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Theoretical Background of Rationale for the Possibility of the Kovalevskaya Gyroscope Usage by a Three-Component Angular Velocity Meter

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Abstract—The TGAVM schemes based on the Kovalevskaya gyroscope with both spherical electrostatic and spring suspensions are described. The differential equations of motion of the gyroscope are given, formulae for the output information on the three components of the angular velocity of MO. The formula for determining the third component includes the first and second derivatives on the coordinates of the translational movements of the gyroscope in the equatorial plane. To determine them, an algorithm is used to filter the interference of derivatives, based on the Luenberger identification device. The results of mathematical simulation by the derivation of the three components of the angular velocity, which confirmed the validity of the premises, are given. An analytical approximate solution of the problem is given for the self-centering mode of the gyroscope rotor and for the resonance mode. It is shown that in the second case the sensitivity of the device can be an order of magnitude higher than in the first. The approximate solution is confirmed by calculations of the third component of the angular velocity based on measuring only the coordinates of the translational movement of the gyroscope, without derivatives.

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1. INTRODUCTION AND FORMULATION OF THE PROBLEM

The problem is to provide a technical solution that makes it possible to measure by the Kovalevskaya single-rotor gyroscope three components of the absolute angular velocity vector of a moving object (MO). This predetermines the imposition of links in five degrees of freedom due to correction and suspension systems providing the point of suspension of the gyroscope to MO and the property of tracking its own rotation axis for one of the MO axes during all its turns practically coinciding with it. This technical solution corresponds to a system of five linearized differential equations of motion and the possibility of removing information, in known way — through the indicated correction systems on two components of angular velocity lying in the equatorial plane of the rotor. The third component is determined by measuring the coordinates of the translational motion of the rotor in the equatorial plane, determining their first and second derivatives in time and using them in the found calculation algorithm using an onboard computer. Considering that information about the angular velocity is contained in additional oscillations of the same frequency as the carrier oscillations from centrifugal forces due to the displacement of the center of mass of the rotor. It is important not only to identify the useful oscillation components, but also to find conditions when they will stand out with greater certainty. Thus, it is necessary to find alternative analytical solutions and use them to measure the influence of the parameters of the gyroscope on the sensitivity and error of solving the problem. Bearing in mind the possibility of using a contact suspension, apply a method of self-centering to eliminate the influence of

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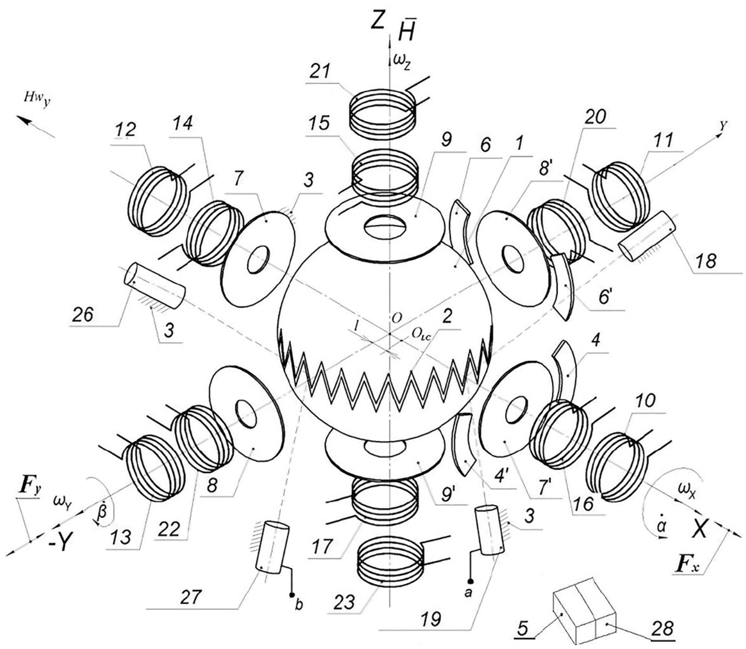

Fig. 1.

centrifugal forces on the suspension supports and analyze the effectiveness of the problem solution in this case. In addition, to produce mathematical modeling of processes in order to illustrate the effectiveness of algorithms based on differential equations. The device and some properties of a three-component gyroscopic angular velocity meter (TGAVM) based on a Kovalevskaya gyroscope (TGAVM-K) with an electrostatic suspension (ESS) are described in [1]. From the studies [1–3] some schematic design solutions, equations and algorithms are used. The gyroscope oscillation method proposed in [2] in the Kovalevskaya gyroscope is solved in a natural way due to the equatorial displacement of the center of mass. Along with the ESS-based scheme, TGAVM can be used on a mechanical suspension, for example, with a rotor in ball bearings enclosed in a gyro camera suspended in the instrument housing on spring supports similar to those used in MMG [4] and performing the same functions as the ESS. To create radial correction systems, traditional angle sensors and magnetoelectric moment sensors can be used.

