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Identification of the Damping Properties of Rigid Isotropic Materials by Studying the Damping Flexural Vibrations of Test Specimens

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Abstract—A technique for determining the damping properties of a rigid isotropic material from the experimental data on the damping capacity of elongated cantilever-fixed test specimens due to the internal and external aerodynamic damping is proposed. The following two methods for eliminating the aerodynamic damping component are considered: the extrapolation of the data on the damping capacity of a series of test specimens of different widths to the point corresponding to the zero width and the theoretical-experimental approach. The damping properties of the material are determined by the vibration logarithmic decrement depending on the amplitude of the linear deformation. This dependence is represented by a power polynomial. The polynomial coefficients are determined from the minimum condition of the goal function for the positive logarithmic decrement of the material vibrations. These coefficients are sought at the reference point by repeatedly solving the direct problem of determining the damping capacity of the test specimen from the given damping properties of the material. An example is considered to illustrate the identification of the damping properties of steel St.3.

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1. INTRODUCTION AND THEORETICAL FOUNDATIONS

According to the currently existing American standard ASTM E-756 [1], an acoustic method in the resonance mode is used to investigate the dynamic behavior of cantilever-fixed test specimens of different structure, which permits experimentally determining the damping properties of materials in the frequency range from 50 to 5000 Hz. If the material is isotropic and rigid (metal, alloy, high-modulus polymer), then the test specimen is completely manufactured from this material. But the results obtained in this case cannot be considered as acceptable for analyzing the dynamical reaction of the structures by the following two reasons: (1) the above-cited standard does not take into account the aerodynamic component of the logarithmic decrement of the test specimen vibrations which can be compared with the internal damping parameters, and for the materials with low damping properties, can decisively influence the amplitudes of the flexural vibrations of the test specimen; (2) the damping characteristics of the materials obtained in the case of flexural vibrations of the test specimens give only a comparative estimate of their damping properties and cannot be used under the conditions other than the conditions in the experiment. Therefore, it is necessary to identify the real damping properties of the material from the internal damping characteristics of the test specimens under the conditions of their flexural vibrations. These characteristics can be obtained by studying the resonance or damping transverse vibrations of the test specimens. But experiments under the resonance conditions require an extremely fine tuning of the vibration source frequency because of the fast response of the test specimen vibration amplitude to small variations in the frequency near the resonance. More reliable data on the damping capacity of the test specimen can be obtained by analyzing its flexural damped vibrations [2, 3].

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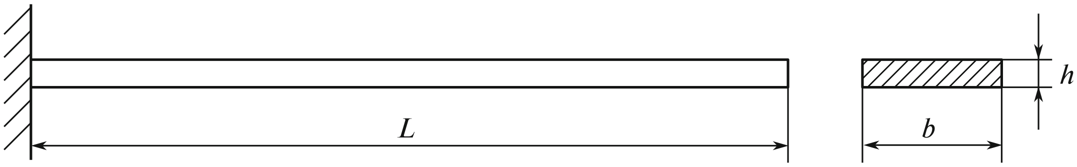

Fig. 1.

The damping capacity of rigid isotropic materials can be determined by using the elongated cantilever-fixed test specimens of rectangular cross-section (Fig. 1). The damping properties of the test specimen can be represented by the vibration logarithmic decrement (OLD) δ depending on the vibration amplitude A of its free end. The dependence $\delta(A)$ can be obtained by processing the experimental vibrorecord of damped vibrations of the test specimen in the air by using the technique presented in [2], where it is shown that the damping properties of the test specimen are significantly influenced by the external aerodynamic damping thus increasing the OLD as the specimen width b increases. This influence can be eliminated by using an approach based on testing a series of test specimens of the same length L and different width b. This permits constructing the dependencies $\delta(A, b)$ and then extrapolating them to obtain the dependence $\delta^*(A)$ for $b = 0$ which is required to identify the damping properties of the material.

There is a theoretical-experimental method for determining the relation $\delta^*(A)$, which significantly decreases the number of necessary experiments:

$$\delta^*(A) = \delta(A) - \delta_a. \tag{1.1}$$

Here $\delta(A)$ is the experimental OLD of the test specimen in the air for a finite width b and δ_a is the OLD computational aerodynamic component determined by the formula [3, 4]

$$\delta_a = \frac{b\rho_f F_A}{h\rho} \tag{1.2}$$

with the notation

$$F_A = \frac{6.14}{\sqrt{\beta}} + 7\sqrt{\kappa}\frac{\xi^2}{\xi^2 + 3.2}, \quad \beta = \frac{b^2 f}{\nu},$$

$$\xi = \kappa[2 + 1.78\ln\Delta - (0.54 + 0.88\ln\Delta)\ln\beta], \quad \kappa = \frac{A}{b}, \quad \Delta = \frac{h}{b}.$$

Here A is the deflexion amplitude of the test specimen free end in vibrations according to the first mode, f is the vibration frequency measured in Hz, and $\nu = 1.5 \times 10^{-5}\ \mathrm{m^2/s}$ is the kinematic viscosity of the air whose viscosity is $\rho_f = 1.29\ \mathrm{kg/m^3}$.

