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On the Modeling of a Prestressed Thermoelectroelastic Half-Space with a Coating

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Abstract—The constitutive equations of nonlinear mechanics of a prestressed electrothermoelastic continuum are linearized in the framework of the theory of small strains imposed on finite strains. Simple and convenient-to-operate formulas of linearized constitutive equations and equations of motion of the medium are obtained. A model of electrothermoelastic half-space with inhomogeneous coating, which is a structure of functionally graded layers, is proposed. It is assumed that each of the medium components is under the action of initial mechanical strains and initial temperature, and the materials of the medium components are orthotropic pyroelectric materials of hexagonal crystal system of class 6 mm. The integral representation of the medium wave field is constructed by a hybrid numerical-analytical method based on a combination of analytical solutions and numerical schemes used to reconstruct the Green function for the inhomogeneous components of the coating and the matrix approach used to satisfy the boundary conditions.

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INTRODUCTION

For the mass usage of contemporary artificial inhomogeneous materials in spacecraft and aircraft construction, in production of high-technological equipment, and electronics, it is required to investigate their physical, technological, and strength characteristics depending on the regimes and conditions of their operation, which stimulated extensive experimental, fundamental and applied studies of the problem. For example, the appearance of first models of functionally graded materials (FGM), i.e., of materials whose properties vary continuously in a certain way, was first related to studies in seismology, foundation engineering, and geophysics [1–6]. The further development of structure modeling by using FGM components was first related to vigorous development of electronics and the possibility of increasing the efficiency of device operation based on the use of surface acoustic waves (Gulyaev–Bluestein waves, Rayleigh waves). Another direction in this field is related to the wide usage of synthetic materials (metal ceramics, bi- and multimaterials, various composites) in machine engineering and aircraft and spacecraft manufacturing. The problem complexity is that, for functionally graded materials or structures with functionally graded coating, it is impossible solve the dynamic problems analytically. The compromise assumption that all properties of the material vary at the same rate according to the same law of variation in the single spatial variable is often used in the literature [7–10]. The functionally graded material is modeled by various approaches from separation into layered elements where the material properties are linear [7] or quadratic functions of thickness to the methods where the functional dependence is represented either as an expansion in power series [8] or as easily differentiable exponential [9, 10], sinusoidal, hyperbolic, and quadratic functions or polynomials [10]. In these models, the properties of either one "supporting" material or of two materials are considered. In [11, 12], the model deals with a bi-material whose properties vary in depth from the values for one material to the values for the second material depending on the value, volume, and localization of the fractions of one

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material in the other material. The assumption that all properties of the material vary in the same way permits obtaining an analytical solution, which is undoubtedly important for estimating the results of complicated numerical or numerical-analytical modeling, but it can efficient be used to investigate the inhomogeneity of the material properties only in some specific cases.

The relationship between thermal, electric, and elastic fields in contemporary artificial high-technological materials ensures a mechanism for determining the thermomechanical perturbations depending on the character of the external load and on the method for the material manufacturing, i.e., on the initial mechanical stresses, induced electric potentials, temperature regimes, etc.

One of the first works, where the constitutive relations and the equations of motion of thermoelectric crystals and plates were obtained, the main theorems were formulated, and the physical laws of thermoelectric materials were studied are [13, 14]. On the basis of Mindlin's theory of thermopiezoelectricity, under the assumption that the thermal perturbations propagate at a finite speed, a generalized linear thermoelastic theory of piezoelectric media was constructed. When solving the dynamic coupled problems of generalized theory of electrothermoelasticity, the specific characteristics of surface wave propagation in semibounded electrothermoelastic media were studied in [15–20], where the pyroelectric materials of hexagonal crystal system of symmetry class 6 mm but the initial stresses were neglected. The influence of initial mechanical and temperature actions on the dynamics of a homogeneous layer of an anisotropic thermoelastic material was investigated in [21, 22]. The three-dimensional Green function of the medium was constructed [21], and the influence of the initial stresses on its dispersion properties was analyzed. The mixed problem of a layer vibrations under the action of a thermal load was solved [22], and the specific features of the thermal flow distribution in the region of contact depending on the character, type, and value of the initial actions were discovered. In [23], in the Lagrange coordinates related to the body natural configuration, the constitutive relations and the equations of motion of nonlinear mechanics of electrothermoelastic medium were successively linearized in the absence of external fields. In the process of linearization, the fourth-order terms with respect to strains and the second-order terms with respect to the temperature deviation were preserved in the expansion of the thermodynamic potential. Such an approach is justified when the influence of various types of the initial stress state on the dynamics of homogeneous electrothermoelastic materials is investigated, because, along with the linearized equations of motion and the constitutive relations determining the law of state of the medium, it is also necessary to consider the nonlinear effects due to the action of mechanical strains and temperatures. But when the specific features of behavior of compound structures and media with inhomogeneous coating are investigated, the higher-order constants taken into account complicate the problem significantly. In the present paper, we linearize the thermodynamic potential and preserve the second-order terms with respect to strains, electric field, and temperature deviations. We obtain simpler and more convenient formulas of linearized constitutive relations and equations of motion of the medium. To construct the Green matrix-function of the prestressed electrothermoelastic half-space with functionally graded coating, we use the hybrid numerical-analytical method, earlier proposed in [24, 25], which is based on a combination of analytical solutions for the homogeneous components of the coating with numerical schemes of reconstructing the Green function for the inhomogeneous components and the matrix approach used to satisfy the boundary conditions. By using the Fourier transform, we reduce solving systems of linearized equations of motion, i.e., systems of second-order partial differential equations with constant and variable coefficients, to solving systems of first-order ordinary differential equations with boundary and initial-boundary conditions with respect to the components of the displacement vector and normal components of the stress vector. The solution of a system of first-order ordinary differential equations is constructed numerically. Such an approach to modeling media with complex inhomogeneous coating allows us to use the distinctions between the initial stress states of the coating elements, and in functional dependence, between the intensity and localization domains of variations in their physical properties, as well as the distinctions between the conditions arising on the interfaces between the medium components.