2. ELEMENTS OF THEORY OF A TGAVM BASED ON THE KOVALEVSKAYA GYROSCOPE

The circuit of the sensitive element and the circuit for connecting two radial correction channels are shown in Figs. 1 and 2. Mathematical models of both constructive options with accuracy up to instrument errors are identical.

A three-component angular velocity meter based on a spherical rotor with an electrostatic suspension, shown in Fig. 1, contains: a spherical rotor 1, made entirely of lightweight conductive material, for example, beryllium. The rotor 1 is hollow, made in the form of the so-called heavy Kovalevskaya gyroscope, the outer surface of which is spherical, and the center of gravity O_{LC} is displaced relative to the center of suspension (center of the sphere "0" in the equatorial plane on the shoulder l perpendicular to the kinetic moment vector H [1, 3]. Fig. 2 is applied on it to retrieve information with the help of lasers and photodetectors mounted on housing 3.

To determine the angular velocity around the OZ axis of the instrument housing, a measuring circuit [1, 3] is inserted into the structure, containing a pair of electrically isolated measuring segment electrodes 4 and 4′, located along the OX axis (electrodes 6 and 6′ are located along the OY axis); 5 is a block of amplifiers of radial correction systems 24, 25 according to Fig. 2 (which shows the equivalent circuit of correction systems). The circuit is made as a measuring. For this purpose, a phase-sensitive rectifier with a filter is used in it, the output of which is designed to determine the angular velocity around the OX axis of the device body and connected to one input of the controller 28 (Fig. 1), 29 (Fig. 2), and the

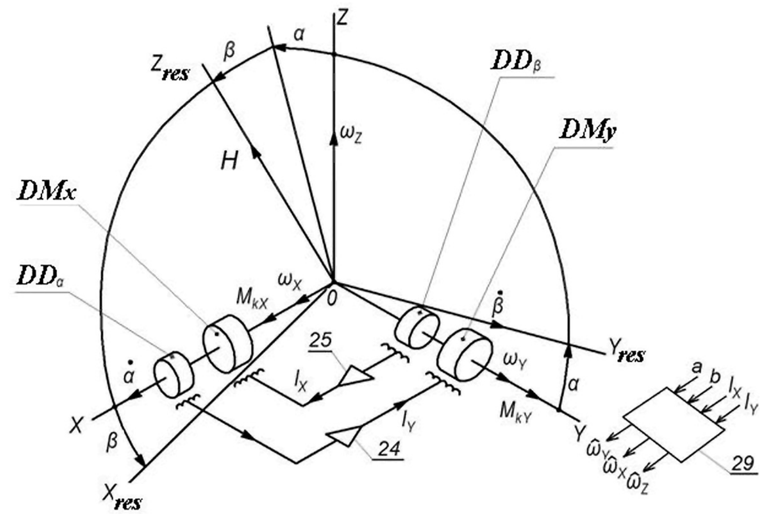

Fig. 2.

output of another PSR with filter OY is connected to the second input of the controller. Elements 7, 7′; 8, 8′; 9, 9′ are the suspension electrodes of the rotor 1 along the axes OX, OY, OZ respectively; 10, 11, 12, 13 are the windings of an asynchronous rotor drive electric motor; 14, 15, 16, 17 are winding of the torque sensor (***DMy***) around the axis *OY*; 18, 19 are a laser **L** and a photodetector **PD**, forming the angle sensor β; 20, 21, 22, 23 are the windings of the torque sensor (***DMx***) around the axis *OX*; 26, 27 are a laser **L** and a photodetector **PD**, forming an angle sensor α. Linearized equations of TGAVM angular motion are given in [5]. Axial moments of inertia of the rotor 1: $J_x = J_y = A$, $J_z = A/2$, where A is the equatorial moment of inertia. Shifts of the center of mass along the rotor axes: $x_{LC} = l$, $y_{LC} = z_{LC} = 0$.

