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# Bending of a Round Plate under Gas Pressure

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**Abstract**—The effect of the average excess pressure of the environment on the linear and nonlinear bending of a round plate is studied. Different values of the gas pressure on both surfaces of the plate form a transverse distributed load, consisting of a differential pressure and the interaction of the average pressure with the curvature of the middle surface. With a small ratio of the average pressure to the elastic modulus of the material and with a large relative thickness, the influence of the second component of the bending load is small. With a large ratio of the average pressure to the elastic modulus and a small relative thickness, this effect is significant.

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## 1. INTRODUCTION

A lot of literature is devoted to the analysis of linear and nonlinear bending of round thin plates and membranes. The same applies to the stability problem of round plates under the action of radially directed compressive forces. We point, for example, to [1—12]. In [13, 14], in addition to the elastic deformation of the plates, elastoplastic bending is considered. In [13], the nonlinear bending of round plates under pressure and temperature was studied from the standpoint of using them as safety membranes; in [14], the results of a study of the nonlinear bending and stability of round plates under the pressure of a heated or cooled compressible working environment are presented.

In all these works, it is assumed that the transverse distributed load $q$ on a thin plate is equal to $q = \gamma h + p_2 - p_1$, where $\gamma$ is the specific gravity of the material, $h$ is its thickness, $p_1$, $p_2$ are the excess gas pressures on the lower and upper surfaces of the plate. In the case of a small relative thickness of the plate and applying pressure to only one of the surfaces or a small ratio of the average pressure $p_m = (p_1 + p_2)/2$ to the elastic modulus of the material $E$, this value of $q$ is quite accurate.

Taking into account the effect of the difference in the areas of the lower and upper surfaces, the average pressure $p_m$ on the cylindrical bending of the elongated plate, leads to the expression for the distributed transverse load [15-17]

$$q = \gamma h - p_1 + p_2 + p_m h\Big(\frac{d^2w}{dx^2}\Big), \tag{1.1}$$

where the deflection function $w(x)$ depends on the reduced stiffness $D(1 + \alpha)$. Here $D$ is the flexural rigidity of the plate, $\alpha$ is the dimensionless parameter, which for a pivotally fixed plate of length $L$ is

$$\alpha \approx \Big(\frac{p_m}{E}\Big)\Big(\frac{L}{h}\Big)^2. \tag{1.2}$$

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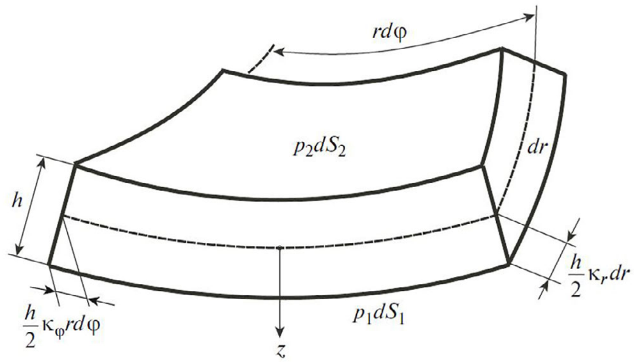


**Fig. 1.**

When obtaining (1.1), (1.2), it is assumed that the edges of the plate are isolated from the action of pressures $p_1, p_2$.

For structures such as reinforced concrete, ship structures at atmospheric pressure or one-sided low pressures, the parameter $\alpha$ is zero or very small. But for the designs of petrochemical equipment, energy, deep-sea vehicles, aerospace technology, there can be considerable values of $\alpha$. If $E = 2\cdot 10^6$ bar (steel), $p_m = 20$ bar, $L/h = 10^2$ , then $\alpha = 0.1$. The reduced stiffness is increased by 10%. In the case of $E = 2\times 10^5$ bar (magnesium alloy), the parameter $\alpha = 1$. In this case, the reduced stiffness is equal to $2D$, the deflection is two times less than in the case when the difference in the areas of the upper and lower surfaces of the plate is not taken into account.