In the case of damped vibrations, the material at any point of its cross-section is in the state of cyclic extension-compression, and the damping properties of the material are determined by the OLD δ depending on the strain amplitude ε_0. It is proposed to seek the dependence $\delta(\varepsilon_0)$ as the power polynomial

$$\delta(\varepsilon_0) = \sum_{k=0}^{n} c_k \varepsilon_0^k. \tag{1.3}$$

To identify the damping properties of the material, it is necessary to seek the coefficients c_k ($k = 0, 1, 2, \ldots, n$) of the polynomial (1.3) from the available dependence $\delta^*(A)$ so that the computational OLD of the test specimen be slightly different from the experimental values of δ^* for a given set of amplitudes A_j ($j = 1, 2, \ldots, m$) in the range of representation of the dependence $\delta^*(A)$. To estimate the result, it is necessary to introduce the norm of discrepancy of the compared quantities, for example, in the form

$$F(c_0, c_1, \ldots, c_n) = \frac{\|\boldsymbol{\delta}^* - \boldsymbol{\delta}\|}{\|\boldsymbol{\delta}^*\|}.$$

Here $F(c_0, c_1, \ldots, c_n)$ is the goal function implicitly depending on the coefficients c_k, $\boldsymbol{\delta}^*$ and $\boldsymbol{\delta}$ are vectors containing the experimental and computational OLD of the test specimen for the amplitudes A_j

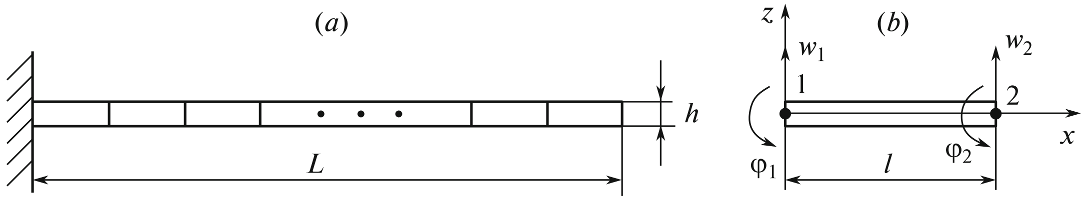

Fig. 2.

$(j = 1, 2, \ldots, m)$, and $\|\boldsymbol{\delta}^* - \boldsymbol{\delta}\|$ and $\|\boldsymbol{\delta}\|$ are the Euclidean norms of the corresponding vectors. The coefficients c_k are determined from the minimum condition for the goal function $F(c_0, c_1, \ldots, c_n)$ under the restriction

$$\delta(\varepsilon_0) = \sum_{k=0}^{n} c_k \varepsilon_0^k > 0.$$

To determine the minimum of the function of several variables, one usually employs the method of coordinate and fastest descent [5], where it is required to calculate the partial derivatives of a given function with respect to each independent variable. But in the absence of the explicit dependence $F(c_0, c_1, \ldots, c_n)$, it is necessary to average the partial derivatives $\partial F/\partial c_k$ numerically. Therefore, it is more preferable to use the direct methods for determining the zeroth-order terms [5] (the simplex method, the Hooke–Jeeves configuration method, Rosenbrock's method), where it is not required to calculate the derivatives $\partial F/\partial c_k$. The most convenient is the Hooke–Jeeves configuration method, which can easily be realized for any dimension of the search space.

The computational OLD of the test specimen can be obtained by using the finite-element method. Since the test specimen shape is elongated in plan and its thickness h is much less that the width b, one can use the computational model (Fig. 2 *a*) composed of beam finite elements (Fig. 2 *b*). The damped vibrations of the test specimen are described by the system of differential equations

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{C}\dot{\mathbf{r}} + \mathbf{K}\mathbf{r} = 0 \tag{1.4}$$

with the initial conditions $\mathbf{r}(0) = \mathbf{r}_{no}$ and $\dot{\mathbf{r}}(0) = 0$, where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, and $\mathbf{r}$ are the mass matrix, the damping matrix, the rigidity matrix, and the vector of angular displacements of the above-described model, $\mathbf{r}_{no}$ is the vector of angular displacements for the initial static deviation w_{no} of the free end of the test specimen. The dot over a symbol denotes its differentiation with respect to time t. The matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are formed by the method of direct rigidity [6] from the corresponding matrices $\mathbf{M}_\mathrm{e}$, $\mathbf{C}_\mathrm{e}$, and $\mathbf{K}_\mathrm{e}$ of finite elements. The form of the matrices $\mathbf{M}_\mathrm{e}$ and $\mathbf{K}_\mathrm{e}$ is well known [7]

$$\mathbf{M}_\mathrm{e} = m \begin{bmatrix} \frac{13l}{35} & \frac{11l^2}{210} & \frac{9l}{70} & -\frac{13l^2}{420} \\ \frac{11l^2}{210} & \frac{l^3}{105} & \frac{13l^2}{420} & -\frac{l^3}{140} \\ \frac{9l}{70} & \frac{13l^2}{420} & \frac{13l}{35} & -\frac{11l^2}{210} \\ -\frac{13l^2}{420} & -\frac{l^3}{140} & -\frac{11l^2}{210} & \frac{l^3}{105} \end{bmatrix}, \quad \mathbf{K}_\mathrm{e} = EI \begin{bmatrix} \frac{12}{l^3} & \frac{6}{l^2} & -\frac{12}{l^3} & \frac{6}{l^2} \\ \frac{6}{l^2} & \frac{4}{l} & -\frac{6}{l^2} & \frac{2}{l} \\ -\frac{12}{l^3} & -\frac{6}{l^2} & \frac{12}{l^3} & -\frac{6}{l^2} \\ \frac{6}{l^2} & \frac{2}{l} & -\frac{6}{l^2} & \frac{4}{l} \end{bmatrix},$$

where m, EI, and l are the linear mass, the bending rigidity, and the length of the element.

The finite element matrices $\mathbf{C}_\mathrm{e}$ depend on the model of nonelastic deformation of the material. If the material of the test specimen has viscoelastic properties, then to describe them, one can use the physical relations between the components of the stress tensor σ_{ij}, the strain tensor ε_{ij}, and the strain rate tensor $\dot{\varepsilon}_{ij} = \partial\varepsilon_{ij}/\partial t$ as follows:

$$\sigma_{ij} = \sigma_{ij}(\varepsilon_{ij}, \dot{\varepsilon}_{ij}).$$

In the case of uniaxial stress state, the simplest of such dependencies, which is most often used in practice, corresponds to the well-known Voigt–Thompson–Kelvin model [8]

$$\sigma = E\varepsilon + \alpha\dot{\varepsilon}, \tag{1.5}$$

where σ, ε, and $\dot{\varepsilon}$ are the normal stress, relative strain, and the rate of its variation in the time t, and E and α are the Young modulus and the viscosity coefficient of the material. The latter is related to the logarithmic decrement of the vibrations $\delta(\varepsilon_0)$ as

$$\alpha = \frac{E\delta(\varepsilon_0)}{\pi\omega},$$

where ω is the frequency of the material. With regard to this fact, the model (1.5) becomes

$$\sigma = E\varepsilon + \frac{E\delta(\varepsilon_0)\dot{\varepsilon}}{\pi\omega}.$$

