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Asymptotic Analysis of the Equilibrium Equation of a Fluid-Saturated Porous Medium by the Homogenization Method

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Abstract—The homogenization of static elasticity equations describing the stress strain state of fluid-saturated porous medium is considered. In this paper, the homogenization method is used to determine the pore pressure transfer tensor, which (a coefficient in the isotropic case) is an important parameter influencing the stress-strain state of fluid-saturated rocks. It shows what a part of the pressure in the fluid is "active" in the formation of macroscopic strains.
The pore pressure transfer tensor is calculated for model and real geological specimens. The dependence of this tensor on the porosity, pore shape, and Poisson ratio is investigated. The use of the computational technique for determining the effective properties of rocks shows that it is practically important in the engineering geology.

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1. INTRODUCTION

The effective moduli of elasticity of soils and rocks are determined by nature experiments or experiments with core specimens and by geographic methods. It is also expedient to use the computational experiments based on solving local problems in the representative volume elements [1, 2]. Such a technique can be used to estimate the effective properties of rocks if the structure of the pore space and the elastic properties of the matrix components are known. As a rule, thin sections are used to study the mineral composition of rocks and the pore space structure, and the properties of the mineral composing the specimens under study are given in appropriate handbooks and cadasters [3–5]. In any case, the computational technique can be used for the preliminary rapid analysis.

The works where the ideas of the homogenization method were used to calculate the stress strain state (SSS) of rocks or soil massifs have been known for several dozen years [6–9]. But these studies deal with the homogenized elastic properties, for example, of stratified soils [6, 7]. In [1, 10], the homogenization method was applied to a "dry" porous medium, which results in developing a technique for calculating the effective models of elasticity. In this paper, we consider the case of fluid-saturated medium, which additionally implies a technique for calculations the pore pressure transfer tensor.

There also exist many approximate formulas [11] based on the Eshelby solution [12], which permit calculating the effective properties.

The scalar coefficient α of transfer of the pore pressure on the matrix solid material was introduce din [13–17], and therefore, in the literature, it is also called the Biot coefficient (parameter) [18–20]. For a statically homogeneous and isotropic porous fluid-saturated soil, the scalar coefficient α enters the expression for the effective stress $\langle \sigma_{ij} \rangle^{\text{eff}}$:

$$\langle \sigma_{ij} \rangle^{\text{eff}} = \langle \sigma_{ij}^{\Pi} \rangle - \alpha \delta_{ij} \langle p \rangle, \tag{1.1}$$

where $\langle \sigma_{ij}^{\Pi} \rangle$ are the homogenized combined stresses, $\langle \sigma_{ij} \rangle^{\text{eff}}$ are the homogenized effective stresses in the soil solid phase which are transferred through the contacts between the rock grains, and $\langle p \rangle$ is the

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homogenized pressure of the liquid. Therefore, the pore pressure transfer coefficient shows what part of the pore pressure must be taken into account in calculations of the effective stresses. The following formula for calculating the coefficient α is given in [14–17]:

$$\alpha = 1 - \frac{\beta_s}{\beta_{\text{eff}}},$$

where β_s is the compressibility of the matrix solid material and β_{eff} is the effective compressibility of the soil; a rigorous derivation of this formula is given in [15, 21, 22], and an experimental verification of the expression for α can be found in [16]. Thus, for isotropic rocks, α is a scalar dimensionless parameter varying from 0 to 1 and depending on the soil properties (mainly on the porosity and the shape of the pores) and the effective stresses [21, 22]. If the effective compressibility of the rock significantly exceeds the compressibility of the matrix solid material ($\beta_{\text{eff}} \gg \beta_s$), then one can assume that the complete value of the seam pressure influences the rock strain ($\alpha \approx 1$). For example, such a situation is observed when studying the stress state of highly porous loose sands and weakly loose sandstones lying near the Earth surface under the conditions of small pressures. For poor-porous strongly compacted rocks with extremely many closed pores, the effective compressibility of the rock turns out to be close in value to the compressibility of the matrix solid material of the soil ($\beta_{\text{eff}} \approx \beta_s$). In this case, a variation in the pore pressure does not lead to strain origination in the rock, and hence it is expedient to set $\alpha \approx 0$ [21]. The high compressibility β_{eff} is also observed in poor-porous rocks such as granite, because they contain narrow cracks with large surface area, while the rocks with the same low content of porosity but with circular pores exhibit a weak compressibility β_{eff} [22]. The studies [23] showed that, for sandstones, the value of α varies from 0.60 to 0.85 as their open porosity varies from several percents to 26%. As the effective strain in the soil increases, its effective compressibility β_{eff} decreases, and hence the coefficient α also decreases.

The fact that the real value of the pore pressure transfer coefficient is taken into account in formula (1.1) allows one correctly to determine the effective stresses, and this is important in geotechnical calculations. Therefore, many researchers use the coefficient α in the models describing the process of deformation of a loaded fluid-saturated soil [13–26].

But it is more difficult to determine the coefficient α than to determine the effective moduli of elasticity. Different authors have currently proposed some experimental methods for determining this coefficient. For example, a method for determining the coefficient α on a special high pressure installation was developed in [16].