If the media below and above the plate are liquids with specific weights $\gamma_1$ and $\gamma_2$, then the term $(\gamma_2 - \gamma_1)w$ appears in expression (1.1). Taking this factor into account leads to the problem of the interaction of elastic and hydrodynamic instabilities [15]. This statement complicates the analysis and is not considered here.

## 2. STATEMENT OF THE PROBLEM

We consider the static elastic bending of a round plate with a diameter of $2c$ and a thickness $h$, on the lower and upper surfaces of which the gas pressures $p_0 + p_1$ and $p_0p_2$ act, where $p_0$ is atmospheric pressure, $p_1, p_2$ are excess pressures. The pressures $p_1$ and $p_2$ can be both positive and negative, and the negative values of $p_1, p_2$ are less than $p_0$. The effect of gas densities on the transverse load ($\gamma_1 = \gamma_2 = 0$) is not taken into account. When the plate bends pressures $p_1$, $p_2$ remain unchanged. The edge of the plate is isolated from the action of excess pressure (only $p_0$ acts). Prior to the application of pressures $p_1, p_2$ the plate under an all-round pressure $p_0$ is in an unstressed plane state. The direction of the $z$ axis, load $q$, and deflection $w(r)$ are positively down. Axisymmetric bending is considered.

Fig. 1 shows a plate element with an area $dS = rd\varphi dr$ of the middle surface. In accordance with Kirchhoff's hypotheses, during deformation, the cross section remains flat and perpendicular to the middle surface, the plate thickness does not change. Under axisymmetric bending, the resulting curvatures along the radius $\kappa_r$ and along the angle $\kappa$ are related to the deflection function $w(r)$ by the equations [10, p. 173]

$$\kappa_r = -\frac{d^2w}{dr^2}, \quad \kappa_\varphi = -\frac{1}{r}\frac{dw}{dr}. \tag{2.1}$$

With axisymmetric bending, the torsion curvature is zero. The areas of the lower and upper surfaces of the element are equal to

$$dS_1 = dr\Big(1 + \frac{h}{2}\kappa_r\Big)rd\varphi\Big(1 + \frac{h}{2}\kappa_\varphi\Big), \quad dS_2 = dr\Big(1 - \frac{h}{2}\kappa_r\Big)rd\varphi\Big(1 - \frac{h}{2}\kappa_\varphi\Big). \tag{2.2}$$

The transverse distributed force acting on the area $dS$ of the middle surface is equal to

$$qdS = \gamma hdS + p_2dS_2 - p_1dS_1. \tag{2.3}$$

Here the dead weight is assigned to the median surface. Substituting expressions (2.2), (2.1) into (2.3) and discarding nonlinear terms, we obtain

$$q = p_e + p_m h \nabla^2 w, \quad p_e = \gamma h + p_2 - p_1, \quad \nabla^2 = \frac{1}{r}\frac{d}{dr}\Big(r\frac{d}{dr}\Big), \quad p_m = \frac{p_1 + p_2}{2}. \tag{2.4}$$

The system of nonlinear equations of axisymmetric bending in the notation [10, p. 178] has the form

$$\begin{gathered} D\frac{d(\nabla^2 w)}{dr} - \frac{h}{r}\frac{d\Phi}{dr}\frac{dw}{dr} = \Psi, \quad \frac{d(\nabla^2 \Phi)}{dr} + \frac{E}{2r}\Big(\frac{dw}{dr}\Big)^2 = 0, \\ D = \frac{Eh^3}{12(1-\nu^2)}, \quad \sigma_r = \frac{d\Phi}{rdr}, \quad \sigma_\varphi = \frac{d^2\Phi}{dr^2}, \quad \Psi = \frac{1}{r}\int_0^r qrdr. \end{gathered} \tag{2.5}$$

Taking into account $q$ from (2.4), the expression $\Psi$ can be given the form

$$\Psi = \frac{p_e r}{2} + \frac{p_m h}{r}\int_0^r r\nabla^2 w dr = \frac{p_e r}{2} + p_m h\frac{dw}{dr}. \tag{2.6}$$

Further down, for the deflection function $w$, the conditions of hinged support of the plate along the contour [10, p. 179]

$$w = 0, \quad \frac{d^2 w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr} = 0 \quad (r = c), \tag{2.7}$$

and pinching

$$w = 0, \quad \frac{dw}{dr} = 0 \quad (r = c), \tag{2.8}$$

will be used.

