, r, R) - f_4(-kr, r, r_0)) = f_1(k, r_0, R), \quad \zeta(m(t_4)) = t_4.$$

For displacements in regions $m(t) \le r \le m(t_3)$ and $r_0 \le r \le m(t)$, dependences (5.3) and (5.4) continue to hold.

At some moment of time $t = t_5$, the elastoplastic boundary $r = m(t)$ coincides with the inner surface $r = r_0$ and the viscoplastic flow in the cylindrical layer ceases. For velocity throughout the layer, the first relation (3.8) will be satisfied, in which the function $c(t)$ will take the form

$$c(t) = \left(\ln\frac{r_0}{R}\left(R^{n-2}\ln\frac{r_0}{R} + 2^n Bnk^{n-2}\mu(r_0^{n-2} - R^{n-2})(t - t_4)\right)^{-1}\right)^{\frac{1}{n-2}}.$$

The displacements in regions $m(t_3) \le r \le R$ and $r_0 \le r \le m(t_3)$ are calculated by relations (3.8) and (5.3), respectively.

The function v_0 in the calculations was chosen as follows:

$$v_0 = \begin{cases} \xi_1 t, & 0 \le t \le t_1, \\ \xi_1 t_1, & t_1 \le t \le t_2, \\ \xi_1 t_1 - \xi_2(t - t_2), & t_2 \le t \le t_4, \\ 0, & t \ge t_4. \end{cases}$$

The calculations were carried out in dimensionless variables $\tilde{r} = r/R$, $\tau = \xi_1 t^2/r_0$ with the following values of the constant parameters $k/\mu = 0.003$, $r_0/R = 0.2$, $n = 3$, $B\mu^2\sqrt{r_0/\xi_1} = 3$, $\xi_2/\xi_1 = 0.5$, $(\mu/\eta)\sqrt{r_0/\xi_1} = 10$, $a_0 = 10^{-6}$. The graph of the elastoplastic boundary $\tilde{m} = m/R$ in the interval from $\tau_0 = 0.0097$ to $\tau_3 = 0.1121$ is shown in Fig. 1a, in the interval from τ_3 to $\tau_5 = 5.756$ is shown in Fig. 1b. Note that $\tau_1 = 0.03$, $\tau_2 = 0.1$, $\tau_4 = 0.4391$. The distributions of the velocities and displacements of points of the medium along the layer at different instants of time are shown in Fig. 2a and 2b, respectively. Figure 3 illustrates the change in irreversible deformations at points of the inner surface $r = r_0$ in the interval $0 \le \tau \le \tau_5$. Relaxation of the stress components σ_{rz} and the largest of the diagonal σ_{zz} is shown in Fig. 4a and 4b, respectively ($\tau_6 = 1000$, $\tau_7 = 10000$).

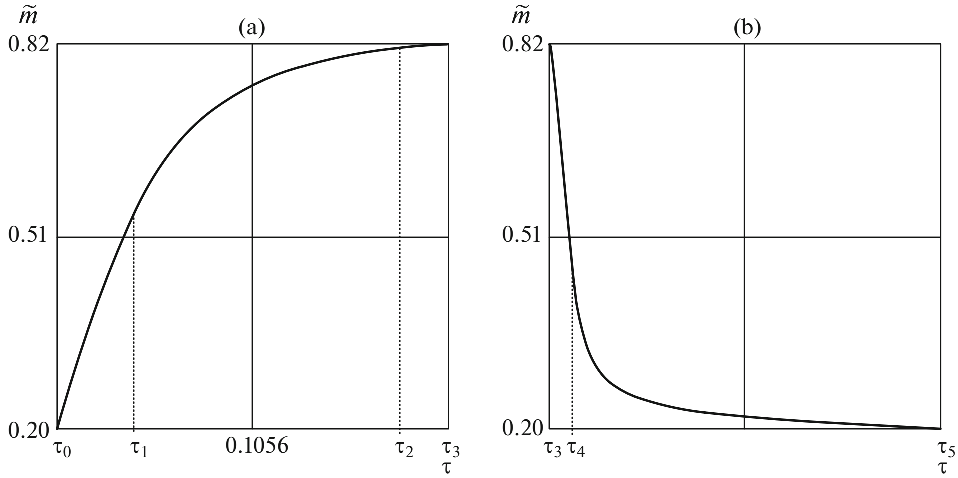

Fig. 1.