In the case of anisotropic media, the formula for calculating the effective stresses contains the pore pressure transfer tensor α_{ij} [19, 20, 24–26]:

$$\langle \sigma_{ij} \rangle^{\text{eff}} = \langle \sigma_{ij}^{\Pi} \rangle - \alpha_{ij} \langle p \rangle.$$

The following formula for determined the tensor α_{ij} for a medium with structure anisotropy, i.e., for a medium whose matrix consist of a homogeneous isotropic solid material, was derived in [25, 20]:

$$\alpha_{ij} = \delta_{ij} - C_{ijkl} S^s_{klmn},$$

where C_{ijkl} is the effective tensor of elasticity moduli of the rock with mutually connected pores and S^s_{klmn} is the tensor of compliance of the matrix solid material. In this case, it is assumed that a hydrostatic state originates in the matrix solid material if it experiences the action of the pressure p. In the case of transversal isotropy and orthotropy, the author of [25] obtained simper formulas for calculating the tensor α_{ij}, and these formulas contain parameters which can directly be determined in experiments. In [20], the final formulas for α_{ij} for the types of symmetry (cubic, hexagonal, and orthorhombic) encountered most often were derived and it was shown that, in the case of isotropic medium, the expression for α_{ij} takes a familiar form just as in [14–17]. In [26], the formula for determining the pore pressure transfer tensor was derived in the case of general anisotropy, and the material tensors contained in the formula for calculating α_{ij} are interpreted in terms of the parameters which can directly be measured in well-known experiments. The cases of isotropy and transversal isotropy are considered as illustrative examples. The author thus draw emphasize the importance of experimental methods for determining the tensor α_{ij} in special cases of anisotropy.

In the present paper, we propose a computational method for determining the tensor α_{ij} which differs from the methods considered above. It is base on solving local problems in the representative volume element and is similar to the method for calculating the effective elastic properties. The difference from the above-cited studies is in the following. In [25, 20, 26], the tensor contained in the expression for strains expressed in terms of stresses was first determined, and α_{ij} was then obtained by inverting this relation, and only in the case of open pores. In the present paper, we initially determine the tensor α_{ij} contained in the expression for the stresses expressed in terms of strains. The situation is the same as in the case of effective moduli of elasticity and compliance. The method represented below can also be used in the cases where the matrix solid material is inhomogeneous and the pores are open or closed.

2. ASYMPTOTIC ANALYSIS

The proposed mathematically rigorous method for determining the effective moduli of elasticity and the pore pressure transfer tensor is based on the asymptotic homogenization of the equilibrium equation for the inhomogeneous elastic porous medium:

$$[C_{ijkl}u_{k,l}]_{,j} + X_i = 0, \quad \mathbf{x} \in V \tag{2.1}$$

with boundary conditions on the pore surface Σ_{pore}:

$$C_{ijkl}u_{k,l}n_j = -pn_i. \tag{2.2}$$

Here we assume that p is a known pore pressure.

We standardly introduce fast coordinates ξ_i as

$$\xi_i = \frac{x_i}{\varepsilon}, \quad \varepsilon = \frac{l}{L} \ll 1,$$

where x_i are the slow coordinates, l is the characteristic dimension of the representative volume element (RVE) of the porous medium, and L is the characteristic global dimension of the entire porous medium. For the periodic medium, the representation volume element is the cell of periodicity [2]. In Eqs. (2.1) and (2.2), the tensor of moduli of elasticity and the normal $\mathbf{n}$ depend on the fact coordinates ξ_i, and the pressure p, on x_i. The point is that the liquid pressure p has the asymptotic representation [27]

$$p = p_0(\mathbf{x}) + \varepsilon p_1(\mathbf{x}, \boldsymbol{\xi}) + \cdots . \tag{2.3}$$

To obtain the first terms of the asymptotic solution of problem (2.1)–(2.2), it suffices to determine the first term which we further denote by $p(\mathbf{x})$. We seek the solution of problem (2.1)–(2.2) as the asymptotic series

$$u_k(\mathbf{x}, \boldsymbol{\xi}) = v_k(\mathbf{x}) + \varepsilon N_{kpq}(\boldsymbol{\xi})v_{p,q}(\mathbf{x}) + \cdots + \varepsilon M_k(\boldsymbol{\xi})p(\mathbf{x}) + \cdots , \tag{2.4}$$

where $N_{kpq_1\ldots q_m}(\boldsymbol{\xi})$ and $M_{kq_1\ldots q_m}(\boldsymbol{\xi})$ are local functions of fast coordinates.

To obtain the effective moduli of elasticity $\mathbf{C}$ and the tensor $\boldsymbol{\alpha}$, it suffices to consider the first terms in (2.4), and precisely these terms are written in (2.4). The boundary conditions on ∂V are not considered in this case.

By linearity, it is natural to represent the solution (2.4) of Eqs. (2.1) with conditions (2.2) as the sum of two solutions:

$$u_k = u_k^{(1)} + u_k^{(2)}, \tag{2.5}$$

$$\begin{gathered} [C_{ijkl}(\boldsymbol{\xi})u_{k,l}^{(1)}]_{,j} + X_i = 0, \quad \boldsymbol{\xi} \in V, \\ C_{ijkl}(\boldsymbol{\xi})u_{k,l}^{(1)}n_j(\boldsymbol{\xi}) = 0, \quad \boldsymbol{\xi} \in \Sigma_{\text{pore}}, \end{gathered} \tag{2.6}$$

$$[C_{ijkl}(\boldsymbol{\xi})u_{k,l}^{(2)}]_{,j} = 0, \quad \boldsymbol{\xi} \in V, \tag{2.7}$$

$$C_{ijkl}(\boldsymbol{\xi})u_{k,l}^{(2)}n_j(\boldsymbol{\xi}) = -p(\mathbf{x})n_i(\boldsymbol{\xi}), \quad \boldsymbol{\xi} \in \Sigma_{\text{pore}}. \tag{2.8}$$

Here we assumed that the modulus of bulk compression of water is significantly less than the modulus of bulk compression of the soil solid material, and hence we assume that there is no fluid in the pores in problem (2.5)–(2.6). We note that the bulk compressibility of the fluid can be taken into account in problem (2.5). In this case, a pore is replaced by a material with the modulus of bulk compression of the fluid and a small shear modulus. The latter is required to ensure the elasticity of the problem. The boundary condition (2.6) is not posed in this case.