1. STATEMENT OF THE PROBLEM OF NONLINEAR VIBRATIONS OF AN ELECTROTHERMOELASTIC MEDIUM

We consider an orthonormal Cartesian vector bases $\mathbf{i}_1$, $\mathbf{i}_2$, $\mathbf{i}_3$ and the reference v and actual V configurations before and after the action of surface and mass forces, respectively. The position of a material point in these configurations is determined by the radius vectors $\mathbf{r} = x_k\mathbf{i}_k$ and $\mathbf{R} = X_k\mathbf{i}_k$ and the nabla operator representation has the form

$$\nabla_0 = \mathbf{i}_m \frac{\partial}{\partial x_m}, \quad \nabla = \mathbf{i}_m \frac{\partial}{\partial X_m}. \tag{1.1}$$

The parameters determining the state of piezoactive medium are the electric potential φ and the vectors of the electric field intensity in the actual configuration

$$\mathbf{E} = -\nabla\varphi, \quad \mathbf{W} = -\nabla_0\varphi. \tag{1.2}$$

The medium strain is characterized by the strain gradient, the Cauchy–Green strain measure, and the Cauchy–Green strain tensor ($\mathbf{I}$ is the unit tensor),

$$\mathbf{C} = \nabla_0\mathbf{R}, \quad \mathbf{G} = \mathbf{C}\cdot\mathbf{C}^T, \quad \mathbf{S} = \frac{1}{2}(\mathbf{G} - \mathbf{I}). \tag{1.3}$$

The thermomechanical properties of the medium depend on the temperature θ and, in the reference configuration, are described by the Piola stress tensor $\mathbf{\Pi}$ and the specific entropy η:

$$\mathbf{\Pi} = \mathbf{P}\cdot\mathbf{C}, \quad \mathbf{P} = \boldsymbol{\chi}_\mathbf{S}, \tag{1.4}$$

$$\eta = -\chi_\theta. \tag{1.5}$$

Representation (1.4) contains the Kirchhoff tensor $\mathbf{P}$. The electrical properties of the medium in the reference configuration are described by the polarization vector

$$\boldsymbol{\pi} = -\boldsymbol{\chi}_\mathbf{W}, \tag{1.6}$$

by the material form of the vector of electric induction (ε_0 is the dielectric permeability of a vacuum and J is the metric multiplier)

$$\mathbf{d} = \varepsilon_0 J\mathbf{C}^{-T}\cdot\mathbf{E} - \boldsymbol{\chi}_\mathbf{W}, \tag{1.7}$$

and by the electric Piola–Maxwell tensor ($\mathbf{I}$ is the unit tensor)

$$\mathbf{m} = \varepsilon_0 J\mathbf{C}^{-T}\cdot\mathbf{\Xi}, \tag{1.8}$$

$$\mathbf{\Xi} = \mathbf{E}\mathbf{E} - \tfrac{1}{2}\mathbf{E}\cdot\mathbf{E}\mathbf{I}. \tag{1.9}$$

The thermal processes are described by the temperature gradient $\mathbf{g}$ and the heat flux vector $\mathbf{h}$ defined in the reference configuration metric:

$$\mathbf{g} = \nabla_0\theta, \tag{1.10}$$

$$\mathbf{h} = -\boldsymbol{\lambda}\cdot\mathbf{g}. \tag{1.11}$$

Here $\boldsymbol{\lambda}(\mathbf{C}, \theta, \mathbf{g})$ is the tensor of coefficients of thermal conductivity. In the case of material of class 6 mm of hexagonal crystal system, this tensor is diagonal, i.e., $\boldsymbol{\lambda} = \|\lambda_{ii}\|_{i=1}^3$, $\lambda_{11} = \lambda_{22} \neq \lambda_{33}$. The tensor $\boldsymbol{\chi}_\mathbf{S}$ the vector $\boldsymbol{\chi}_\mathbf{W}$, and the scalar quantity χ_θ which participate in representations (1.4)–(1.6) are the derivatives of the thermodynamic potential $\chi = \chi(\mathbf{S}, \mathbf{W}, \theta)$ which is a scalar function determining the energy accumulated in the deformation of a thermoelastic body [26–28].