2.1. 2.1. Differential Equations of Gyroscope Angular Motion. Algorithms for Determining the Equatorial Angular Velocity ω_x, ω_y of a MO

Differential equations of angular motion are of the form:

$$\begin{aligned} &J\ddot{\alpha} + n_\alpha\dot{\alpha} + C_\alpha\alpha + H\dot{\beta} + K_\beta\beta = -H\omega_{\mathrm{y}} + mlW_z\sin\varphi + m_\alpha,\\ &J\ddot{\beta} + n_\beta\dot{\beta} + C_\beta\beta - H\dot{\alpha} - K_\alpha\alpha = H\omega_{\mathrm{x}} - mlW_z\cos\varphi + m_\beta, \end{aligned} \tag{2.1}$$

where K_α, K_β are the transfer coefficients of the radial correction; C_α, C_β are the suspension stiffness coefficients; m_α, m_β are the moments of disturbing forces. The device uses axial radial correction. By virtue of this, we have:

$$Mky = K_\alpha\alpha = K_{DM}P_{DMy} = -H\omega_x + C_\beta\beta,$$

from where

$$\omega_x^* = -\frac{K_{DM}}{H}P_{DMy} + \frac{C_\beta\beta}{H},$$

where P_{DMy} is the effective electromagnetic power of the torque sensor (axis OY) based on an asynchronous motor in the inhibited mode, defined as the product of the alternating voltage at the output of the amplifier 24 and the current [7]. Similarly, for the *OX* axis, we have:

$$Mkx = K_\beta\beta = K_{DM}P_{DMx} = -H\omega_y + C_\alpha\alpha,$$

from where

$$\omega_y^* = -\frac{K_{DM}}{H}P_{DMx} - \frac{C_\alpha\alpha}{H}.$$

For magnetoelectric TS we have:

$$Mky = K_\alpha\alpha = K_{DMy}I_y = -H\omega_x + C_\beta\beta, \quad Mkx = K_\beta\beta = K_{DMx}I_x = -H\omega_y + C_\alpha\alpha,$$

We have:

$$\omega_x^* = -\frac{K_{DM}}{H} I_y + \frac{C_\beta \beta}{H}, \quad \omega_y^* = -\frac{K_{DM}}{H} I_x - \frac{C_\alpha \alpha}{H}. \tag{2.2}$$

Here K_{DM} is the TS transfer ratio of the asynchronous type; K_{DMx}, K_{DMy}, I_x, I_y are the TS transfer ratio and the currents of the corresponding TS of the magnetoelectric type. Thus, signals about angular velocities around the axes *OX* and *OY* are determined directly by the work of the radial correction. The main problem is to determine the third component of the angular velocity ω_z. It is solved on the basis of a system of differential equations describing the translational motion of the rotor of the Kovalevskaya gyroscope relative to the body.

2.2. Equations of Translational Motion of a Gyroscope. The Algorithm for Determining the Third Component ω_z of the Angular Velocity of the MO

Differential equations of relative motions have the form:

$$\begin{aligned} m\ddot{x} + n_x\dot{x} + C_x x &= m\Omega^2 l \cos\Omega t + 2m\omega_z\dot{y} - mW_x - 2\mathrm{m}\dot{z}\omega_\mathrm{y}, \\ m\ddot{y} + n_y\dot{y} + C_y y &= m\Omega^2 l \sin\Omega t - 2m\omega_z\dot{x} - mW_y + 2\mathrm{m}\dot{z}\omega_\mathrm{x}, \\ m\ddot{z} + n_z\dot{z} + C_z z &= -mW_z - 2m\omega_y\dot{x} - 2\mathrm{m}\dot{y}\omega_\mathrm{x}, \end{aligned} \tag{2.3}$$