The asymptotic solution (2.4) is divided into two solutions:

$$u_k^{(1)}(\mathbf{x}, \boldsymbol{\xi}) = v_k(\mathbf{x}) + \varepsilon N_{kpq}(\boldsymbol{\xi}) v_{p,q}(\mathbf{x}) + \cdots\,, \tag{2.9}$$

$$u_k^{(2)}(\mathbf{x}, \boldsymbol{\xi}) = \varepsilon M_k(\boldsymbol{\xi}) p(\mathbf{x}) + \cdots\,. \tag{2.10}$$

Substituting (2.9) into (2.5)–(2.6) and (2.10) into (2.7)–(2.8), we obtain the following results. In the RVE, the functions N_{kpq} and M_k satisfy the equations and the boundary conditions

$$[C_{ijkl} N_{kpq,l} + C_{ijpq}]_{,j} = 0, \quad \boldsymbol{\xi} \in V_{\mathrm{RVE}}, \tag{2.11}$$

$$[C_{ijkl} N_{kpq,l} + C_{ijpq}] n_j = 0, \quad \boldsymbol{\xi} \in \Sigma_{\mathrm{pore}}, \tag{2.12}$$

$$[C_{ijkl} M_{k,l} + C_{ijpq}]_{,j} = 0, \quad \boldsymbol{\xi} \in V_{\mathrm{RVE}}, \tag{2.13}$$

$$C_{ijkl} M_{k,l} n_j = (-n_i), \quad \boldsymbol{\xi} \in \Sigma_{\mathrm{pore}}. \tag{2.14}$$

The boundary conditions on the boundary Σ_{RVE} have different form depending on the medium in question, i.e., periodic or nonperiodic.

If the porous medium is nonperiodic, then the following condition can be posed on the RVE boundary:

$$N_{kpq} = 0, \quad M_k = 0, \quad \boldsymbol{\xi} \in \Sigma_{\mathrm{RVE}}. \tag{2.15}$$

In this case, there arise a boundary layer near the boundary Σ_{RVE}, but this boundary layer does not affect the values of the averages over the RVE within the accepted accuracy. This property simply follows from the definition of RVE.

But if the RVE is the cell of periodicity, then, on its boundary Σ_{RVE}, the following periodicity conditions for the functions N_{kpq} and M_k are posed:

$$N_{kpq}\big|_{\xi_\alpha = l_\alpha/2} = N_{kpq}\big|_{\xi_\alpha = -l_\alpha/2}, \quad -\frac{l_\alpha}{2} \le \xi_\alpha \le \frac{l_\alpha}{2}, \quad \alpha = 1, 2, 3, \tag{2.16}$$

$$(C_{ijkl} N_{kpq,l} + C_{ijpq}) n_j\big|_{\xi_\alpha = l_\alpha/2} = -(C_{ijkl} N_{kpq,l} + C_{ijpq}) n_j\big|_{\xi_\alpha = -l_\alpha/2}, \tag{2.17}$$

$$M_k\big|_{\xi_\alpha = -l_\alpha/2} = M_k\big|_{\xi_\alpha = l_\alpha/2}, \tag{2.18}$$

$$C_{ijkl} M_{k,l} n_j\big|_{\xi_\alpha = -l_\alpha/2} = -C_{ijkl} M_{k,l} n_j\big|_{\xi_\alpha = l_\alpha/2}. \tag{2.19}$$

The following additional conditions are required for local problems with conditions (2.16)–(2.19):

$$\langle N_{kpq} \rangle = 0, \quad \langle M_k \rangle = 0 \tag{2.20}$$

(the angular brackets denote the value averaged over the RVE). We note that, to calculate the effective properties, the periodic conditions are also often posed on the RVE boundary in the case of nonperiodic structure of the medium.

The mean stress has the form

$$\langle \sigma_{ij}^{\Pi} \rangle = C_{ijkl}^{\mathrm{eff}} v_{k,l} + \langle C_{ijkl} M_{k,l} \rangle p,$$

where the effective moduli of elasticity become

$$C_{ijpq}^{\mathrm{eff}} = \langle C_{ijkl} N_{kpq,l} + C_{ijpq} \rangle.$$

Thus, an asymptotic analysis implies the pore pressure transfer tensor α_{ij}:

$$\alpha_{ij} = -\langle C_{ijkl} M_{k,l} \rangle.$$

In this case, the homogenized equation of equilibrium has the form

$$C_{ijkl}^{\mathrm{eff}} v_{k,lj} + X_i = \alpha_{ij} p_{,j}, \tag{2.21}$$

where C_{ijkl}^{eff} and α_{ij} are calculated by solving local problems (2.11)–(2.14) in the RVE with conditions (2.15) or (2.16)–(2.20). It is assumed that these problems must be solved by the finite element method.

Thus, the derivation of the equilibrium equation (3.32) (the second equation of the Biot coupled model of poroelasticity [24]) completes the derivation of this model on the basis of the asymptotic approach. The first equation of the Biot model (equation of filtration) was obtained asymptotically in [1, 27], where the homogenization of the viscous liquid flow in a porous medium was given, which leads to the Darcy law and actually to the filtration equation.