Boundary Value Problem of Vibrations of Electrothermoelastic Medium in the Reference Configuration

We consider the problem of vibrations of an electrothermoelastic medium occupying a volume V bounded by the surface $o = o_1 + o_2 = o_3 + o_4 = o_5 + o_6$. We assume that the displacement vector $\mathbf{u}^*$ is given on the surface part o_1 and the mechanical stresses $\mathbf{f}^*$ are given on the other part o_2. The electric potential φ^* is given on the metallic part of the surface o_3 and the charge distribution g^* is given on the surface part o_4 which can also be partially metallized. The temperature distribution θ^* is given on the surface part o_5 and the heat flux h^* is given on the remaining part of the surface o_6. The boundary value problem of vibrations of the prestresses thermoelasic medium in the coordinates of the reference configuration is described by the equation of motion

$$\nabla_0\cdot(\mathbf{\Pi} + \mathbf{m}) = \rho_0\ddot{\mathbf{u}}, \tag{1.12}$$

the equation of forced electrostatics

$$\nabla_0 \cdot \mathbf{d} = 0, \tag{1.13}$$

the heat conduction equation

$$\nabla_0 \cdot \mathbf{h} - \rho_0 r + \theta\dot{\eta} = 0, \tag{1.14}$$

and the boundary conditions

$$\mathbf{u}\big|_{o_1} = \mathbf{u}^*, \quad \mathbf{n}\cdot\mathbf{\Pi}\big|_{o_2} = \mathbf{f}^*, \quad \varphi\big|_{o_3} = \varphi^*, \quad \mathbf{n}\cdot\mathbf{d}\big|_{o_4} = -g^*, \quad \theta\big|_{o_5} = \theta^*, \quad \mathbf{n}\cdot\mathbf{h}\big|_{o_6} = -h^*. \tag{1.15}$$

Equations (1.12) and (1.14) involve the undeformed body density ρ_0 and the intensity r of the bulk heat sources.

2. LINEARIZATION OF THE INITIAL STRESS STATE OF AN ELECTROTHERMOELASTIC BODY

We assume that there is a certain initial equilibrium of the electrothermoelastic body determined by the parameters

$$\mathbf{R} = \mathbf{R}(\mathbf{r}), \quad \varphi = \varphi_0(\mathbf{r}), \quad \theta = T_1(\mathbf{r}). \tag{2.1}$$

The stress state of the body is described by the system of equations

$$\nabla_0 \cdot (\mathbf{\Pi}_0 + \mathbf{m}_0) = 0, \quad \nabla_0 \cdot \mathbf{d}_0 = 0, \quad \nabla_0 \cdot \mathbf{h}_0 = 0. \tag{2.2}$$

We assume that, under the action of surface and mass forces, this configuration experiences small mechanical $\varepsilon\mathbf{u}$, electrical $\varepsilon\varphi$, and thermal $\varepsilon\theta$ perturbations, where ε is a small parameter. The perturbed state characteristics are denoted by the index $*$:

$$\mathbf{R}^\times = \mathbf{R} + \varepsilon\mathbf{u}, \quad \mathbf{w}^\times = \mathbf{w} + \varepsilon\mathbf{u}, \quad \varphi^\times = \varphi_0 + \varepsilon\varphi, \quad \theta^\times = T_1 + \varepsilon T. \tag{2.3}$$

Following [27], we write the tensor and vector quantities in the perturbed state as

$$\begin{aligned} &\mathbf{\Pi}^\times = \mathbf{\Pi}_0 + \varepsilon\mathbf{\Pi}^\bullet + o(\varepsilon^2), \quad \mathbf{m}^\times = \mathbf{m}_0 + \varepsilon\mathbf{m}^\bullet + o(\varepsilon^2), \quad \eta^\times = \eta_0 + \varepsilon\eta^\bullet + o(\varepsilon^2), \\ &\mathbf{d}^\times = \mathbf{d}_0 + \varepsilon\mathbf{d}^\bullet + o(\varepsilon^2), \quad \mathbf{h}^\times = \mathbf{h}_0 + \varepsilon\mathbf{h}^\bullet + o(\varepsilon^2), \end{aligned} \tag{2.4}$$

where $\mathbf{\Pi}^\bullet$, $\mathbf{m}^\bullet$, $\eta^\bullet$, $\mathbf{d}^\bullet$, and $\mathbf{h}^\bullet$ are the convective derivatives of the corresponding functions defined by the formula

$$\mathbf{f}^\bullet = \frac{d}{d\varepsilon}\mathbf{f}(\mathbf{R} + \varepsilon\mathbf{u}, \varphi_0 + \varepsilon\varphi, T_1 + \varepsilon T)\bigg|_{\varepsilon=0}. \tag{2.5}$$

The parameters (2.4) determining the perturbed state of the body (2.3) must satisfy Eqs. (1.12)–(1.14):

$$\nabla_0 \cdot (\mathbf{\Pi}^\times + \mathbf{m}^\times) = \rho_0\ddot{\mathbf{w}}6\times, \quad \nabla_0 \cdot \mathbf{d}^\times = 0, \quad \nabla_0 \cdot \mathbf{h}^\times + \theta^\times\dot{\eta}^\times = 0. \tag{2.6}$$

We introduce representations (2.4) into the system of Eqs. (2.6) and take (2.2) into account. We preserve only the terms linear in ε and obtain the linearized equations of motion, electrostatics, and heat conduction

$$\nabla_0 \cdot (\mathbf{\Pi}^\bullet + \mathbf{m}^\bullet) = \rho_0\ddot{\mathbf{u}}, \quad \nabla_0 \cdot \mathbf{d}^\bullet = 0, \quad \nabla_0 \cdot \mathbf{h}^\bullet + T_1\dot{\eta}^\bullet = 0. \tag{2.7}$$