The boundary conditions for the stress function $\Phi$ in the absence of displacement of the plate contour along the radius ($u = 0$) and free movement ($\sigma_r = 0$) respectively have the form:

$$\frac{d^2\Phi}{dr^2} - \frac{\nu}{r}\frac{d\Phi}{dr} = 0, \quad \frac{1}{r}\frac{d\Phi}{dr} = 0 \quad (r = c). \tag{2.9}$$

The question is about the effect on the bending of the plate of the second term in (2.6), which appears as a result of taking into account the difference in the areas of the lower and upper surfaces when determining the transverse distributed force. To do this, we first consider a linear problem.

## 3. LINEAR BENDING

From (2.5), (2.6) we have the equation

$$\frac{d(\nabla^2 w)}{dr} - \frac{p_m h}{D}\frac{dw}{dr} = \frac{p_e r}{2D}. \tag{3.1}$$

a) In the case of pinching the contour, making an approximate solution satisfying the conditions (2.8), in the form [10, p. 186]

$$w = f\Big(1 - \frac{r^2}{c^2}\Big)^2. \tag{3.2}$$

and integrating (3.1) by the Bubnov–Galerkin method, we obtain the following expression for the relative deflection amplitude

$$\xi = \frac{f}{h} = \frac{3(1-\nu^2)q^*}{16(1+\alpha)}, \quad q^* = \frac{p_e}{E}\Big(\frac{c}{h}\Big)^4, \quad \alpha = \frac{3(1-\nu^2)p_m c^2}{4Eh^2}. \tag{3.3}$$

Thus, the dimensionless parameter $\alpha$ determines the contribution of the second term in equation (3.1). At $E = 2 \times 10^6$ bar (steel), $\nu = 0.3$, $c/h = 10^2$, $p_m = 2$ bar, the parameter $\alpha = 0.0068$. If for those values of $E$, $\nu$ we take $c/h = 10^3$, $p_m = 20$ bar, then $\alpha = 6.8$. In the first case, the average pressure does

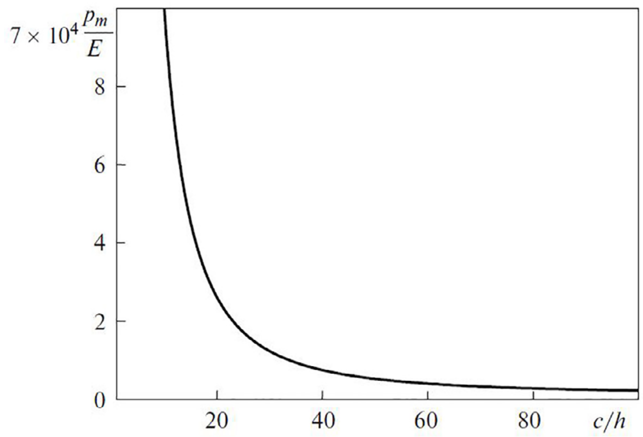


**Fig. 2.**

not play any role, the classical theory of bending of thin plates gives the correct result. In the second case, the average pressure $p_m$ exerts greater bending resistance than the elastic modulus $E$ (or rather, the value $p_m c^2$ than $Eh^2$). This is because the intrinsic flexural rigidity of the plate is proportional to $h^3$, and the influence of average pressure is proportional to $h$. At $p_1 = p_2 = p_m = 20$ bar, the plate bent under its own weight $\gamma h$ is straightened, while the deflection decreases by almost seven times.

Fig. 2 shows the dependence of the average pressure on the ratio of the radius to the plate thickness for the parameter $\alpha = 10^{-2}$ in (3.3), which means a one percent correction to the deflection amplitude. If with these input parameters the display point falls into the region below the curve, then there is no influence of the average pressure on the bend, above the curve this effect becomes noticeable.