Solving the local problems which is necessary to calculating the effective properties, one can simultaneously estimate the local distribution of stresses by the formula

$$\sigma_{ij} = (C_{ijkl}N_{kpq,l} + C_{ijpq})v_{p,q} + C_{ijkl}M_{k,l}p.$$

In this case, there is no restriction on the inhomogeneity of matrix solid material of the porous medium and on the presence of closed pores containing liquid or gas under the (locally distributed) pressure or not.

3. PORES UNIFORMLY ORIENTED IN SPACE

One of possible techniques for calculating the effective tensor of pore pressure transfer for media with arbitrarily oriented elongated pores is that the homogenization process consists of two stages [29]. At the first stage, the material with parallel pores is considered. In this case, the effective tensor is transversally isotropic (axis 1 is directed along the axis of elongation of the holes)

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & 0 \\ \alpha_{12} & \alpha_{22} & 0 \\ 0 & 0 & \alpha_{22} \end{bmatrix}.$$

These coefficients are calculated by solving the problem in the representative volume element (cell of periodicity in the case of periodic media).

The spatial homogenization permits obtaining the final formulas. The homogenization procedure corresponds to the process where the parallel fibers acquire all possible directions in the space with respect to a given coordinate system. Under the assumption that the holes are uniformly distributed over the directions in the space, to obtain the final formulas, it is necessary to calculate the integrals

$$\frac{1}{4\pi}\int\limits_0^{2\pi}\int\limits_0^{p} i\alpha'_{IJ}(\varphi,\theta)\sin\theta\, d\theta\, d\varphi,$$

where the components $\alpha'_{IJ}(\varphi,\theta)$ are the components of the pore pressure transfer tensor in the case of parallel fibers directed along the axis $1'$ of the coordinate system with primes, which is given by two Euler angles φ and θ with respect to the basic (without primes) coordinate system. The coordinates α'_{IJ} are transformed by the law

$$\begin{aligned}
&\alpha'_{IJ}(\varphi,\theta) = A_{Ii}A_{Jj}\alpha_{ij},\\
&A_{11} = \cos\theta, \quad A_{12} = \sin\theta\sin\varphi, \quad A_{13} = -\cos\varphi\sin\theta,\\
&A_{21} = 0, \quad A_{22} = \cos\varphi, \quad A_{23} = \sin\varphi,\\
&A_{31} = \sin\theta, \quad A_{32} = -\sin\varphi\cos\theta, \quad A_{33} = \cos\varphi\cos\theta.
\end{aligned}$$

After the required calculations, we obtain the following formulas for the effective tensor of the pore pressure transfer

$$\begin{bmatrix} \frac{1}{3}\alpha_{11} + \frac{2}{3}\alpha_{22} & 0 & 0 \\ 0 & \frac{1}{3}\alpha_{11} + \frac{2}{3}\alpha_{22} & 0 \\ 0 & 0 & \frac{1}{3}\alpha_{11} + \frac{2}{3}\alpha_{22} \end{bmatrix}.$$

As expected, the obtained tensor is isotropic, and we can say that the coefficient of the pore pressure transfer α is equal to the arithmetic mean of the components of the transversal tensor.

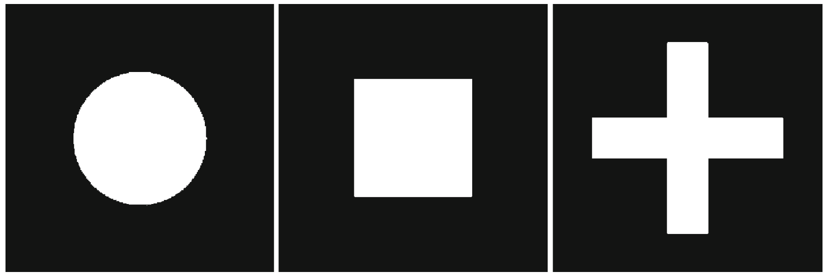

Fig. 1.

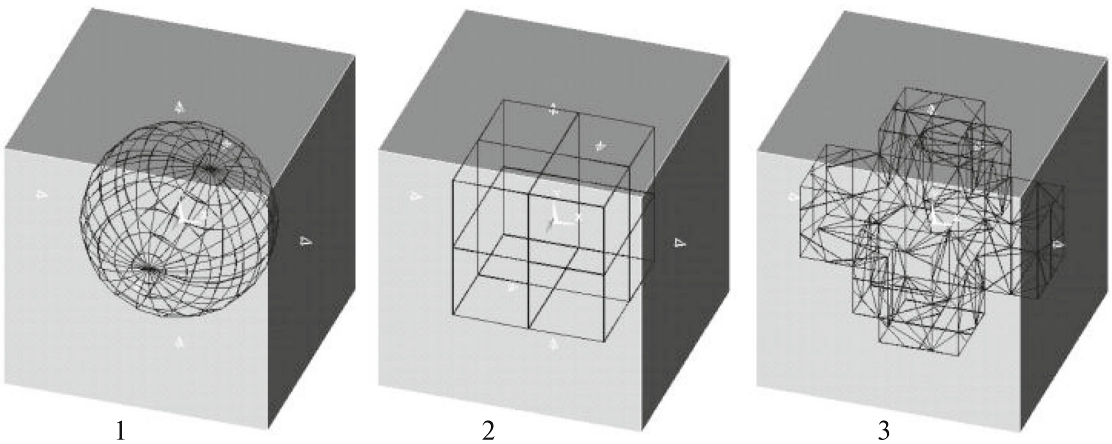

Fig. 2.

4. ANALYSIS OF THE RESULTS AND EXAMPLES

The above-described technique based on solving some local problems was applied to calculate the effective elastic moduli and the pore pressure transfer tensor for some model and geological specimens to study their dependence on different parameters.