We write an infinitely small increment of the work of the inelastic part of the stress σ on the corresponding strain increment $d\varepsilon$ over the finite element volume

$$dA = -\frac{E}{\pi\omega}\int\limits_F\int\limits_0^l d\varepsilon\delta(\varepsilon_0)\dot{\varepsilon}\, dF\, dx. \tag{1.6}$$

The strain ε at the point z in the element cross-section can be determined in terms of its angular displacements $\mathbf{r}_\mathrm{e} = \{w_1\ \varphi_1\ w_2\ \varphi_2\}$ by using the geometric dependence

$$\varepsilon = -zw'' = -z\mathbf{f}''^T\mathbf{r}_\mathrm{e}, \tag{1.7}$$

where $\mathbf{f}''$ is the vector of the second derivatives of the basis function f_i $(i = 1, 2, 3, 4)$ of the finite element with respect to its local coordinates x (Fig. 2 *b*). The superscript T in (1.7) and further denotes the operation of transposition. The functions f_i of the beam elements have the form

$$f_1 = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}, \quad f_2 = x - \frac{2x^2}{l} + \frac{x^3}{l^2}, \quad f_3 = \frac{3x^2}{l^2} - \frac{2x^3}{l^3}, \quad f_4 = -\frac{x^2}{l} + \frac{x^3}{l^2}. \tag{1.8}$$

The quantity ε_0 can be related to the curvature amplitude $\chi_0(x)$ of the element axis

$$\varepsilon_0 = |z|\chi_0(x), \quad \chi_0(x) = \mathbf{f}''^T\mathbf{r}_{0,\mathrm{e}}. \tag{1.9}$$

Here $\mathbf{r}_{0,\mathrm{e}}$ is the vectors of amplitudes of the nodal displacements of this element. It follows from (1.8) and (1.9) that the dependence $\chi_0(x)$ is linear within the element and it can be written as

$$\chi_0(x) = \chi_{0,1}\left(1 - \frac{x}{l}\right) + \chi_{0,2}\frac{x}{l} = a\left(1 + s\frac{x}{l}\right). \tag{1.10}$$

Here $\chi_{0,1}$ and $\chi_{0,2}$ are the respective amplitudes of nodes 1 and 2 of the finite element, and $a = \chi_{0,1}$, $s = (\chi_{0,2} - \chi_{0,1})/\chi_{0,1}$. The quantities $\chi_{0,1}$ and $\chi_{0,2}$ are determined by the expression

$$\{\chi_{0,1}\chi_{0,2}\} = \left[\begin{array}{c|c|c|c} -\dfrac{6}{l^2} & -\dfrac{4}{l} & \dfrac{6}{l^2} & -\dfrac{2}{l} \\ \hline \dfrac{6}{l^2} & \dfrac{2}{l} & -\dfrac{6}{l^2} & \dfrac{4}{l} \end{array}\right]\mathbf{r}_{0,\mathrm{e}}.$$

With regard to (1.9) and (1.10), the dependence (1.3) becomes

$$\delta(\varepsilon_0) = \delta(a, s) = \sum_{k=0}^{n} c_k|z|^k a^k\left(1 + s\frac{x}{l}\right)^k. \tag{1.11}$$

Substituting (1.7) and (1.11) into (1.6), we obtain

$$dA = -\frac{E}{\pi\omega} d\mathbf{r}_{\rm e}^T \sum_{k=0}^{n} c_k a^k \int\limits_F |z|^{k+2} dF \int\limits_0^l \mathbf{f}'' \mathbf{f}''^T \left(1 + s\frac{x}{l}\right)^k dx\, \dot{\mathbf{r}}_{\rm e}.$$

The quantity dA can be represented as

$$dA = -\frac{EI}{\pi\omega} d\mathbf{r}_{\rm e}^T \left(\sum_{k=0}^{n} \frac{c_k \mathbf{H}_k I_k}{I}\right) \dot{\mathbf{r}}_{\rm e}. \qquad (1.12)$$

The matrices $\mathbf{H}_k$ and the quantities I_k and I are determined by the expressions

$$\mathbf{H}_k = a^k \int\limits_0^l \mathbf{f}'' \mathbf{f}''^T \left(1 + s\frac{x}{l}\right)^k dx, \quad I_k = \int\limits_F |z|^{k+2} dF, \quad I = \int\limits_F z^2 dF.$$

Relation (1.12) can be rewritten as

$$dA = -d\mathbf{r}_{\rm e}^T \mathbf{C}_{\rm e} \dot{\mathbf{r}}_{\rm e}, \quad \mathbf{C}_{\rm e} = \frac{EI}{\pi\omega} \sum_{k=0}^{n} \frac{c_k \mathbf{H}_k I_k}{I}, \qquad (1.13)$$

where $\mathbf{C}_{\rm e}$ is the matrix of the finite element damping.