At large and close pressures $p_1$ and $p_2$, when the average pressure pm is also large and the difference $p_2 - p_1$ is small, the greatest influence of $p_m$ and parameter $\alpha$ on bending is realized (for the same values of $E$, $c$, $h$). Their least influence takes place with one-way pressure. Let $p_1 = 0$, $p_m = p_2/2$ and ignore weight ($\gamma = 0$). We limit the value of $p_2$ to the applicability of linear equation (3.1) and its solution (3.3). Setting $\xi \leq 1$ in (3.3), we obtain

$$\frac{p_2}{E} \leq \frac{16(1+\alpha)}{3(1-\nu^2)}\left(\frac{h}{c}\right)^4. \tag{3.4}$$

Taking into account (3.4) and $p_m = p_2/2$ from (3.3), we have the largest possible value of the parameter

$$\alpha = \frac{2}{(c/h)^2 - 2}. \tag{3.5}$$

Assuming here that $\alpha = 10^{-2}$, we find that for $c/h \leq 14$ the display point in Fig. 2 is above the curve, and for $c/h > 14$ it is below the curve (there is no influence of average pressure). This is explained by the fact that with a decrease in the relative thickness, the bending stiffness and the permissible one-sided pressure quickly fall. Moreover, the ratio $p_m/E$ is also small.

There is an apparent contradiction between the general parameter $\alpha$ according to (3.3) , which is proportional to $(c/h)^2$, and the parameter $\alpha$ according to (3.5), which is proportional when $(c/h)^2 \gg 2$ to the value $(h/c)^2$. In the first case, the average pressure $p_m$ can vary over a wide range with a constant difference $p_2 - p_1$, in the second case $p_1 = 0$, therefore $p_m$ is rigidly determined by the pressure $p_2$, the value of which is limited by the deflection not exceeding the plate thickness.

b) The conditions of hinged support (2.7) are satisfied with the deflection function [10, p. 191]

$$w = f\left(1 - 2a\frac{r^2}{c^2} + ab\frac{r^4}{c^4}\right), \quad a = \frac{3+\nu}{5+\nu}, \quad b = \frac{1+\nu}{3+\nu}. \tag{3.6}$$

Relative deflection equals to

$$\xi = \frac{f}{h} = \frac{3q^*(1-\nu^2)}{16ab(1+\alpha)}, \quad \alpha = \frac{3(1-\nu^2)p_m c^2}{4Eh^2\beta}, \quad \beta = \frac{b(3-2b)}{6-8b+3b^2}. \tag{3.7}$$

Thus, the parameter $\alpha$ in the case of articulation is equal to the value (3.3) for the case of a clamped edge of the plate divided by the coefficient $\beta$, which depends only on $\nu$. For $\nu = 0.3$, the coefficient $\beta = 0.26$. Therefore, by (3.7), the parameter $\alpha$ is almost four times larger than by (3.3) . The influence of average pressure on the bend of a pivotally fixed plate is four times greater than in the case of pinching. This is explained by the fact that, according to (2.4), the transverse force $q$ also depends on the curvature formed during bending. In the case of articulation, the sign of curvature does not change over the entire area; therefore, the contribution of the last term of $q$ to the solution is the largest. In the case of pinching near the support and in the central part of the plate, the signs of curvature are different. According to approximation (3.2), the inflection point is located at a radius $r \approx 0.58$c.

The largest value of unilateral pressure $p_2$, giving a relative deflection $\xi = 1$, is

$$\frac{p_2}{E} = \frac{16(1+\alpha)}{3(1-\nu)(5+\nu)}\left(\frac{h}{c}\right)^4, \tag{3.8}$$

which at $\nu = 0.3$ is four times less than according to (3.4). Instead of (3.5) we get

$$\alpha = \frac{2}{(c/h)^2(5+\nu)\beta/2(1+\nu)-2}, \tag{3.9}$$

where the factor at $(c/h)^2$ is 0.53 ($\nu = 0.3$).

These estimates show that in the case of hinging the plate, the average pressure $p_m$ has a greater effect on bending than with a pinched edge. As indicated above, a positive average pressure $p_m$ reduces the deflection. A negative value of $p_m$ (degassing) and, accordingly, a negative value of the parameter $\alpha$ lead to an increase in the deflection at the same value of its own weight and the difference in excess pressure $p_2 - p_1$.