where m is the mass of the rotor; n_x, n_y, n_z, C_x, C_y, C_z are damping and stiffness coefficients. In the case of a TGAVM-K with a mechanical suspension with the second derivatives ($\ddot{x}$,...) the multiplier is the mass M of the rotor with the gyro camera. The process of determining the signal about the angular velocity ω_z is explained below. When the rotor rotates with an angular velocity $\dot{\varphi} = \Omega$ due to the displacement l of the center of mass of the rotor O_{LC} relative to the center of suspension O, centripetal acceleration $-\Omega^2 l$ occurs and, accordingly, the centrifugal force $F^C = m\Omega^2 l$ (m is the mass of the rotor 1). Due to the rotation of the rotor, this force is projected on the axis of the Resal's coordinate system OX_res, OY_res, OZ_res, and each of the projections has a periodic character: $F_x^C = lm(\Omega)^2\cos\phi$, $F_y^C = lm(\Omega)^2\sin\phi$, $\phi = \Omega t$. These forces cause oscillations of the center of mass of the rotor 1 along the OX and OY axes with relative speeds $\dot{x}$ and $\dot{y}$, and since there is a moving angular velocity ω_z, corresponding accelerations and forces F_x^K and F_y^K occur: $F_x^K = 2m\dot{y}\omega_z - 2m\dot{z}\omega_y$, $F_y^K = -2m\dot{x}\omega_z + 2m\dot{z}\omega_x$. These forces are summed with the forces F_x^C and F_y^C and create additional oscillations of the rotor, in which information about ω_z is hidden. The differential equations (2.3) of the motion of the center of mass of the rotor contain the above-mentioned forces and inertial forces mW_x, mW_y of the apparent acceleration of the object. Let us turn to the equations in the dimensions of the accelerations, which are obtained by dividing the left and right parts by the mass of the rotor:

$$\begin{aligned} \ddot{x} + n_x\dot{x} + Cx &= m\Omega^2 l \cos\Omega t + 2\omega_z\dot{y} - W_x - 2\dot{z}\omega_y, \\ \ddot{y} + n_y\dot{y} + Cy &= \Omega^2 l \sin\Omega t - 2\omega_z\dot{x} - W_y + 2\dot{z}\omega_x, \\ \ddot{z} + n_z\dot{z} + Cz &= -W_z - 2\omega_y\dot{x} - 2\dot{y}\omega_x. \end{aligned} \tag{2.4}$$

Adopted to simplify the calculations: $C_x/m = C_y/m = C$, $n_x/m = n_y/m = n$. Angular speeds ω_x, ω_y, speed $\dot{z}$ are measured in the device. Assuming that apparent accelerations are measured by additional accelerometers, these variables are taken into account in one way or another, and terms with these variables are excluded from equations (2.4). In this case, the third equation describes a perturbed motion along the OZ axis, the ESS stiffness in which is large, for the ω_z calculation algorithm, its influence is insignificant, and therefore is not considered. The analysis of the solutions (2.4) revealed the periodic nature of the established motions; therefore, the algorithm for calculating ω_z given in [5] is not quite correct, since it involves dividing the numerator by a periodic value passing through zero. Therefore, we use a stable algorithm in the form of a phase detector for signals from the first two equations of system (2.4). It is obtained by solving an inverse problem, in which the coordinates x and y and their derivatives with respect to time are considered known, and the angular velocity ω_z is unknown. It is derived from (2.4) by multiplying the first equation by $\dot{y}$, the second equation by $\dot{x}$ and subtracting the results. As a result, we obtain an algorithm:

$$\omega_z = \frac{\ddot{x}\dot{y} - \ddot{y}\dot{x} + n(\dot{x}\dot{y} - \dot{y}\dot{x}) + C(x\dot{y} - y\dot{x}) - \Omega^2 l(\cos\Omega t\dot{y} - \sin\Omega t\dot{x})}{2(\dot{y}^2 + \dot{x}^2)} \tag{2.5}$$

The formula is an algorithm for the problem solution at all stages of the instrument operation, except the initial one, when the translational velocities are very small, at which the error is large. To eliminate the division operation by a value close to zero, the following algorithm is used:

$$\omega_z = \frac{\ddot{x}\dot{y} - \ddot{y}\dot{x} + C(x\dot{y} - y\dot{x}) - \Omega^2 l(\cos\Omega t\,\dot{y} - \sin\Omega t\,\dot{x})}{2(\dot{y}^2 + \dot{x}^2 + e^{-a\cdot t})} \tag{2.6}$$

The exponent introduced to prevent the solution divergence at the initial stage of the transition process quickly decreases when the speed of the translational movements of the rotor is very small. The constant a is determined based on the time of the transition process.