The numerical experiments showed that to identify the dependence $\delta(\varepsilon_0)$ of the material, it suffices to consider only $n = 3$ in (1.13). The ratios I_k/I and the matrices $\mathbf{H}_k$ then become

$$\mathbf{H}_0^{(11)} = \frac{2}{l^3}\left[\begin{array}{c|c} 6 & 3l \\ \hline 3l & 2l^2 \end{array}\right], \quad \mathbf{H}_0^{(12)} = \frac{2}{l^3}\left[\begin{array}{c|c} -6 & 3l \\ \hline -3l & 2l^2 \end{array}\right], \quad \mathbf{H}_0^{(22)} = \frac{2}{l^3}\left[\begin{array}{c|c} 6 & -3l \\ \hline -3l & 2l^2 \end{array}\right],$$

$$\mathbf{H}_1^{(11)} = \frac{a}{l^3}\left[\begin{array}{c|c} 6(s+2) & 2l(s+3) \\ \hline 2l(s+3) & l^2(s+4) \end{array}\right], \quad \mathbf{H}_1^{(12)} = \frac{a}{l^3}\left[\begin{array}{c|c} -6(s+2) & 2l(s+3) \\ \hline -2l(s+3) & l^2(s+4) \end{array}\right],$$

$$\mathbf{H}_1^{(22)} = \frac{a}{l^3}\left[\begin{array}{c|c} 6(s+2) & -2l(s+3) \\ \hline -2l(2s+3) & l^2(3s+4) \end{array}\right],$$

$$\mathbf{H}_2^{(11)} = \frac{a^2}{15l^3}\left[\begin{array}{c|c} 36(2s^2+5s+5) & 3l(7s^2+20s+30) \\ \hline 3l(7s^2+20s+30) & 2l^2(4s^2+15s+30) \end{array}\right],$$

$$\mathbf{H}_2^{(12)} = \frac{a^2}{15l^3}\left[\begin{array}{c|c} -36(2s^2+5s+5) & 3l(17s^2+40s+30) \\ \hline -3l(7s^2+20s+30) & l^2(13s^2+30s+30) \end{array}\right],$$

$$\mathbf{H}_2^{(22)} = \frac{a^2}{15l^3}\left[\begin{array}{c|c} 36(2s^2+5s+5) & -3l(17s^2+40s+30) \\ \hline -3l(17s^2+40s+30) & 2l^2(19s^2+45s+30) \end{array}\right],$$

$$\mathbf{H}_3^{(11)} = \frac{a^3}{5l^3}\left[\begin{array}{c|c} 3(7s^3+24s^2+30s+20) & 3l(2s^3+7s^2+10s+10) \\ \hline 3l(2s^3+7s^2+10s+10) & l^2(2s^3+8s^2+15s+20) \end{array}\right],$$

$$\mathbf{H}_3^{(12)} = \frac{a^3}{5l^3}\left[\begin{array}{c|c} -3(7s^3+24s^2+30s+20) & 3l(5s^3+17s^2+20s+10) \\ \hline -3l(2s^3+7s^2+10s+10) & l^2(4s^3+13s^2+15s+10) \end{array}\right],$$

$$\mathbf{H}_3^{(22)} = \frac{a^3}{5l^3}\left[\begin{array}{c|c} 3(7s^3+24s^2+30s+20) & -3l(5s^3+17s^2+20s+10) \\ \hline -3l(2s^3+7s^2+10s+10) & l^2(11s^3+38s^2+45s+20) \end{array}\right].$$

The matrices $\mathbf{H}_k^{(21)}$ which are not written above are the transposed matrices $\mathbf{H}_k^{(12)}$.

To solve system (1.4), it is necessary to use the stepwise integration methods recalculating the finite element matrices $\mathbf{C}_{\rm e}$ at each vibration cycle according to the attained values $\chi_{0,1}$ and $\chi_{0,2}$ which determine the values of a and s in the matrices $\mathbf{H}_k$. Therefore, the integration step Δt necessary to trace these values must be sufficiently small, 100–120 steps per vibration cycle, and the number of solutions of system (1.4) in the search of the coefficients c_k of the power polynomial (1.3) by direct methods can be large (several hundreds of thousands). This leads to unacceptable computer times costs and the inevitable accumulation of computational errors. But the vibration amplitudes A of the test specimen are measured not from the initial static deviation w_{no} but some time later, when the process of transition from the static deflection shape to the lowest natural shape $\mathbf{f}_1$ is terminated. This permits passing from system (1.4) to the equation for the generalized coordinate $q_1(t)$ of the shape $\mathbf{f}_1$:

$$m\ddot{q}_1(t) + c\dot{q}_1(t) + kq_1(t) = 0 \tag{1.14}$$

with the initial conditions

$$q_1(0) = \frac{A_{\max}}{f_w}, \quad \dot{q}_1(0) = 0.$$

Here $A_{\max}$ is the maximal amplitude A in the range of representation of the experimental dependence $\delta^*(A)$, f_w is the component of the shape $\mathbf{f}_1$ corresponding to the deflection w of the free end of the test specimen, m, c, and k are the generalized mass, the generalized damping coefficient, and the generalized rigidity of the test specimen:

$$m = \mathbf{f}_1^T\mathbf{M}\mathbf{f}_1, \quad c = \mathbf{f}_1^T\mathbf{C}\mathbf{f}_1, \quad k = \mathbf{f}_1^T\mathbf{K}\mathbf{f}_1.$$

The replacement of system (1.4) by the Eq. (1.14) significantly decreases the time necessary to obtain the computational OLD of the test specimen but does not completely solve the problem of determining the coefficients c_k, because the work content required to determine them is still rather large. Therefore, it is necessary to develop faster methods for computing the test specimen OJD, which would permit efficiently performing computational experiments for determining the character of the obtained dependencies $\delta(\varepsilon_0)$ for different experimental dependencies $\delta^*(A)$. For this, we write Eq. (1.14) in the form

$$\ddot{q}_1(t + 2n\dot{q}_1(t) + \omega^2 q_1(t) = 0, \tag{1.15}$$

where $n = c/(2m)$ and $\omega^2 = k/m$. The dynamic deflection of the free end of the test specimen is described by the function $w(t) = q_1(t)f_w$. Then instead of (1.15), we obtain the equation

$$\ddot{w}(t) + 2n\dot{w}(t) + \omega^2 w(t) = 0 \tag{1.16}$$

with the initial conditions $w(0) = A_{\max}$ and $\dot{w}(0) = 0$. The damping parameter n implicitly depends on the coefficients c_k of the polynomial (1.3) in terms of the damping matrices $\mathbf{C}_{\rm e}$ of finite elements and must be recalculated at each vibration cycle i according to the attained amplitude A_i of the test specimen. But between the two successive amplitudes A_i and A_{i+1}, this parameter can be assumed to be constant and equal to the value n_i for the amplitude A_i. This permits using the well-known analytic solution for the envelope $A(\tau)$ per one vibration cycle

$$A(\tau) = A_i \exp(-n_i\tau), \tag{1.17}$$

where $0 \le \tau \le \bar{T}$ is the local time. The vibration period $\bar{T}$ must be determined with regard to the parameter n_i:

$$\bar{T} = \frac{2\pi}{\sqrt{\omega^2 - n_i^2}}.$$

But the numerical experiments showed that $\bar{T}$ practically coincides with the vibration period T of the ideally elastic test specimen. Therefore, we must further set $\bar{T} = T$.