As can be seen from (3.3) and (3.7), these solutions are valid only for $\alpha > -1$. The value $\alpha = -1$ can be considered critical when the linear solution increases unboundedly. The corresponding critical values of the mean overpressure according to (3.3) and (3.7) are

$$p_m^* = -\frac{4Eh^2}{3(1-\nu^2)c^2}, \quad p_m^* = -\frac{4Eh^2\beta}{3(1-\nu^2)c^2}. \tag{3.10}$$

It is of interest to find out if the real valuesof the input parameters $E$, $h$, $c$ can give values $p_m^* > -p_0 = -1$ bar. The first version of the numerical data adopted above ($E = 2\cdot 10^6$ bar, $\nu = 0.3$,$c/h = 10^2$) leads to large values of the critical average overpressure in both cases of fixing. They are devoid of physical meaning. In the second variant of the numerical data ($E = 2\cdot 10^6$ bar, $\nu = 0.3$, $c/h = 10^3$), from (3.10) we obtain $p_m^* = -2.93$ bar and $p_m^* = -0.76$ bar, respectively. With the received data, the critical value of the average pressure for the clamped plate is devoid of physical meaning, while for the case of articulation, it seems quite acceptable. Naturally, the given linear solution is valid only for deflections smaller than the plate thickness.

## 4. NON-LINEAR BENDING

We take the same approximating functions for deflection $w$ as in the linear solution.

a) In the case of a clamped edge, substituting functions (3.2) in the second equation (2.5), we obtain the following expression bounded at r = 0 [10, p. 187]

$$\frac{d\Phi}{dr} = -\frac{Ef^2r^3}{c^4}\left(1 - \frac{2r^2}{3c^2} + \frac{r^4}{6c^4}\right) + \frac{Cr}{2}. \tag{4.1}$$

In the case when the edge of the plate freely moves along the radius, $C = Ef^2/c^2$ follows from the second condition (2.9). We substitute in the first equation (2.5) and in (2.6) the expressions

$$\frac{d(\nabla^2 w)}{dr} = \frac{32fr}{c^4}, \quad \frac{dw}{dr} = -\left(\frac{4fr}{c^2}\right)\left(1 - \frac{r^2}{c^2}\right).$$

**Table 1**

| *GU* | (2.7) | | (2.8) | |
|---|---|---|---|---|
| $E$ bar | $\alpha = 0$ | $\alpha \neq 0$ | $\alpha = 0$ | $\alpha \neq 0$ |
| $2 \times 10^4$ | 1.513 | 3.478 | 0.636 | 0.853 |
| | 1.276 | 1.805 | 0.611 | 0.770 |
| $2 \times 10^5$ | 0.308 | 0.348 | 0.082 | 0.085 |
| | 0.301 | 0.337 | 0.082 | 0.085 |
| $2 \times 10^6$ | 0.034 | 0.035 | 0.008 | 0.008 |
| | 0.034 | 0.035 | 0.008 | 0.008 |

After integration by the Bubnov—Galerkin method, we obtain the equation

$$\frac{6}{7}\xi^3 + \frac{16(1+\alpha)}{3(1-\nu^2)}\xi = q^*, \tag{4.2}$$

where $q^*$ and $\alpha$ are presented in (3.3) .

In a nonlinear solution, the influence of average pressure can be significant even in the case of one-way pressure. Assuming, as before, $\nu = 0.3$, $\gamma = 0$, $p_1 = 0$, $p_m = p_2/2$, we represent (4.2) in the form

$$0.857\xi^3 + 5.862(1+\alpha)\xi = q^*, \quad q^* = \frac{p_2}{E}\left(\frac{c}{h}\right)^4, \quad \alpha \approx \frac{p_2}{3E}\left(\frac{c}{h}\right)^2. \tag{4.3}$$