Further, differential equations in normal form are used, which for the initial system (2.4) take the form

$$\begin{aligned}
&\dot{x}_0 = x_1,\\
&\dot{x}_1 = -nx_1 - Cx_0 + \Omega^2 l\cos\Omega t + 2\omega_z y_1 + x_2,\\
&\dot{x}_2 = 0,\\
&\dot{y}_0 = y_1,\\
&\dot{y}_1 = -ny_1 - Cy_0 + \Omega^2 l\sin\Omega t - 2\omega_z x_1 + y_2,\\
&\dot{y}_2 = 0.
\end{aligned} \tag{2.7}$$

Here we have the following notation: $\dot{x}_0 = \dot{x}$, $\dot{y}_0 = \dot{y}$, the equations also contain the terms denoted as $W_x = x_2$, $W_y = y_2$. They are introduced to extend the order of the system and serve to obtain second derivatives of the coordinates. Differential equations of the Luenberger observer identification device (OID) [5] for the initial system (2.7) are:

$$\begin{aligned}
&\dot{x}_0^* = x_1^* + K_1(x_0 - x_0^*),\\
&\dot{x}_1^* = -nx_1^* - Cx_0^* + \Omega^2 l\cos\Omega t + x_2^* + K_2(x_0 - x_0^*),\\
&\dot{x}_2^* = K_3(x_0 - x_0^*),\\
&\dot{y}_0^* = y_1^* + K_1(y_0 - y_0^*),\\
&\dot{y}_1^* = -ny_1^* - Cy_0^* + \Omega^2 l\sin\Omega t + y_2^* + K_2(y_0 - y_0^*),\\
&\dot{y}_2^* = K_3(y_0 - y_0^*),
\end{aligned} \tag{2.8}$$

where $\hat{x}_0$, ..., $\hat{y}_0$ are estimates of the coordinates of the state spatial vector; $K_i (i = 1, 2, 3)$ are OID transmission coefficients; input variables are the x_0 and y_0 coordinates. To determine the coefficients of the observing device, relations are applied based on the standard Butterworth form for the characteristic polynomial of system (2.8):

$$\frac{\tau^3}{8}s^3 + \frac{\tau^2}{2}s^2 + \tau s + 1, \quad K_1 = \frac{4}{\tau} - n, \quad K_2 = \frac{8}{\tau^2} - K_1 n - C, \quad K_3 = \frac{8}{\tau^3}, \quad \tau = \frac{2}{\omega_C}, \tag{2.9}$$

where τ and ω_C are the time constant and cutoff frequency of the OID; s is the differentiation symbol. Algorithm (2.5) for the accepted assumptions is exact, formula (2.6) with a small a quickly converges to it in calculations.

2.3. Mathematical Modeling of the Device Exploitation

The results of the simulation of operation of the TGAVM-K based on the Kovalevskaya gyroscope, produced by equations (2.1), (2.7), (2.8) and algorithm (2.6), which was performed in the MathCad environment are discussed below. Instrument parameters and OID: $m = 50g$; $n_x = n_y = n_z = 10\,\mathrm{cN\cdot s/cm}$; $l = 10^{-4}$ cm; $\Omega = 400$ 1/s; $Cx = Cy = Cz = 500$ cN/cm; $H = 25\,\mathrm{cN\cdot cm\cdot s}$; $\omega_x = 0.6$ rad/s; $\omega_y = 0.4$ rad/s; $\omega_z = 0.6$ rad/s; $\tau = 10^{-3}$ s; $K_1 = 3990$ 1/s; $K_2 = 8\times10^6\ 1/\mathrm{s}^2$; $K_3 = 8\times10^9\ 1/\mathrm{s}^3$; $K_\alpha = K_\beta = 10^4\,\mathrm{cN\cdot cm/rad}$; $Kdm = 1\,\mathrm{cN\cdot cm/mA}$.

The simulation results are presented in figure 3 in the form of graphs ω_x, ω_y. (rad/s) depending on time t (s) for given values of ω_x, ω_y. Fig. 4 *a, b* shows the simulation results for given laws of motion along ω_z.