The computational OLD of the test specimen with the amplitude A_i is related to the parameter n_i by the well-known dependence $\delta_i = n_iT$. Substituting $n_i = c_i/(2m)$ and $T = 2\pi/\omega$ into this dependence, we obtain

$$\delta_i = \frac{c_i\pi}{m\omega}. \tag{1.18}$$

Expressions (1.17) and (1.18) allow us to construct the sweep method algorithm for determining the vibration amplitudes and the corresponding OLD of the test specimen avoiding the procedure of step integration of Eq. (1.16):

$$A_{i+1} = A_i \exp(-n_i T), \quad \delta_{i+1} = \frac{c_{i+1}\pi}{m\omega}. \tag{1.19}$$

The generalized damping coefficient c_{i+1} of the test specimen is calculated by the damping matrix $\mathbf{C}$ of the test specimen for the vibration amplitude A_{i+1}. The required values of the curvature amplitude $\chi_{0,1}^{(i+1)}$ and $\chi_{0,2}^{(i+1)}$ at the finite element nodes are determined by the expression

$$\left\{\chi_{0,1}^{(i+1)} \chi_{0,2}^{(i+1)}\right\} = \left[\begin{array}{c|c|c|c} -\dfrac{6}{l^2} & -\dfrac{4}{l} & \dfrac{6}{l^2} & -\dfrac{2}{l} \\ \hline \dfrac{6}{l^2} & \dfrac{2}{l} & -\dfrac{6}{l^2} & \dfrac{4}{l} \end{array}\right] \mathbf{f}_{\mathrm{e}} \frac{A_{i+1}}{f_w},$$

where $\mathbf{f}_{\mathrm{e}}$ is the vector containing the shape components $\mathbf{f}_1$ corresponding to the current element. The reliability of algorithm (1.19) is confirmed by numerical experiments for determining the computational dependence $\delta(A)$ of the test specimen according to the well-known dependence $\delta(\varepsilon_0)$ of the material.

When the coefficients c_k ($k = 0, 1, 2, \ldots, n$) of the polynomial (1.3) are determined by the Hooke–Jeeves configuration method, one meets the problem of choice of the initial (reference) point in the space of given coefficients, which arises due to the difference in their values, i.e., each successive coefficient turns out to be greater than the preceding one by three-four orders and higher. One of the versions for solving this problem may consist in the recalculation of the given dependence $\delta^*(A)$ of the test specimen into the dependence $\delta^*(\varepsilon_0)$, where ε_0 is the maximal strain amplitude at the fixation point corresponding to the vibration amplitude A. The amplitude ε_0 can be calculated from the amplitude A by the formula

$$\varepsilon_0 = \frac{\chi_F(0)Ah}{2f_w}, \tag{1.20}$$

where $\chi_F(0)$ is the curvature of the shape $\mathbf{f}_1$ at the fixed end of the test specimen. The dependence $\delta^*(\varepsilon_0)$ is approximated by the power polynomial of the same order as in the search of the coefficients c_k of the polynomial (1.3):

$$\delta^*(\varepsilon_0) = \sum_{k=0}^{n} c_k^* \varepsilon_0^k. \tag{1.21}$$

Relation (1.20) permits determining $\varepsilon_{0,j}$ for given values of the vibration amplitudes A_j ($j = 1, 2, 3, \ldots, m$) of the test specimen. Substituting $\varepsilon_{0,j}$ and $\delta^*(\varepsilon_{0,j})$ into the approximation (1.21), we obtain a system of m linear algebraic equations for the coefficients c_k^* ($k = 0, 1, 2, \ldots, n$). The usual number of experimental points is equal to $m > n + 1$. Therefore, the obtained system is overdetermined. Such a system can be solved by the least square method. This gives the coefficients c_k^* of the polynomial (1.21) which are assumed to be the coordinates of the reference point.

But the performed numerical experiments showed that the process of search of the coefficients c_k is sometimes completed before attaining the minimum of the goal function. The possible cause of this effect may be the already noted significant difference between the values of the coefficients $F(c_0, c_1, \ldots, c_n)$, which can lead to incorrect determination of these coefficients. Therefore, an approach for avoiding this difficulty is proposed. Let us determined the strain amplitudes $\varepsilon_{0,k}$ for the chosen amplitudes A_k ($k = 0, 1, 2, \ldots, n$) of the test specimen in the range $[A_{\min}, A_{\max}]$ of representation of the experimental dependence $\delta^*(A)$:

$$\varepsilon_{0,k} = \frac{\chi_F(0)A_k h}{2f_w}.$$

The amplitudes A_k can be chosen with equal step $h_A = (A_{\max} - A_{\min})/n$ including the extreme points of the range $[A_{\min}, A_{\max}]$. Substituting $\varepsilon_{0,k}$ into (1.3), we obtain the system of equations

$$\mathbf{Ec} = \boldsymbol{\delta}, \tag{1.22}$$

$$\mathbf{E} = \begin{bmatrix} 1 & \varepsilon_{0,0} & \varepsilon_{0,0}^2 & \cdots & \varepsilon_{0,0}^n \\ 1 & \varepsilon_{0,1} & \varepsilon_{0,1}^2 & \cdots & \varepsilon_{0,1}^n \\ 1 & \varepsilon_{0,2} & \varepsilon_{0,2}^2 & \cdots & \varepsilon_{0,2}^n \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \varepsilon_{0,n} & \varepsilon_{0,n}^2 & \cdots & \varepsilon_{0,n}^n \end{bmatrix}, \quad \mathbf{c} = \begin{Bmatrix} c_0 \\ c_1 \\ c_2 \\ \cdots \\ c_n \end{Bmatrix}, \quad \boldsymbol{\delta} = \begin{Bmatrix} \delta_0 \\ \delta_1 \\ \delta_2 \\ \cdots \\ \delta_n \end{Bmatrix}.$$