The largest possible values of $p_2/E$ and $\alpha$ are determined, for example, from the condition $\xi = 4$. Then it follows from (4.3)

$$\frac{p_2}{E} \approx \left(\frac{h}{c}\right)^4 [54.8 + 23.4(1+\alpha)], \quad \alpha \approx \frac{26.07}{(c/h)^2 - 7.8}. \tag{4.4}$$

According to (4.4), we conclude that with $c/h \geq 51$ ($\alpha = 10^{-2}$ is assumed) there is no influence of the average pressure on the bend, while, for example, with $c/h = 4$, there is a strong influence ($\alpha \approx 3.2$). Moreover, according to the linear theory (3.5), $\alpha \approx 0.14$. Thus, in the case of nonlinear bending under unilateral pressure, the influence of the average pressure is greater than in the case of linear bending.

Let us consider an example of a bend when, at the same pressure drop $p_2 - p_1$ (or $q^*$), the average overpressure is $p_m = \pm 0.9$ bar. Suppose that the values of $E$, $c/h$ are such that $q^* = 2$, $\alpha = \pm 1.2$. If at $\alpha = 1.2$ the relative deflection $\xi < 1$ is realized, then the linear solution of (3.3) $\xi = 0.155$ is valid. For $\alpha = -1.2$, the deflection cannot be determined by (3.3); it is necessary to turn to the nonlinear solution (4.3), from which we find $\xi = 1.570$. Note, according to the classical theory of bending, $\xi = 0.341$ ($\alpha = 0$). As you can see, there is not only a quantitative difference between these solutions. When evacuating the surface of the plate, when the coefficient $\alpha$ approaches $(-1)$ or less, the deflection must be determined on the basis of a nonlinear theory.

b) When hinged instead of (4.1) we have

$$\frac{d\Phi}{dr} = -\frac{Ef^2a^2r^3}{c^4}\left(1 - \frac{2br^2}{3c^2} + \frac{b^2r^4}{6c^4}\right) + \frac{Cr}{2}. \tag{4.5}$$

From the second condition (2.9) it follows that $c = (Ef^2a^2/3c^2)(6 - 4b + b^2)$. In the same way as above, we obtain the equation for $\xi$, which for $\nu = 0.3$ reduces to

$$0.377\xi^3 + 1.437(1+\alpha)\xi = q^*, \quad \alpha = 2.6\frac{p_m}{E}\left(\frac{c}{h}\right)^2. \tag{4.6}$$

In the case $\alpha = -1 \pm \epsilon$ ($\epsilon \ll 1$), the solution of equations (4.3) and (4.6) in two approximations

$$\xi \approx [1.17q^* \mp 7.18\varepsilon(q^*)^{1/3}]^{1/3}, \quad \xi = [2.657q^* \mp 5.26\varepsilon(q^*)^{1/3}]^{1/3}. \tag{4.7}$$

The table shows the values of the deflection in the center of the plate under the considered boundary conditions (BC) for a number of Young's modulus values taking into account ($\alpha \neq 0$) and without taking into account ($\alpha = 0$) the effect of average pressure on linear and nonlinear bending (underlined data) of