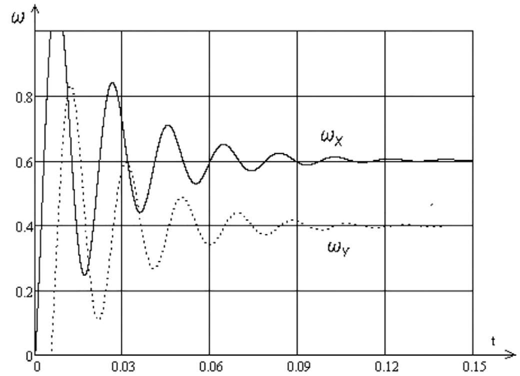

Fig. 3.

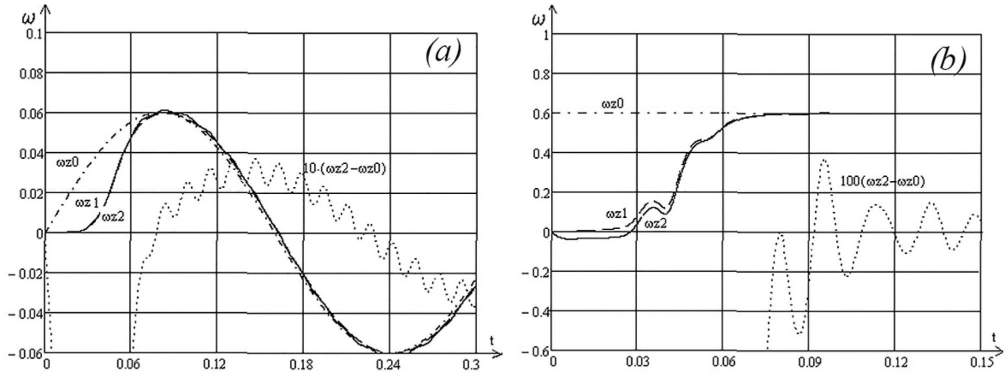

Fig. 4.

The graphs of angular velocities ω_x, ω_y reflect the processes of TGAVM functioning with radial correction systems based on TS of the magnetoelectric type according to equations (2.1) and relations (2.2). The graphs for harmonic and step changes given by ω_{z0}, defined for ω_{z1} by algorithm (2.5) and for ω_{z2} by the algorithm (2.6) using equations (2.7) and Lewinberger's OID (equations (2.8)) by mathematical simulation of the instrument, are shown in Fig. 4 *a, b*.

It also presents errors in the form of the difference ω_{z2} and ω_{z0} with an increased by an order of magnitude and two scale, dotted line. They indicate that the angular velocity MO around the axis *OZ* is determined from the measured translational oscillations of the rotor along the axes *OX* and *OY*, followed by their differentiation. On Fig. 4 *a, b* the simulation results for algorithms (2.7), (2.8) and (2.6) are presented as a solid line. In the same figure, for comparison, the result from [3] is shown for ordinary differentiation algorithms as a dashed line. Dash-dotted lines denote the specified angular velocities MO. The graphs confirm the effectiveness of the proposed algorithms.

3. AN APPROXIMATE ANALYTICAL SOLUTION OF THE PROBLEM OF DETERMINING THE ANGULAR VELOCITY ω_z

For a simplified system with the same coefficients for the two coordinates of the rotor motion, we have a system of differential equations:

$$\begin{aligned} \ddot{x} + n\dot{x} + Cx &= \Omega^2 l \cos\Omega t + 2\omega_z \dot{y}, \\ \ddot{y} + n\dot{y} + Cy &= \Omega^2 l \sin\Omega t - 2\omega_z \dot{x}. \end{aligned} \tag{3.1}$$

The solution is sought by the method of successive approximations in the form of rows:

$$x = x^0 + x^1 + x^2 + \cdots, \quad y = y^0 + y^1 + y^2 + \cdots \tag{3.2}$$

Zero Approximation Equations

$$\ddot{x}^0 + n\dot{x}^0 + Cx^0 = \Omega^2 l \cos\Omega t,$$
$$\ddot{y}^0 + n\dot{y}^0 + Cy^0 = \Omega^2 l \sin\Omega t. \tag{3.3}$$