System (1.22) permits determining the coefficients c_k ($k = 0, 1, 2, \ldots, n$) of the polynomial (1.3) if the right-hand side $\boldsymbol{\delta}$ of this system is known:

$$\mathbf{c} = \mathbf{E}^{-1}\boldsymbol{\delta}.$$

This allows one to vary not the coefficients c_k in the search by the material OLD δ_0, δ_1, δ_2, …, δ_n for given $\varepsilon_{0,k}$ which, in contrast to the coefficients c_k, are of approximately the same order. As the coordinates of the reference point, one can take the test specimen OLD for the vibration amplitudes A_k ($k = 0, 1, 2, \ldots, n$) or the average value $\delta^*(A)$ in the range of amplitudes $[A_{\min}, A_{\max}]$. The vector $\mathbf{c}$ is determined at each point of the space of decrements thus ensuring the computation of the goal function F in the search of the coefficients c_k ($k = 0, 1, 2, \ldots, n$) of the polynomial (1.3).

For the practical realization of this procedure, we propose to use the search matrix

$$\mathbf{P} = \left[\begin{array}{c|c|c|c} \delta_0 & \delta_1 & \cdots & \delta_n \\ \hline \delta_0 + d_0 & \delta_1 & \cdots & \delta_n \\ \hline \delta_0 - d_0 & \delta_1 & \cdots & \delta_n \\ \hline \delta_0 & \delta_1 + d_1 & \cdots & \delta_n \\ \hline \delta_0 & \delta_1 - d_1 & \cdots & \delta_n \\ \hline \cdots & \cdots & \cdots & \cdots \\ \hline \delta_0 & \delta_1 & \cdots & \delta_n + d_n \\ \hline \delta_0 & \delta_1 & \cdots & \delta_n - d_n \end{array}\right]. \tag{1.23}$$

The first row in the matrix $\mathbf{P}$ contains the coordinates δ_0, δ_1, δ_2, …, δ_n of the reference point. The other rows contain the coordinates of points located at the one step distance from the reference point in each coordinate direction of the space $\boldsymbol{\delta}$. The successive verification of the content of the rows of the matrix $\mathbf{P}$ is used to determine the coefficients c_k and the goal function F for each row. Then the row which gives the minimal value of F is chosen. The content of this row is rewritten into the first row of the matrix $\mathbf{P}$ and becomes a new reference point. Then all other rows of the matrix $\mathbf{P}$ are formed as is shown in expression (1.23) and this procedure is repeated. If the current minimal values of F in the process of linear search corresponds to the first row of the matrix $\mathbf{P}$, then the new level of searching with smaller steps d_k ($k = 0, 1, 2, \ldots, n$) begins for the already attained value of F, and the entire procedure described above is repeated. Such transitions with decreasing the step d_k are carried out until the function F ceases to decrease. It is recommended to decrease the steps d_k in each transition approximately by a factor of 5–10 compared with their previous values. The performed numerical experiments exhibited high reliability and precision of the proposed version of the algorithm for searching the coefficients c_k of the polynomial (1.3).

2. EXPERIMENTAL FACILITY FOR STUDYING THE DAMPED FLEXURAL VIBRATIONS OF TEST SPECIMENS

A special experimental facility (Fig. 3) was designed to obtain the vibrorecords of damped vibrations of the test specimens. The facility consists of a rigidly connected basement *1* and a power wall *2*. The test specimen *3* is fixed as a cantilever to the wall. The clamping is ensured by spaced planks connected with the wall by two rows of bolted joints which do not permit the test specimen rotation in the cross-section

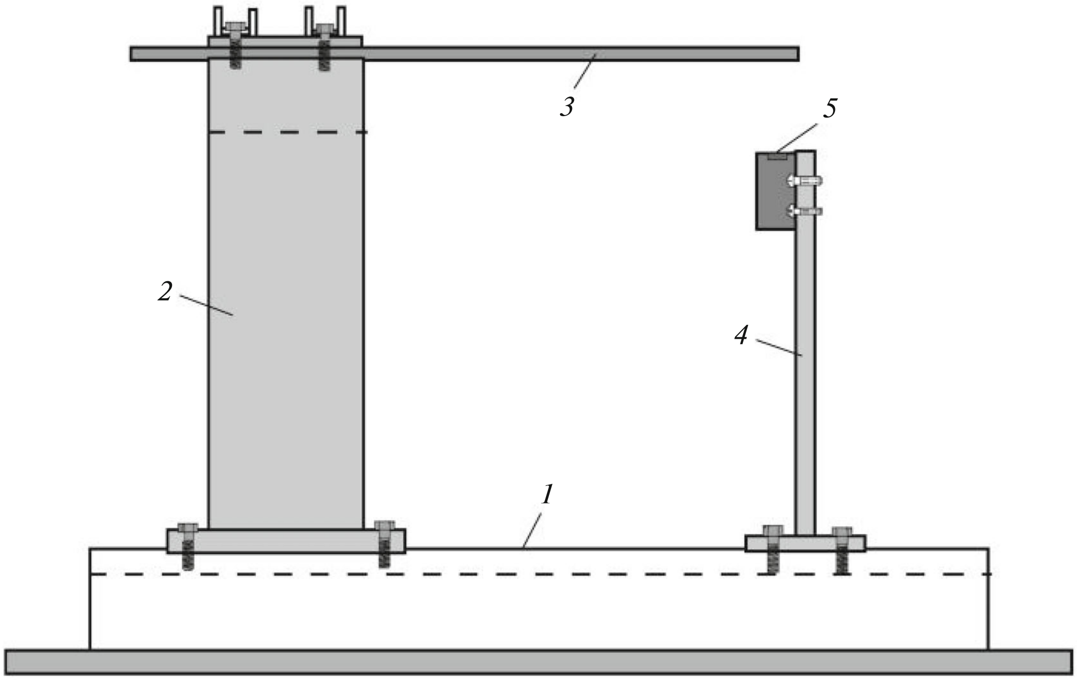

Fig. 3.