We use the differentiation rules [27] with regard to representations (1.4)–(1.11) and formula (2.5) for the quantities participating in Eqs. (2.7) and obtain

$$\begin{aligned} &\mathbf{\Pi}^\bullet = \mathbf{P}^\bullet \cdot \mathbf{C} + \mathbf{P} \cdot \overset{\circ}{\nabla}\mathbf{u}, \quad \mathbf{m}^\bullet = (J\mathbf{C}^{-T})^\bullet \cdot \mathbf{M} + J\mathbf{C}^{-T} \cdot \mathbf{M}^\bullet, \\ &\mathbf{d}^\bullet = \varepsilon_0(J\mathbf{C}^{-T})^\bullet \cdot \mathbf{E} + \varepsilon_0 J\mathbf{C}^{-T} \cdot \mathbf{E}^\bullet + \boldsymbol{\pi}^\bullet, \\ &\mathbf{h}^\bullet = \frac{\partial\mathbf{h}}{\partial\mathbf{S}} \circ \mathbf{S}^\bullet + \frac{\partial\mathbf{h}}{\partial\mathbf{W}} \cdot \mathbf{W}^\bullet + \frac{\partial\mathbf{h}}{\partial\theta}\theta^\bullet + \frac{\partial\mathbf{h}}{\partial\mathbf{g}} \cdot \mathbf{g}^\bullet, \\ &\eta^\bullet = \frac{\partial\eta}{\partial\mathbf{S}} \circ \mathbf{S}^\bullet + \frac{\partial\eta}{\partial\mathbf{W}} \cdot \mathbf{W}^\bullet + \frac{\partial\eta}{\partial\theta}\theta^\bullet, \end{aligned} \tag{2.8}$$

$$\mathbf{P}^\bullet = \frac{\partial \mathbf{P}}{\partial \mathbf{S}} \circ \mathbf{S}^\bullet + \frac{\partial \mathbf{P}}{\partial \mathbf{W}} \cdot \mathbf{W}^\bullet + \frac{\partial \mathbf{P}}{\partial \theta}\theta^\bullet, \quad \mathbf{M}^\bullet = \varepsilon_0 \mathbf{E}^\bullet \mathbf{E} + \varepsilon_0 \mathbf{E}\mathbf{E}^\bullet - \varepsilon_0(\mathbf{E}\cdot\mathbf{E}^\bullet)\mathbf{I},$$
$$\mathbf{E}^\bullet = -\nabla\mathbf{u}\cdot\mathbf{E} - \nabla\varphi, \quad \boldsymbol{\pi}^\bullet = \frac{\partial \boldsymbol{\pi}}{\partial \mathbf{S}} \circ \mathbf{S}^\bullet + \frac{\partial \boldsymbol{\pi}}{\partial \mathbf{W}} \cdot \mathbf{W}^\bullet + \frac{\partial \boldsymbol{\pi}}{\partial \theta}\theta^\bullet. \tag{2.9}$$

The symbol "$\circ$" denotes the operation of complete multiplication [27]. The function $\chi = \chi(\mathbf{S}, \mathbf{W}, \theta)$ determines the energy accumulated in the deformation of electrothermoelastic body. The constitutive relations are constructed under the assumption that the state with parameters

$$\mathbf{S} = 0, \quad \mathbf{W} = 0, \quad \theta = T_0 \tag{2.10}$$

is the state with minimal free energy. In the expansion of the function $\chi = \chi(\mathbf{S}, \mathbf{W}, \theta)$ near the state (2.10), we preserve only the second-order terms with respect to strain, electric field, and the temperature deviation and obtain the thermodynamic potential representation [28]

$$\chi = \frac{1}{2}{}^4\mathbf{C}^{\mathbf{W}} \cdot\cdot\, \mathbf{S} \cdot\cdot\, \mathbf{S} - {}^3\mathbf{e}\cdot\mathbf{W} \cdot\cdot\, \mathbf{S} - \frac{1}{2}C_\varepsilon \rho_0 \tau^2 T_0^{-1} - {}^2\mathbf{Q} \cdot\cdot\, \mathbf{S}\tau - {}^1\mathbf{p}\cdot\mathbf{W}\tau. \tag{2.11}$$

Here ${}^4\mathbf{C}^{\mathbf{W}}$ is the tensor of rank IV of elastic constants of order II characterizing the linear strain at a constant temperature and a constant electric field, ${}^2\boldsymbol{\beta}$ is the symmetric tensor of rank II of dielectric susceptibility constants defined for constant temperature and strain, and for a material of class 6 mm of hexagonal crystal system, this tensor is diagonal [29]: ${}^2\boldsymbol{\beta} = \|\beta_{ii}\|_{i=1}^3$, $\beta_{11} = \beta_{22} \neq \beta_{33}$. In the linear approximation, its components are related to the components of the tensor of dielectric permeability ${}^2\boldsymbol{\varepsilon}$ by the relations $\varepsilon_{kn} = \varepsilon_0\delta_{kn} + \beta_{kn}$; ${}^3\mathbf{e}$ is the tensor of rank III of piezoelectric constants related to electroacoustic effects; ${}^2\mathbf{Q}$ is the tensor coefficient of thermoelasticity, and for a material of class 6 mm of hexagonal crystal system [29], this tensor has the form ${}^2\mathbf{Q} = \|q_{ii}\|_{i=1}^3$, $q_{11} = q_{22} \neq q_{33}$; ${}^1\mathbf{p}$ is the pyroelectric vector, C_ε is the specific heat capacity, ρ_0 is the material density, and $\tau = \theta - T_0$ is the temperature perturbation. The tensor constant of the thermodynamic potential are determined by the formulas