First approximation equations

$$\ddot{x}^1 + n\dot{x}^1 + Cx^1 = \Omega^2 l \cos\Omega t + 2\omega_z \dot{y}^0,$$
$$\ddot{y}^1 + n\dot{y}^1 + Cy^1 = \Omega^2 l \sin\Omega t - 2\omega_z \dot{x}^0. \tag{3.4}$$

3.1. Solution for Self-Centering Mode ($\Omega^2 \gg C$)

For steady motion, applying the principle of self-centering, practically removing the radial load on the supports for TGAVM with a contact hanger, we obtained a solution for two members of the expansion in series (3.2):

$$x = -l\cos\Omega t + \frac{2l\omega_z}{\Omega}\cos\Omega t, \tag{3.5}$$

$$y = -l\sin\Omega t + \frac{2l\omega_z}{\Omega}\sin\Omega t. \tag{3.6}$$

The ratio of the amplitudes of the carriers and information oscillations is $\Omega/2\omega_z$.

For values of $\Omega = 100\,1/\mathrm{s}$ and $\omega_z = 0.5\,\mathrm{rad/s}$, we have a ratio of amplitudes equal to 100. Thus, the amplitude of the carrier oscillations is 100 times greater than the amplitude of the information, that is, useful oscillations. The solution of the problem considered in this subsection is applicable when accurately measuring the amplitudes of oscillations along two axes and subtracting from them the amplitudes of the original oscillations, when $\omega_z = 0$.

3.2. Solution for Resonant Mode ($\Omega^2 = C$)

For steady motion, the solution is:

$$x = \frac{l\Omega}{n}\sin\Omega t - \frac{2l\Omega\omega_z}{n^2}\cos\Omega t, \tag{3.7}$$

$$y = -\frac{l\Omega}{n}\sin\Omega t - \frac{2l\Omega\omega_z}{n^2}\sin\Omega t. \tag{3.8}$$

Multiplying (3.8) by $\sin\Omega t$, and (3.7) by $\cos\Omega t$, we obtain the formulae for the detected variables xs and ys, which are used below for calculations. From the results of multiplication, the average value was obtained:

$$x^* = y^* = -\frac{l\Omega\omega_z}{n^2}. \tag{3.9}$$

The gain in the resonant mode is higher than with self-centering, but the error is greater because of the instability of the damping coefficient n. The ratio of the amplitudes of the carrier oscillations, due to centrifugal forces, to the amplitudes useful, due to the angular velocity ω_z, is equal to $n/2\omega_z$. With $n = n_i/m = 200$; ω_z= 1 rad/s, this ratio is 100. With $n = 20$, the amplitude ratio is 10. Thus, in the resonant mode, the amplitudes of the useful, that is, information oscillations can be made an order of magnitude higher than in the self-centering mode. However, the damping coefficient n due to greater instability than the shoulder l, will lead to a greater error in determining ω_z than in the first case. For the contact suspension, the load on the supports will also be greater (for an electrostatic suspension this is not observed).

3.3. Formulae for the Approximate Determination of the Angular Velocity ω_z

We write the formulae for the detected signals:

$$xs = x\cos\Omega t, \quad ys = y\sin\Omega t. \tag{3.10}$$

For the self-centering mode, taking into account formulae (3.5), (3.6), we have:

$$xs = -l\Big(\frac{1}{2} - \frac{\omega_z}{\Omega}\Big) - l\Big(\frac{1}{2} - \frac{\omega_z}{\Omega}\Big)\cos 2\Omega t,$$
$$ys = -l\Big(\frac{1}{2} - \frac{\omega_z}{\Omega}\Big) + l\Big(\frac{1}{2} - \frac{\omega_z}{\Omega}\Big)\cos 2\Omega t.$$

Added up, we got:

$$xs + ys = -l + 2l\frac{\omega_z}{\Omega}. \tag{3.11}$$

To estimate the angular velocity, we have the following algorithm:

$$\omega_z = (xs + ys + l)\frac{\Omega}{2l}. \tag{3.12}$$

We derive equations for the resonant mode. To do this, we expand the expressions for xs and ys based on the formulas (3.7), (3.8) and, multiplying (3.7) by $\cos\Omega t$, and (3.8) by $\sin\Omega t$, obtained:

$$\begin{aligned} x^1 s &= l\frac{\Omega}{2n}\sin 2\Omega t - l\frac{\Omega\omega_z}{2n^2} - l\frac{\Omega\omega_z}{2n^2}\cos 2\Omega t,\\ y^1 s &= -l\frac{\Omega}{2n}\sin 2\Omega t - l\frac{\Omega\omega_z}{2n^2} + l\frac{\Omega\omega_z}{2n^2}\cos 2\Omega t. \end{aligned} \tag{3.13}$$