In the case of model structures, the medium is assumed to be periodic and the local problems were solved on the cell of periodicity. It was first investigate how the coefficient of the pore pressure transfer α depends on the pore shape. The pore with symmetry such that $\alpha_{11} = \alpha_{22} = \alpha_{33} = \alpha$ were considered, and two-dimensional (Fig. 1) and three-dimensional (Fig. 3) specimens with the same porosity $n = 18.9\%$ (Poisson's ratio of the matrix solid material $\nu = 0.3$) were studied. The point is that the structure of geological materials is more often known in the form of two-dimensional rather than three-dimensional images. The following values were calculated for the coefficient α for the specimens shown in Fig. 1: (1) 0.4495, (2) 0.4940, (3) 0.6649; and in Fig. 2, (1) 0.3662; (2) 0.4052, (3) 0.4435. One can see that the specimens with a cross pore have the greatest value of α, and the specimens with a circular pore have the least value. These calculations allows us to conclude that the closer the pore shape to the circular shape, the less the coefficient α. It is also useful to note that, for similar pore shapes, the values of α for plane specimens are greater than those for volume specimens approximately by 20–30%. This observation can be used to analyze the two-dimensional images of real three-dimensional geological structures. The possibility of such comparison for model structures means that it is expedient to consider them.

Figure 3 shows how the coefficient α depends on the porosity n (in percents) for two-dimensional (*1*, *2*) and three-dimensional (*3*, *4*) specimens with a pore shapes as (*1*) square, (*2*) circle, (*3*) cube, (*4*) ball. One can see the regular increase in α as the porosity increases. We again note that, for similar shapes of the pores, the values of α are greater for plane models.

For the isotropic matrix solid material, the coefficient α depends only on the Poisson ratio. This dependence is shown in Figs. 4 and 5. The coefficient of the pore pressure transfer α increases with the Poisson ratio ν, as is illustrated by the graphs of α versus ν for plane specimens with a circular pore (Fig. 4) and volume specimens with spherical pore (Fig. 5). The numbers of the graphs in Figs. 4

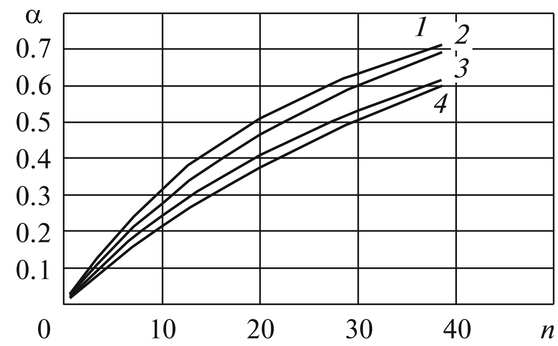

Fig. 3.

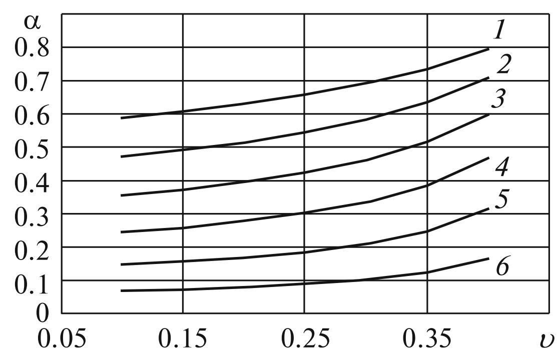

Fig. 4.

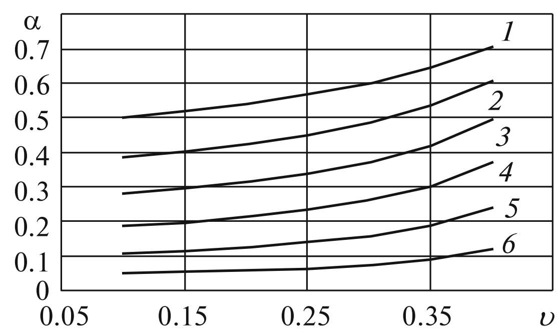

Fig. 5.

and 5 correspond to specimens of different porosity: (1) 38.48%, (2) 28.27%, (3) 19.83%, (4) 12.57%, (5) 7.07%, (6) 3.14%. For the same values of ν, the coefficient α increases with the specimen porosity.

Further, we consider the cases where the tensor $\boldsymbol{\alpha}$ is not spherical. The results of investigation of the pore pressure transfer tensor for model specimens with a pore shaped as an ellipsoid (Fig. 6) and an ellipsoidal pore (Fig. 7) are given in Tables 1 and 2, respectively. All specimens have the same porosity $n = 11.31\%$. For each specimen, the tensor component α_{22} has the greatest value, and axis 2 is parallel to the direction of the least elongated axis of the ellipse or ellipsoid. The nonmonotone dependence for α_{11} is very remarkable. As the pore elongation increases, the values of α_{22} increase, while the values of the other components of the tensor $\boldsymbol{\alpha}$ first decrease and then increase.