$${}^4\mathbf{C} = C_{ijkl}\mathbf{i}_i\mathbf{i}_j\mathbf{i}_k\mathbf{i}_l, \quad {}^3\mathbf{e} = e_{ijk}\mathbf{i}_i\mathbf{i}_j\mathbf{i}_k, \quad {}^2\boldsymbol{\beta} = \beta_{ij}\mathbf{i}_i\mathbf{i}_j, \quad \mathbf{Q} = Q_{ij}\mathbf{i}_i\mathbf{i}_j, \quad \boldsymbol{\lambda} = \lambda_{ij}\mathbf{i}_i\mathbf{i}_j, \quad \mathbf{p} = p_i\mathbf{i}_i. \tag{2.12}$$

Introducing (2.11) into expressions (1.4), (1.5), (1.7), and (1.9), we obtain

$$\mathbf{P} = {}^4\mathbf{C}^{\mathbf{W}} \cdot\cdot\, \mathbf{S} - {}^3\mathbf{e}\cdot\mathbf{W} - {}^2\mathbf{Q}\tau, \quad \boldsymbol{\pi} = {}^3\mathbf{e} \cdot\cdot\, \mathbf{S} + {}^2\boldsymbol{\beta}\cdot\mathbf{W} + {}^1\mathbf{p}\tau,$$
$$\eta = C_\varepsilon\rho_0\tau T_0^{-1} + {}^2\mathbf{Q} \cdot\cdot\, \mathbf{S} + {}^1\mathbf{p}\cdot\mathbf{W}. \tag{2.13}$$

The derivatives contained in formulas (2.8) and (2.9) have the form

$$\frac{\partial \mathbf{P}}{\partial \mathbf{S}} = {}^4\mathbf{C}^{\mathbf{W}}, \quad \frac{\partial \mathbf{P}}{\partial \mathbf{W}} = -{}^3\mathbf{e}, \quad \frac{\partial \mathbf{P}}{\partial \theta} = -{}^2\mathbf{Q},$$
$$\frac{\partial \boldsymbol{\pi}}{\partial \mathbf{S}} = {}^3\mathbf{e}, \quad \frac{\partial \boldsymbol{\pi}}{\partial \mathbf{W}} = {}^2\boldsymbol{\beta}, \quad \frac{\partial \boldsymbol{\pi}}{\partial \theta} = {}^1\mathbf{p}, \quad \frac{\partial \eta}{\partial \theta} = \frac{C_\varepsilon\rho_0}{T_0}. \tag{2.14}$$

Substituting expressions (2.13) and (2.14) into formulas (2.8) with regard to expressions (2.9) and (1.6)–(1.11), we obtain

$$\boldsymbol{\Pi}^\bullet = ({}^4\mathbf{C}^{\mathbf{W}} \circ \mathbf{S}^\bullet + {}^3\mathbf{e}\cdot\nabla_0\varphi - {}^2\mathbf{Q}T)\cdot\mathbf{C} + \mathbf{P}\cdot\nabla_0\mathbf{u},$$
$$\mathbf{m}^\bullet = \varepsilon_0 J\mathbf{C}^{-T}\cdot[((\nabla\cdot\mathbf{u})\mathbf{I} - \nabla\mathbf{u}^T)\cdot\boldsymbol{\Xi} + \boldsymbol{\Xi}^\bullet],$$
$$\mathbf{d}^\bullet = \varepsilon_0 J\mathbf{C}^{-T}[((\nabla\cdot\mathbf{u})\mathbf{I} - \nabla\mathbf{u}^T)\cdot\mathbf{E} - \nabla\mathbf{u}\cdot\mathbf{E} - \nabla\varphi] + \pi^\bullet, \tag{2.15}$$
$$\boldsymbol{\pi}^\bullet = {}^3\mathbf{e}\circ\mathbf{S}^\bullet - {}^2\boldsymbol{\beta}\cdot\nabla_0\varphi + {}^1\mathbf{p}T, \quad \mathbf{h}^\bullet = -\lambda\cdot\nabla_0 T,$$
$$\eta^\bullet = {}^2\mathbf{Q}\circ\mathbf{S}^\bullet - {}^1\mathbf{p}\cdot\nabla_0\varphi + C_\varepsilon\rho_0 T T_0^{-1}.$$