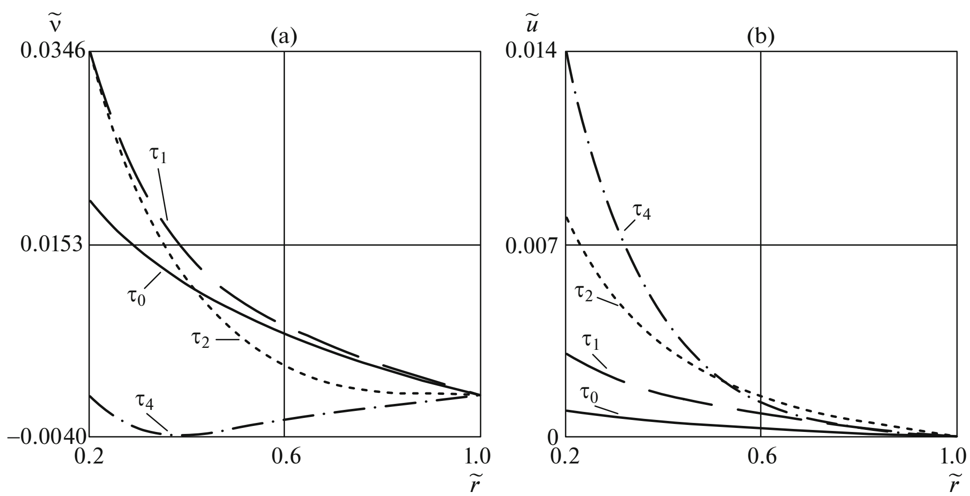

Fig. 2.

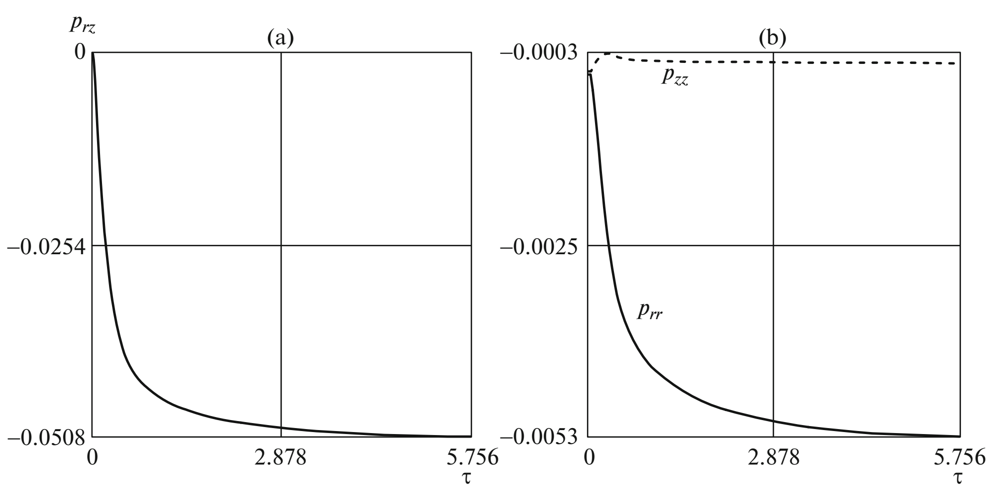

Fig. 3.

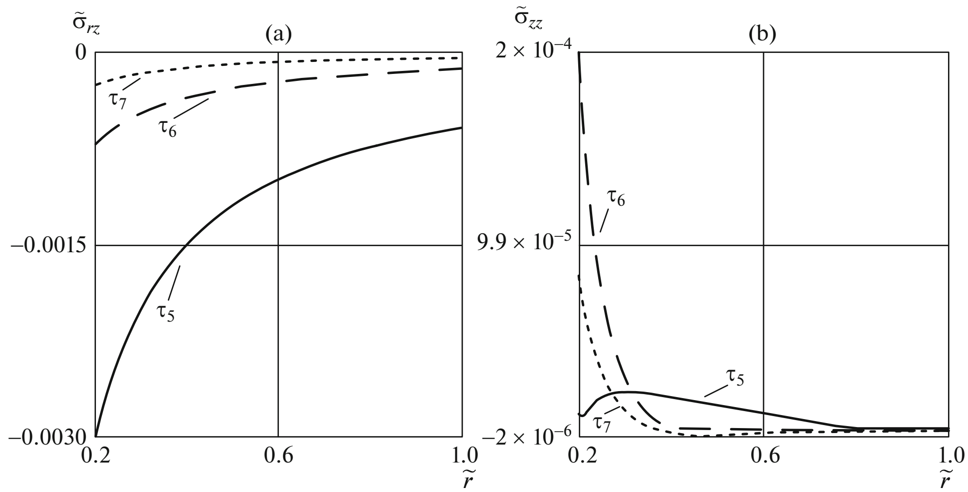

Fig. 4.

6. CONCLUSIONS

The solution to the problem of the deformation of the material of the cylindrical layer that occurs during rectilinear uniformly accelerated movement of the inner cylinder, its subsequent movement with constant speed and equally slow motion until it stops completely, while the outer cylinder remains stationary, is constructed. Under conditions of rigid adhesion of the material to the boundary walls, the case is considered when the accumulated irreversible deformations are both creep and plastic deformations. The progress of the elastoplastic boundary separating the regions with different acting laws of accumulation of irreversible deformations is studied, the parameters of the stress-strain state of the medium are calculated, and stress relaxation occurring after the cylinder stops is observed.

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