The calculations of the pore pressure transfer tensor for transversally isotropic specimens with cylinder-shaped pores (Fig. 8, Table 3) and with pores shaped as elongated parallelepipeds (Fig. 9, Table 4) permit using the methods for calculating the effective tensor of the pore pressure transfer for media with arbitrarily oriented pores on the basis of the already available tensor $\boldsymbol{\alpha}$ for the media with pores located in parallel. As was already noted, this tensor turns out to be isotropic, i.e., we can speak about the coefficient of the pore pressure transfer α. The results of calculations of the tensor $\alpha_{\beta\beta}$

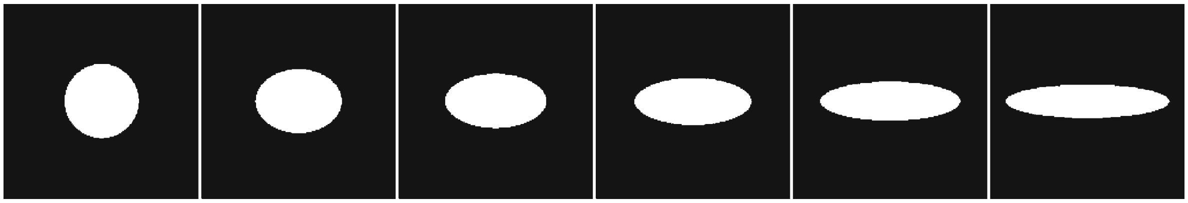

Fig. 6.

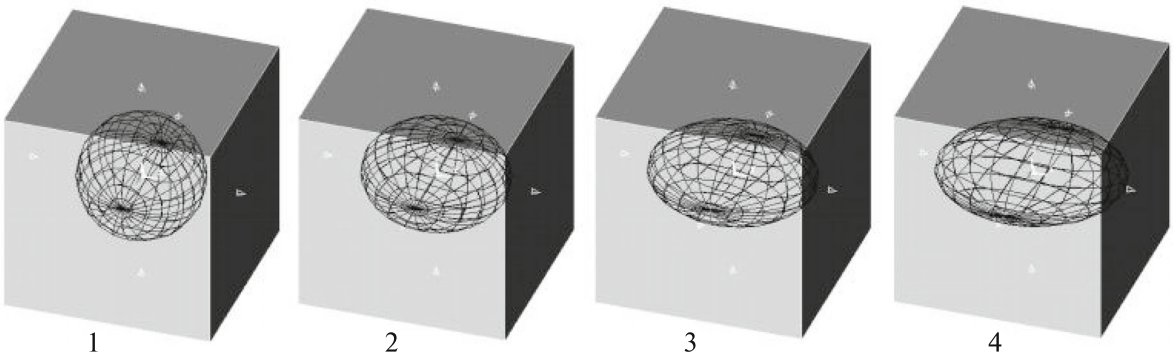

Fig. 7.

Table 1

	1	2	3	4	5	6
α_{11}	0.31	0.29	0.29	0.30	0.33	0.37
α_{22}	0.31	0.34	0.40	0.47	0.58	0.69

Table 2

	1	2	3	4
α_{11}	0.24	0.23	0.23	0.25
α_{22}	0.24	0.26	0.31	0.40
α_{22}	0.24	0.24	0.24	0.26

Table 3

	1	2	3	4	5
n	2.83	3.85	5.03	6.36	7.39
α_{11}	0.18	0.26	0.36	0.47	0.57
$\alpha_{22} = \alpha_{33}$	0.10	0.14	0.19	0.24	0.29
α	0.13	0.18	0.24	0.32	0.38

($\beta = 1, 2, 3$) in the case of parallel pores and of the coefficient α calculated from them in the case of chaotically located pores are shown in Tables 3 and 4. These tables allow us to conclude that, for disks, the coefficient α is approximately two times greater if it is recalculated for the same porosity. Moreover, the dependence on the porosity is nearly linear in both cases.

The above-developed technique was used to study the effective properties of real rocks. Some specific specimens of magmatic rocks, hyaloclastites and volcanic tuff, were investigated. In these calculations, the properties of minerals and rocks contained in these specimens were taken from the reference literature [3–5].

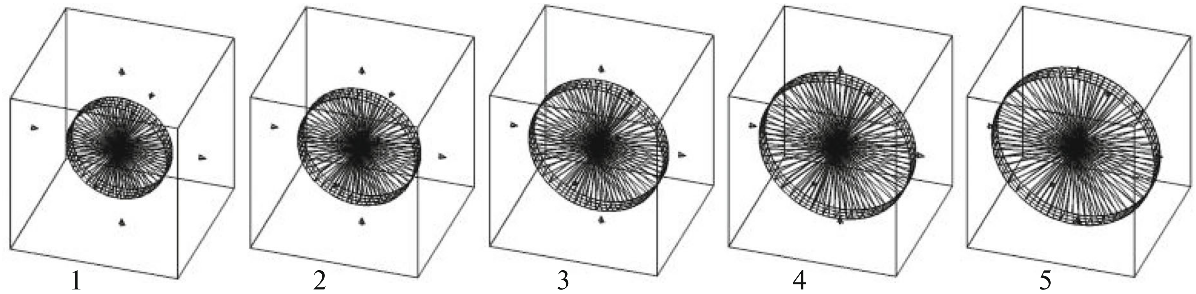

Fig. 8.

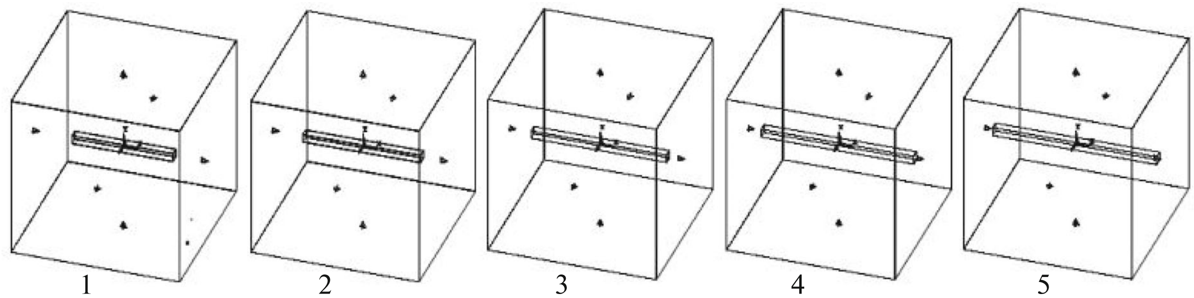

Fig. 9.