Here we used the following differentiation formulas [27]:

$$(J\mathbf{C}^{-T})^\bullet = J\mathbf{C}^{-T}\cdot((\nabla\cdot\mathbf{u})\mathbf{I} - \nabla\mathbf{u}^T), \quad \nabla = \mathbf{C}^{-1}\cdot\nabla_0,$$
$$\frac{\partial \eta}{\partial \mathbf{S}} = \frac{\partial}{\partial \mathbf{S}}\left(-\frac{\partial \chi}{\partial \theta}\right) = -\frac{\partial \mathbf{P}}{\partial \theta}, \quad \mathbf{S}^\bullet = \frac{1}{2}(\nabla_0\mathbf{u}\cdot\mathbf{C}^T + \mathbf{C}\cdot\nabla_0\mathbf{u}^T),$$

$$J^\bullet = J\nabla\cdot\mathbf{u},\quad \mathbf{C}^\bullet = \nabla_0\mathbf{u},\quad (\mathbf{C}^{-1})^\bullet = -\nabla\mathbf{u}\cdot\mathbf{C}^{-1},$$
$$(\mathbf{C}^{-T})^\bullet = -\mathbf{C}^{-T}\cdot\nabla\mathbf{u}^T,\quad \mathbf{W}^\bullet = -\nabla_0\varphi,\quad \theta^\bullet = T,\quad \mathbf{g}^\bullet = \nabla_0 T.$$

Further, we introduce the notation

$$\mathbf{\Theta} = \mathbf{\Pi}^\bullet + \mathbf{m}^\bullet,\quad \mathbf{\Delta} = \mathbf{d}^\bullet,\quad \mathbf{H} = \mathbf{h}^\bullet,\quad v = \eta^\bullet..$$

In this notation, the linearized equations of motion, electrostatics, and heat conduction (2.7), which describe the dynamics of prestressed electrothermoelastic body, become

$$\nabla_0\cdot\mathbf{\Theta} = \rho_0\ddot{\mathbf{u}},\quad \nabla_0\cdot\mathbf{\Delta} = 0,\quad \nabla_0\cdot\mathbf{H} + T_0\dot{v} = 0. \tag{2.16}$$

Equations (2.16) were constructed in general form with respect to the initial stress state. In what follows, we specify it and consider the homogeneous initial stress state.

Homogeneous Initial Stress State

We assume that the initial stress state (2.1) in an electrothermoelastic medium is homogeneous and determined by the conditions

$$\mathbf{R} = \mathbf{\Lambda}\cdot\mathbf{r},\quad \mathbf{\Lambda} = v_k\mathbf{i}_k\mathbf{i}_k,\quad \theta = T_1. \tag{2.17}$$

In condition (2.17), $v_k = 1 + \delta_k$, where δ_k $(k = 1,2,3)$ are relative elongations of the fibers directed along the coordinate axes x_k coinciding in the natural configuration with the Cartesian coordinates. We also assume that the external electrostatic field is absent, and the electric field intensity in the material is small and appears exceptionally due to the piezo- or pyroeffect. In this case, the terms which contain the vector $\mathbf{E}$ as a multiplier can be neglected in formulas (2.15). With regard to formulas (1.1)–(1.3), representations (2.11), (2.12), (2.17), and notation (2.16), the stress tensor, the induction and heat flux vectors, which participate in linearized equations (2.16), can be written in component form

$$\mathbf{\Theta} = \Pi^*_{ij}\mathbf{i}_i\mathbf{i}_j,\quad \mathbf{\Delta} = d^*_i\mathbf{i}_i,\quad \mathbf{H} = h^*_i\mathbf{i}_i,$$
$$\Pi^*_{ij} = \left(C_{ijkl}v_k\frac{\partial u_k}{\partial x_l} + e_{ijk}\frac{\partial\varphi}{\partial x_k} - q_{ij}T\right)v_j + P_{ik}\frac{\partial u_j}{\partial x_k}, \tag{2.18}$$
$$d^*_i = e_{ijk}v_ju_{j,k} - (\beta_{ii} + \varepsilon_0 J v_i^{-2})\varphi_{,i} + p_iT, \tag{2.19}$$
$$h^*_i = -\lambda_{ii}T_{,i}, \tag{2.20}$$
$$\eta^\bullet = q_{ii}v_i\frac{\partial u_i}{\partial x_i} - p_i\frac{\partial\varphi}{\partial x_i} + C_\varepsilon\rho_0 T T_0^{-1}. \tag{2.21}$$

It follows from (2.17) that the tensors determining the initial stress state are diagonal:

$$\mathbf{C} = v_k\mathbf{i}_k\mathbf{i}_k,\quad \mathbf{S} = S_k\mathbf{i}_k\mathbf{i}_k,\quad S_k = \tfrac{1}{2}(v_k^2 - 1),\quad \mathbf{P} = P_k\mathbf{i}_k\mathbf{i}_k.$$

If we introduce the notation

$$C^*_{ijkl} = C_{ijkl}v_kv_j + P_{il}\delta_{jk},\quad e^*_{ijk} = e_{ijk}v_j,\quad q^*_{ij} = q_{ij}v_j,\quad \beta^*_{ii} = \beta_{ii} + \varepsilon_0 J v_i^{-2}, \tag{2.22}$$

then representations (2.18) and (2.19) become

$$\Pi^*_{ij} = C^*_{ijkl}\frac{\partial u_k}{\partial x_l} + e^*_{ijk}\frac{\partial\varphi}{\partial x_k} - q^*_{ij}T, \tag{2.23}$$
$$d^*_i = e^*_{ijk}\frac{\partial u_j}{\partial x_k} - \beta^*_{ii}\frac{\partial\varphi}{\partial x_i} + p_iT. \tag{2.24}$$