of the clamping. The displacement transducer *5* is jointed to the upright *4* mounted on the basement. The upright position along the basement can be varied to measure the amplitude of the test specimen vibrations as its overhang is changed.

A triangular laser transducer produced by the firm REFTEK (RF603-X/100) ensuring he precision of the vibration amplitude measurements up to 0.01 mm is used in the facility. The digital results of measurements enter a personal computer. The measurements start with a certain delay in time which is necessary for the transition from the initial (static) deflection state to the lowest mode shape of the test specimen. The developed software permits establishing up to 1000 measurements of the deflection per second, and thus, obtaining the real vibrorecord of damped vibrations of the test specimen with a rather high accuracy.

3. NUMERICAL EXPERIMENTS

The experiments were performed for a series of test specimens manufactured from the steel St.3 of lengths $L = 150$ mm, $L = 200$ mm, and $L = 300$ mm for the width b varying from 10 mm to 50 mm with the step of 10 mm and the thickness $h = 1$ mm. The vibrorecords of damped vibrations of these test specimens were processed according to the technique described in [2], and the amplitude dependencies of the test specimens OLD in the air were obtained for the above-cited set of b. Table 1 shows the values of the OLD δ^* of the test specimens of lengths $L = 150$ mm, $L = 200$ mm, and $L = 300$ mm for six amplitudes A, which were obtained by extrapolation of the experimental dependence $\delta(A, b)$ to the width $b = 0$.

The Young modulus E of the steel St.3 was determined from the experimental frequency f by using the formula for calculating the basic frequency of free vibrations of an ideally elastic cantilever beam

$$f_0 = \frac{1}{2\pi}\left(\frac{1.875}{L}\right)^2\sqrt{\frac{EI}{m}}. \tag{3.1}$$

This is justified by the weak dependence of the frequency on the parameters of the internal and external aerodynamic damping [9–12], which is also confirmed by the earlier experiments [2]. The density of the material required to determine the linear mass m of the test specimens was taken from the reference book [13]: $\rho = 7870\,\mathrm{kg/m^3}$. Table 2 shows the experimentally determined frequencies f of the test specimens of lengths $L = 150$ mm, $L = 200$ mm, and $L = 300$ mm and Young moduli E calculated by

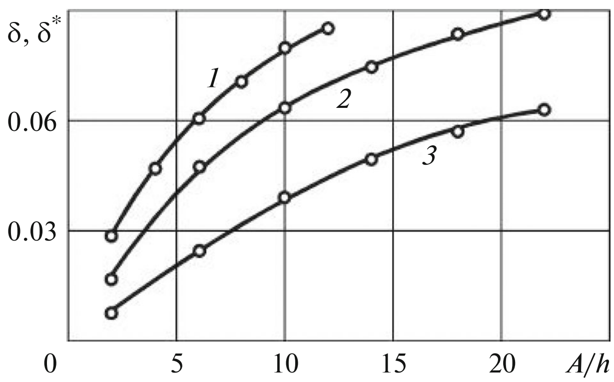

Fig. 4.

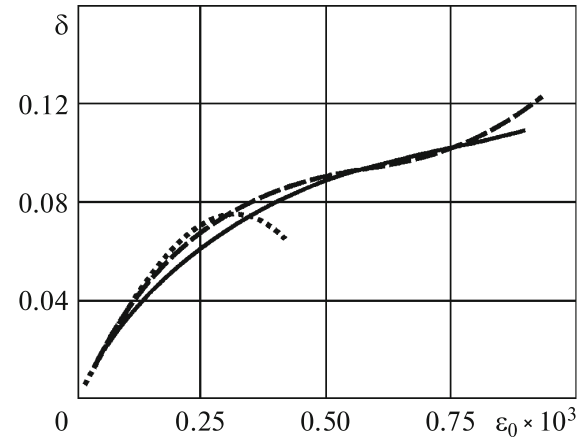

Fig. 5.

Table 1

$L = 150\,\text{mm}$		$L = 200\,\text{mm}$		$L = 300\,\text{mm}$	
A, mm	δ^*	A, mm	δ^*	A, mm	δ^*
2	0.0286	2	0.0167	2	0.0074
4	0.0473	6	0.0471	6	0.0245
6	0.0603	10	0.0632	10	0.0389
8	0.0710	14	0.0747	14	0.0496
10	0.0795	18	0.0830	18	0.0573
12	0.0855	22	0.0894	22	0.0629

Table 2

L mm	f, Hz	E, Pa
150	33.1	16.731×10^{10}
200	18.9	17.241×10^{10}
300	8.8	18.922×10^{10}

formula (3.1). The averaged modulus $E = 17.631 \times 10^{10}$ Pa was taken to identify the dependence $\delta(\varepsilon_0)$ of the material.