Table 4

	1	2	3	4	5
n	0.15	0.18	0.20	0.22	0.24
α_{11}	0.0036	0.0042	0.0047	0.0054	0.0058
$\alpha_{22} = \alpha_{33}$	0.0050	0.0057	0.0065	0.0074	0.0080
α	0.0045	0.0052	0.0059	0.0067	0.0073

The hyaloclastites are volcanic sedimentary rocks formed in the processes of underwater volcanic eruption. Due to the fast cooling of the lava contacting with water, small fragments of volcanic glass are formed, which are then hardened under the action of various postgenetic processes [30]. Two specimens of Iceland hyaloclastites were investigated: with angular chaotic pores ($n = 15\%$) (Fig. 10) and with circular pores ($n = 33\%$) (Fig. 11). Figures 10 and 11 show thin sections of the specimens and their models used in calculations. In the calculations of elastic moduli, the specimen‘s were assumed to be dry. The following properties of volcanic glass were accepted in the calculations: $E = 13.4 \times 10^3$ MPa and $\nu = 0.24$ for the first specimen, $E = 12.3 \times 10^3$ MPa and $\nu = 0.24$ for the second specimen. The calculated values of components of the elasticity modulus tensor ($n \times 10^3$ MPa) and the pore pressure transfer tensor are given in Table 5. We note that $C_{1111} \approx C_{2222}$ and $\alpha_{11} = \alpha_{22} = \alpha$ for both specimens, because the pores are uniformly located in them and are chaotically oriented in the first specimen. The values of components of the tensor $\mathbf{C}$ were used to calculate the effective Young moduli and Poisson ratios. The calculated values of the effective elastic properties turned out to be close to the experimentally determined values of the corresponding factors (Table 5). This fact proves the efficiency of the proposed technique for calculating the tensor $\mathbf{C}$, and indirectly, the tensor $\boldsymbol{\alpha}$. For both specimens, $\alpha = 0.58$ despite the fact that the porosity of the first specimen ($n = 15\%$) is significantly lower than that of the second specimen ($n = 33\%$). This again confirms the law previously revealed by studying the model specimens, i.e., α depends on the pore shape, and this coefficient is lower for media with circular pore than for media with angular (slot-like) pores.

The thin-section and the model of the specimen under study are shown in Fig. 12. The micropsephitic lithoclastic tuff consists of basalt-compound lava fragments and plagioclase fragments ($n = 7\%$). The

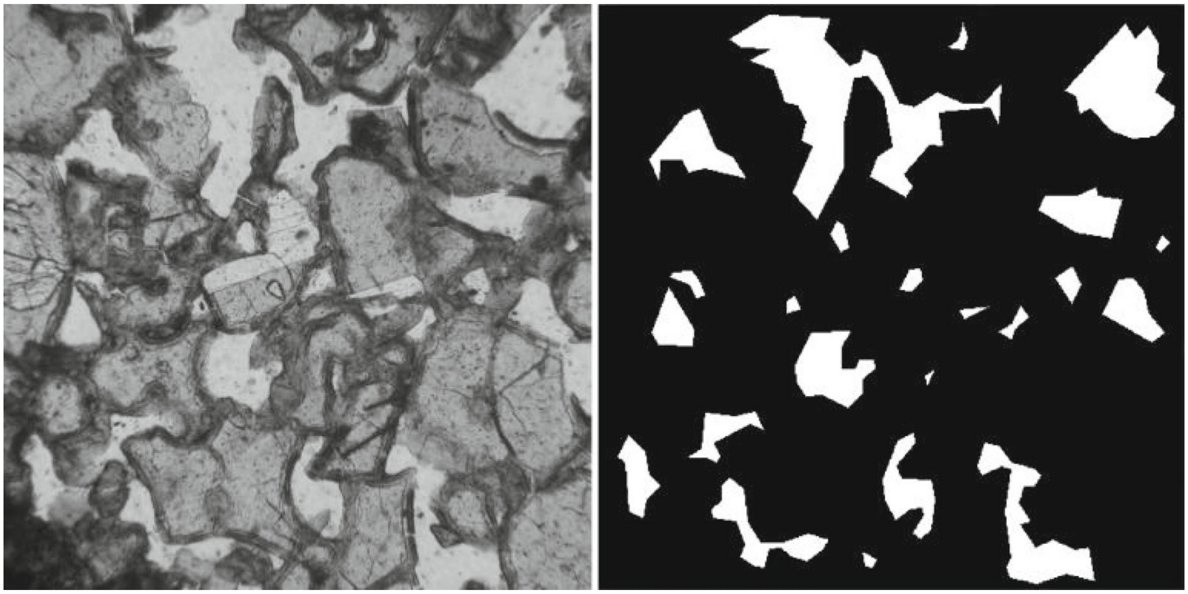

Fig. 10.