With representations (2.23), (2.24), (2.20), and (2.21) taken into account, Eqs. (2.7) in component form become

$$\begin{gathered}\frac{\partial}{\partial x_i}\left(C^*_{ijkl}\frac{\partial u_k}{\partial x_l} + e^*_{ijk}\frac{\partial\varphi}{\partial x_k} - q^*_{ij}T\right) = \rho_0\frac{\partial^2 u_j}{\partial t^2},\quad j = 1,2,3,\\ \frac{\partial}{\partial x_i}\left(e^*_{ijk}\frac{\partial u_j}{\partial x_k} - \beta^*_{ii}\frac{\partial\varphi}{\partial x_i} + p_iT\right) = 0,\\ \frac{\partial}{\partial x_i}\left(\lambda_{ii}\frac{\partial T}{\partial x_i}\right) - T_1 q^*_{ii}\frac{\partial}{\partial t}\left(\frac{\partial u_i}{\partial x_i}\right) + T_1 p_i\frac{\partial}{\partial t}\left(\frac{\partial\varphi}{\partial x_i}\right) - T_1\frac{C_\varepsilon\rho_0}{\theta_0}\frac{\partial T}{\partial t} = 0.\end{gathered} \tag{2.25}$$

Equations (2.25) describe the vibrations of a prestressed electrothermoelastic medium and permit studying the influence of initial stresses on the dynamic processes in this medium under the conditions of coupled elastic, electric, and thermal fields.

3. DYNAMIC PROBLEM FOR A LAMINAR INHOMOGENEOUS PRESTRESSED ELECTROTHERMOELASTIC HALF-SPACE

As an example, we consider harmonic vibrations of na inhomogeneous prestressed medium consisting of a package $N = M - 1$ of inhomogeneous layers $0 \le x_3 \le H$, $H = h_1 \ge h_2 \ge \ldots \ge h_M = 0$, $|x_1, x_2| \le \infty$, lying on an inhomogeneous half-space $x_3 \le 0$, $|x_1, x_2| \le \infty$. We assume that the half-space and all its structure elements are pyroelectrics of class 6 mm of hexagonal crystal system. The initial stress state of the medium components is homogeneous and determined by conditions (2.17); the external electric field is absent.

The linearized equations for the compound electrothermoelastic medium have the form

$$\nabla_0 \cdot \mathbf{\Theta}^{(n)} = \rho^{(n)}\ddot{\mathbf{u}}^{(n)}, \quad \nabla_0 \cdot \mathbf{\Delta}^{(n)} = 0, \quad \nabla_0 \cdot \mathbf{H}^{(n)} + T_0\dot{\eta}^{(n)} = 0. \tag{3.1}$$

The boundary conditions (1.15) in the linearized form are represented by the expressions

$$\begin{aligned} &\mathbf{u}^{(1)} = \mathbf{u}^*\big|_{o_1}, \quad \mathbf{n} \cdot \mathbf{\Theta}^{(1)} = \mathbf{f}^*\big|_{o_2}, \\ &\varphi^{(1)} = \varphi^*\big|_{o_3}, \quad \mathbf{n} \cdot \mathbf{\Delta}^{(1)} = -d^*\big|_{o_4}, \\ &\theta^{(1)} = \theta^*\big|_{o_5}, \quad \mathbf{n} \cdot \mathbf{H}^{(1)} = -h^*\big|_{o_6}. \end{aligned} \tag{3.2}$$

Here $\mathbf{\Theta}^{(n)}$, $\mathbf{\Delta}^{(n)}$, $\mathbf{H}^{(n)}$, and $\eta^{(n)}$ are the linearized stress tensor and the vectors of electric induction, heat flux, and entropy of the layer with number n ($n = 1, \ldots, N$, $N = M - 1$) or of the half-space ($n = M$) defined by formulas (2.20), (2.21), (2.23), and (2.24). The quantities $\mathbf{u}^*$, $\mathbf{f}^*$, and $\mathbf{n}$ contained in conditions (3.2) are the respective vectors of displacements, stresses, and the outer normal to the medium surface, which are defined in natural coordinates, $\rho^{(n)}$ is the material density, $\theta^{(n)}$ is the temperature distribution in the layer with number n, and d^*, φ^*, h^*, and T^* are the charge density distribution and the electric potentia, the heat flux and the temperature on the medium interface.

We assume that all parameters of the functionally graded components of the coating obey the law

$$\begin{aligned} &\rho^{(n)} = \rho_0^{(n)} f_\rho^{(n)}(x_3), \quad c_{lksm}^{(n)} = c_{lksm}^{0(n)} f_c^{(n)}(x_3), \quad e_{lkm}^{(n)} = e_{lkm}^{0(n)} f_e^{(n)}(x_3), \quad \varepsilon_{lm}^{(n)} = \varepsilon_{lm}^{0(n)} f_\varepsilon^{(n)}(x_3), \\ &q_{lk}^{(n)} = q_{lk}^{0(n)} f_q^{(n)}(x_3), \quad p_l^{(n)} = p_l^{0(n)} f_p^{(n)}(x_3), \quad \lambda_{lk}^{(n)} = \lambda_{lk}^{0(n)} f_\lambda^{(n)}(x_3), \quad c_\varepsilon^{(n)} = c_\varepsilon^{0(n)} f_{c_\varepsilon}^{(n)}(x_3), \end{aligned} \tag{3.3}$$

where the index “0” denotes the constants of some “reference” material.