Add up the expressions in (3.13), we have:

$$x^1 s + y^1 s = -2l\frac{\Omega\omega_z}{2n^2}.$$

To estimate the angular velocity, the following expression was obtained:

$$\omega_z = \frac{x^1 s + y^1 s}{2l\Omega}. \tag{3.14}$$

When accounting for TGAVM-K errors, errors will appear in the output signal.

So, the algorithms (3.12) and (3.14) of the approximate calculation of the angular velocity estimates ω_z of TGAVM-K for self-centering and resonant modes, respectively, are derived. The second mode provides greater sensitivity, but also slightly larger error values. These algorithms are much simpler than algorithms (2.5) or (2.6). Most importantly, when using them, it is not necessary to calculate the first and second derivatives of the coordinates of the translational oscillatory movements of the gyroscope along two equatorial axes.

4. Conclusion

1. A technical solution for TGAVM based on the Kovalevskaya gyroscope is described, in which the center of suspension is provided either by a non-contact suspension or by a contact mechanical suspension [10]. Two channels of radial inter-axial correction were used, providing with high accuracy the alignment of the axis of proper rotation with one of the axes of the moving object during all its rotations (in modeling, the error was 5 arc min). This indicates the limitations of the two angular degrees of freedom of the Kovalevskaya gyroscope and the small observable (measured) movements along the x and y axes. An electric drive similar to the gyro instruments was used, which in the case of ESS after the acceleration of the rotor is turned off, and its rotation occurs by inertia due to the high vacuum in the rotor cavity.

2. By virtue of the described technical solution, the TGAVM-K mathematical model is a system of five differential linearized second-order equations with variable coefficients describing small angular and translational movements relative to the instrument body, containing in the right-hand parts members with portable angular velocities and apparent accelerations of the MO. This fundamentally distinguishes the mathematical model of the TGAVM-K from the mathematical model of the classical gyroscope of S. V. Kovalevskaya.

3. The formula (2.5) is derived, as well as the algorithm (2.6) for calculating (using the on-board computer) the third component around the axis of its own rotation of the gyroscope, which almost coincides with the oz axis of the MO. This algorithm contains the first and second order derivatives on time from the two coordinates of the translational relative movements of the center of suspension of the rotor relative to the body of the device.

4. For the development of these derivatives in the on-board computer provides for the use of algorithms for observing the Lewinberger identification device, in which interference filtering is performed. This is done by proper selection of the cutoff frequency OID and the coefficients of the characteristic Butterworth polynomial.

5. Approximate analytical (in the form of finite relations) solutions for oscillatory motions of TGAVM-K in x and y coordinates are given for the conditions of self-centering and resonance, indicating a greater sensitivity of the second variant compared to the first to the action of angular velocity The formulas indicate that the useful oscillations caused by the angular velocity ω_z have the same frequencies as the carrier oscillations from the centrifugal forces due to the displacement of the center of mass. This fact predetermines the difficulty of extracting information from the total oscillations about the third component of the angular velocity of the MO. The detected signals about the indicated oscillations contain coefficients, with a change in which the above-mentioned sensitivity can be adjusted. They also indicate a greater stability of the transmission coefficient in the self-centering method.

6. Mathematical modeling of TGAVM-K operation was performed using full equations and algorithm (2.6), which confirmed the theoretical background on the possibility of creating a three-component angular velocity meter based on a Kovalevskaya single-rotor gyro.

7. The algorithms (3.12) and (3.14) of the approximate calculation of the angular velocity estimates ω_z of TGAVM-K for self-centering and resonant modes are derived. These algorithms are much simpler than algorithms (2.5) or (2.6). The main thing is that when using them it is not necessary to calculate the first and second derivatives of the coordinates of the translational movements of the gyroscope along two equatorial axes. A calculation based on the formulas of these analytical solutions demonstrated a simpler method for solving the problem of determining the third component of the angular velocity ω_z of the MO.

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