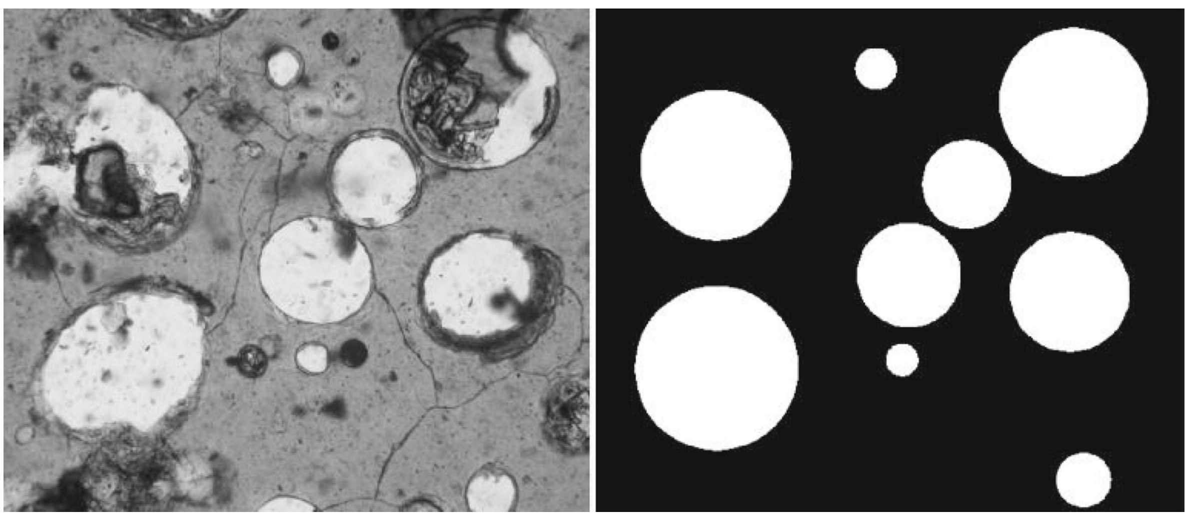

Fig. 11.

Table 5

Rock	Hyaloclastites with angular pores	Hyaloclastites with circular pores	Volcanic tuff
α_{11}	0.58	0.58	0.33
α_{22}	0.58	0.58	0.35
C_{1111}	7.0	6.1	18.2
C_{2222}	7.3	6.1	17.4
C_{1122}	1.7	1.8	3.9
$E_{\rm calc}$	6.3	5.0	18.0
$\nu_{\rm calc}$	0.20	0.21	0.20
$E_{\rm exper}$	6.5	5.3	16.8
$\nu_{\rm exper}$	0.19	0.23	0.18

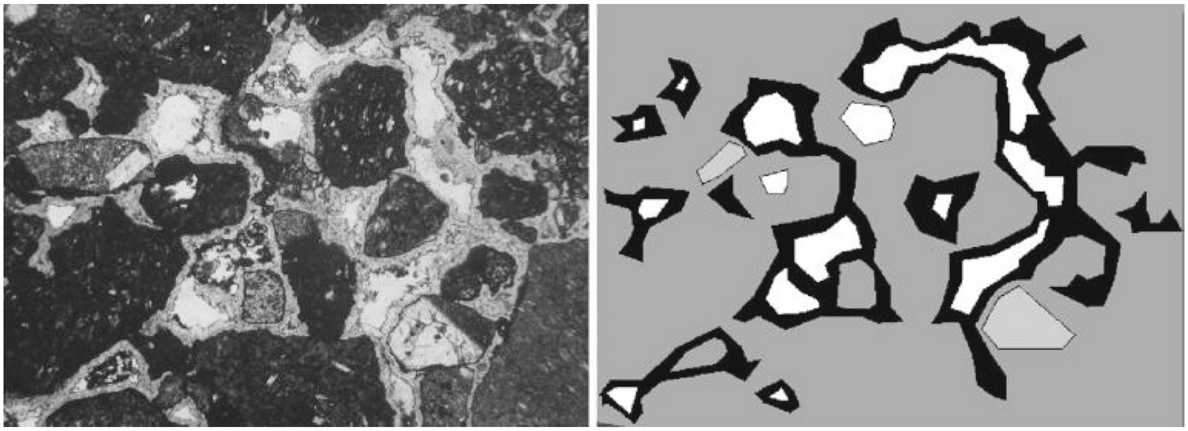

Fig. 12.

pores in lave fragments are edged by zeolites crystals. The specimen was obtained from the Mutnovsky geothermal region in Kamchatka. The following properties of minerals and rock composing the specimen were used in the calculations: lava (dark grey color in the mode in Fig. 12): $E = 22.0 \times 10^3$ MPa and $\nu = 0.2$; zeolites (black color): $E = 28.0 \times 10^3$ MPa and $\nu = 0.22$; plagioclases (light grey color) $E = 80.0 \times 10^3$ MPa and $\nu = 0.28$. (The white color in the model in Fig. 12 indicates the pores.) The obtained values of components of the elasticity modulus tensor and the tensor $\boldsymbol{\alpha}$ are given in Table 5. One can see that the calculated values of the elastic properties for the tuff specimen are also close to the experimental values.

5. CONCLUSION

In the present paper, the equilibrium equation for the fluid-saturated porous medium contained in the Biot model was obtained on the basis of the homogenization method. As a result, a computational method for determining the tensor of pore pressure transfer to the matrix solid material by solving local problems in the representative volume element (RVE) was proposed. The technique for calculating the effective tensor of the pore pressure transfer was considered for media with uniformly oriented pores, namely, this tensor was calculated on the representative volume element with parallel-oriented pores and then homogenized across the space. This technique was illustrated for model specimens and for real structures of geological materials. It was investigated how the pore pressure transfer tensor depends on the porosity, pore shape and orientation, and Poisson ratio. The results of calculations of the properties of rock specimens selected in geothermal deposits in Kamchatka and Iceland coincided with the experimental data, which confirms the possibility of using the proposed technique in the engineering geology.

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