The dependence $\delta(\varepsilon_0)$ is represented by the cubic polynomial

$$\delta(\varepsilon_0) = c_0 + c_1\varepsilon_0 + c_2\varepsilon_0^2 + c_3\varepsilon_0^3. \tag{3.2}$$

Table 3 presents the coefficients c_0, c_1, c_2, and c_3 of the polynomial (3.2) which were obtained by the search according to the Hooke–Jeeves configuration method for the test specimens of lengths $L = 150$ mm, $L = 200$ mm, and $L = 300$ mm. The computational models of all three test specimens were represented 20 finite elements. The accuracy of determining these coefficients was estimated by comparing the computational and experimental OLD δ and δ^* of the above-mentioned test specimen represented depending on the dimensionless amplitude of vibrations A/h (Fig. 4). The digits 1, 2, 3 correspond to the respective test specimens of lengths $L = 150$ mm, $L = 200$ mm, and $L = 300$ mm. The computational OLD δ of the test specimens (solid lines) are sufficiently close to the experimental data δ^* (marked by small circles).

Figure 5 illustrates the dependencies $\delta(\varepsilon_0)$ for the steel St.3 for the coefficients c_0, c_1, c_2, and c_3 shown in Table 3. The dependencies $\delta(\varepsilon_0)$ obtained for the test specimens of lengths $L = 150$ mm (solid line) and $L = 200$ mm (dashed line) are sufficiently close and lie approximately in the same ranges of strain amplitudes ε_0, while the dependence $\delta(\varepsilon_0)$ for $L = 300$ mm (dotted line) differs from the two

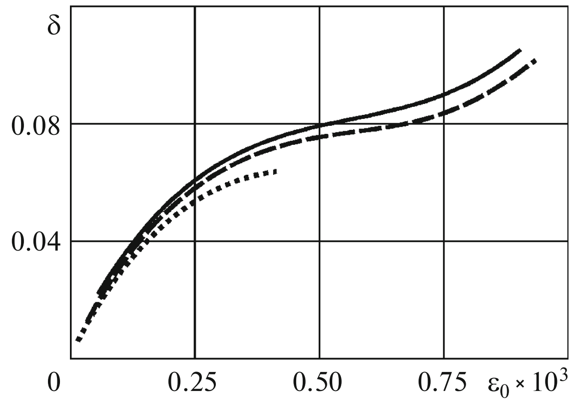

Fig. 6.

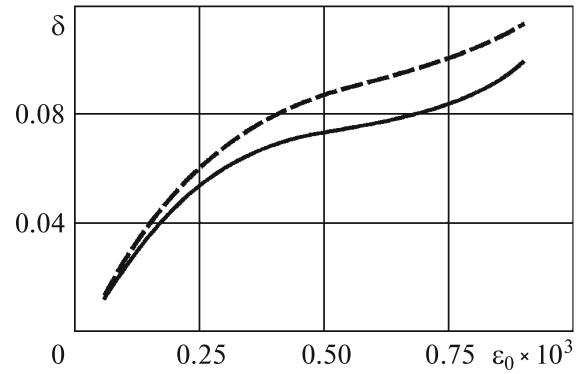

Fig. 7.

Table 3

L mm	c_0	c_1	c_2	c_3
150	5.467×10^{-3}	3.021×10^{2}	-3.534×10^{5}	1.622×10^{8}
200	-5.178×10^{-5}	4.092×10^{2}	-6.396×10^{5}	3.671×10^{8}
300	-2.266×10^{-5}	4.028×10^{2}	-2.995×10^{5}	-7.121×10^{8}

Table 4

$L = 150$ mm		$L = 200$ mm		$L = 300$ mm	
A, mm	δ^*	A, mm	δ^*	A, mm	δ^*
2	0.0290	2	0.0163	2	0.0073
4	0.0492	6	0.0401	6	0.0192
6	0.0583	10	0.0535	10	0.0294
8	0.0665	14	0.0627	14	0.0372
10	0.0726	18	0.0691	18	0.0437
12	0.0775	22	0.0737	22	0.0487

others. The final dependence $\delta(\varepsilon_0)$ was obtained by averaging the results over the test specimens of lengths $L = 150$ mm and $L = 200$ mm:

$$\delta(\varepsilon_0) = 2.708 \times 10^{-3} + 3.557 \times 10^{2}\varepsilon_0 - 4.965 \times 10^{5}\varepsilon_0^2 + 2.647 \times 10^{8}\varepsilon_0^3.$$

In what follows, we present the results of identification of the dependence $\delta(\varepsilon_0)$ for the same material, i.e., steel St.3, which were obtained for the same three test specimens of lengths $L = 150$ mm, $L = 200$ mm, and $L = 300$ mm and the width $b = 10$ mm. The aerodynamic component of the damping was eliminated by using formulas (1.1) and (1.2). Table 4 presents the obtained OLD δ^* of test specimens in a vacuum for the previously considered six amplitudes A.

Figure 6 illustrates the obtained dependencies $\delta(\varepsilon_0)$ for the steel St.3 test specimens of lengths $L = 150$ mm (solid line), $L = 200$ mm (dashed line), and $L = 300$ mm (dotted line). The final dependence $\delta(\varepsilon_0)$ is obtained by averaging the results over the first two test specimens

$$\delta(\varepsilon_0) = 2.519 \times 10^{-3} + 3.440 \times 10^{2}\varepsilon_0 - 5.523 \times 10^{5}\varepsilon_0^2 + 3.237 \times 10^{8}\varepsilon_0^3.$$

For comparison, Fig. 7 shows the obtained averages dependencies $\delta(\varepsilon_0)$ for the two versions of eliminating the aerodynamic component of the damping of the test specimens: (1) by extrapolating the OLD dependence in the air on the width b of the test specimen till the point $b = 0$ (dashed line); (2) by applying the computational component of the aerodynamic damping for the test specimen width $b = 10$ mm (solid line). In the second version of eliminating the aerodynamic component of the damping, the damping properties were by 12.7% lower at the average than in the first version.

CONCLUSION

It is shown that it is principally possible to identify the damping properties of rigid isotropic materials (illustrated by an example of the steel St.3) from the experimental amplitude dependence of the test specimen OLD in the air when the external aerodynamic component of the damping is eliminated from this dependence. The following two versions of eliminating this component are possible: (1) by extrapolating the OLD dependence in the air on the width b of the test specimen till the point $b = 0$; (2) by applying the computational component of the aerodynamic damping for the finite width b of the test specimen.

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