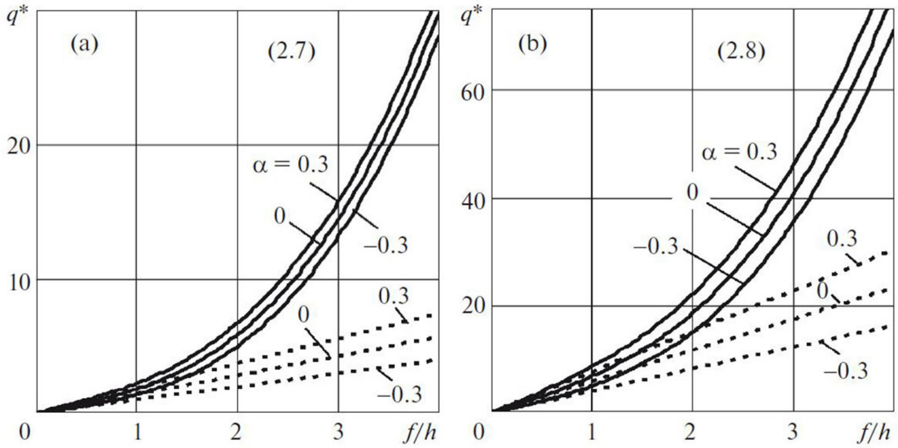


**Fig. 3.**

the plate. Close pressures $p_1 = 1$ bar, $p_2 = 1.00110$ bar are given for the parameters: $c/h = 100$, $\nu = 0.3$, $\gamma = 0$. The table shows that in this problem, the influence of the average pressure on the deflection in the center of a Plexiglas plate for a linear solution with pivoting reaches 56%, in the case of a clamped edge — 25%, and in the nonlinear solution 29% and 21%, respectively. It should be noted that the influence of the average pressure on the bending of plates made of both magnesium alloy and steel is not observed during BC pinching, and when the edges of a magnesium alloy plate are hinged, it does not exceed 11%.

Fig. 3,*a, b* shows the dependences of the dimensionless load parameter $q^*$ and deflection in the center of the plate for conditions (2.7) and (2.8), respectively. The solid line indicates the solutions taking into account geometric nonlinearity for $\alpha = -0.3;\ 0;\ 0.3$, and the dashed line indicates the linear solutions. It can be seen that with a fixed value of $q^*$, with increasing $\alpha$, the maximum deflection decreases, and with a negative value of $\alpha$ it increases, while the deflection of the plate in the case of articulated support is almost four times greater than when the edge of the plate is jammed for all the considered values of the parameter $\alpha$ of the linear solution.

## 5. CONCLUSION

1. In the classical theory of bending of thin round plates, the ratio of the deflection arrow $f$ to the thickness $h$ is determined by the dimensionless load parameter $q^* = (p_e/E)(h/c)^4$, which depends on the ratio of the distributed transverse force $p_e = \gamma h + p_2 - p_1$ to the elasticity modulus $E$ of the material and on the relative thickness $h/c$ ($\gamma$ is the specific weight, $p_1$, $p_2$ are excess pressures from the bottom and top of the plate). Taking into account the difference between the areas of the convex and concave surfaces of the plate, the average overpressure $p_m = (p_1 + p_2)/2$, also leads to the dependence of $f/h$ on the dimensionless parameter $\alpha = (\kappa p_m/E)(c/h)^2$. Here, the number $\kappa$ depends on the conditions of plate fixation (for $\nu = 0.3$ for pinching, $\kappa = 0.68$, articulation $\kappa = 2.62$).

2. At a constant value of $q^*$, an increase in the parameter $\alpha$ leads to a decrease in the deflection ($\alpha = 0$ corresponds to the classical theory). Excessive pressures can also have negative values, which takes place during the degassing of plate surfaces (should be $p_0 + p_1 > 0$, $p_0 + p_2 > 0$, where $p_0$ is atmospheric pressure). Therefore, $p_m$ and $\alpha$ can also be negative, with $\alpha > -1$ in the linear problem. In a nonlinear problem, this restriction is removed. With a negative value of $\alpha$, the deflection increases. From the condition $\alpha = -1$, the critical value of the mean overpressure $p_m^*$ is determined during the degassing of the plate surfaces. Supercritical bending occurs at $\alpha < -1$, however, $p_m$ should remain less than atmospheric pressure $p_0$.

3. The greatest influence of the average pressure $p_m$ on bending occurs when the large pressures $p_1$ and $p_2$ are equal, and the smallest — with one-sided pressure. In the latter case, the correction for the value of the deflection due to the average pressure can be at small ratios $c/h$. This is explained by the limitation of the admissible value of the difference $p_2 - p_1$ for a given plate.

4. There is a strong dependence of the deflection of the plate on the action of pressure on its edge. The above results were obtained when only atmospheric pressure $p_0$ was applied to the edge. If, for example,

at $p_1 = p_2 = p_m$, the average excess pressure $p_m$ acts on the edge, then its effect on bending disappears. The absolute stability of the shape of the plate, curved under its own weight, is realized. In general, further study of the effect of average overpressure on the bending of a round plate and its experimental study is required.

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