Under the above assumptions, with representations (2.20)–(2.24) taken into account, the components of the tensor $\mathbf{\Theta}^{(n)}$, vectors $\mathbf{\Delta}^{(n)}$ and $\mathbf{H}^{(n)}$, and the scalar quantity $\eta^{(n)}$ are determined by the formulas (differentiation with respect to coordinates is denoted by indices after the comma)

$$\begin{aligned} &\Theta_{lk}^{(n)} = c_{lksm}^{*(n)} u_{s,m}^{(n)} + e_{lkm}^{*(n)} \varphi_{,m}^{(n)} - q_{lk}^{*(n)} T^{(n)}, \quad d_l^{(n)} = e_{lsm^{*(n)}} u_{s,m}^{(n)} + \varepsilon_{lm}^{*(n)} \varphi_{,m}^{(n)} + p_l^{(n)} T^{(n)}, \\ &h_l^{(n)} = -\lambda_{ll}^{(n)} T_{,l}^{(n)}, \quad \eta^{(n)} = q_{sp}^{*(n)} u_{s,p}^{(n)} - p_m^{(n)} \varphi_{,m}^{(n)} + \rho^{(n)} c_\varepsilon^{(n)} T^{(n)} T_0^{-1}, \end{aligned} \tag{3.4}$$

$$\begin{aligned} &c_{lksp}^{*(n)} = P_{lp}^{(n)} \delta_{ks} + \nu_k^{(n)} \nu_s^{(n)} c_{lksp}^{(n)}, \quad e_{lsp}^{*(n)} = \nu_s^{(n)} e_{lsp}^{(n)}, \quad q_{lk}^{*(n)} = \nu_k^{(n)} q_{lk}^{(n)}, \\ &\varepsilon_{lp}^{*(n)} = \varepsilon_0 \nu_1^{(n)} \nu_2^{(n)} \nu_3^{(n)} (\nu_l^{(n)})^{-2} \delta_{lp} + \beta_{lp}^{(n)}. \end{aligned} \tag{3.5}$$

In the framework of the homogeneous initial stress state (2.17), the components of the tensor $P_{ij}^{(n)}$ contained in representation (3.5) determine the initial stress states of the medium component with number n:

$$P_{ij}^{(n)} = c_{ijkk}^{(n)} S_{kk}^{(n)} - e_{ijk}^{(n)} W_k^{(n)} - (T_1^{(n)} - T_0) q_{ij}^{(n)}. \tag{3.6}$$

The initial stress state of the thermoelectroelastic medium determines the vector of electric induction which can be represented in material form as

$$d_k^{(n)} = e_{kll}^{(n)} S_{ll}^{(n)} + (\varepsilon_0 \nu_1^{(n)} \nu_2^{(n)} \nu_3^{(n)} (\nu_l^{(n)})^{-2} \delta_{mk} + \beta_{mk}^{(n)}) W_m^{(n)} + p_k^{(n)} (\theta^{(n)} - T_0), \quad k = 1, 2, 3. \tag{3.7}$$

It follows from conditions (2.7) that the components $P_{kk}^{(n)}$ in formulas (3.6) are different from zero. The absence of the initial electrostatic action $\mathbf{d}^{(n)} = 0$ (3.7) implies the condition $W_1^{(n)} = W_2^{(n)} = 0$ for the materials of the initial symmetry class 6 mm. Thus, to determine the initial stress state parameters of the nth component of the medium, we have four equations connecting three components of the stress tensor, three components of the strain tensor, one component of the electric field intensity, and one component of the electric induction vector.

In what follows, we introduce the extended vectors of displacement $\mathbf{u}_{e\tau}^{(n)} = \{u_1^{(n)}, u_2^{(n)}, u_3^{(n)}, u_4^{(n)}, u_5^{(n)}\}$ ($u_4^{(n)} = \varphi^{(n)}$, $u_5^{(n)} = T^{(n)}$) and of load $\mathbf{f}_{e\tau} = \{f_1, f_2, f_3, f_4, f_5\}$ ($f_4 = -d^*$, $f_5 = -h^*$) and use the notation

$$
\begin{aligned}
&\theta_{lksp}^{(n)} = c_{lksp}^{*(n)}, \quad \theta_{lk4p}^{(n)} = v_k^{(n)} e_{plk}^{(n)}, \quad \theta_{l4sp}^{(n)} = v_s^{(n)} e_{lsp}^{*(n)}, \quad \theta_{l44p}^{(n)} = -\varepsilon_{lp}^{*(n)}, \quad k,l,s,p = 1,2,3, \\
&\theta_{lk55}^{(n)} = -q_{lk}^{*(n)}, \quad \theta_{l555}^{(n)} = p_l^{(n)}, \quad \theta_{5555}^{(n)} = -c_\varepsilon^{(n)} \rho^{(n)} T_0^{-1}.
\end{aligned}
\tag{3.8}
$$

Representations (3.5) and (3.8) and the properties of the "reference" material in natural state [29] imply the relations

$$
\theta_{2342}^{(n)} = \theta_{1341}^{(n)} = \theta_{1431}^{(n)} = \theta_{2432}^{(n)}, \quad \theta_{1143}^{(n)} = \theta_{3411}^{(n)}, \quad \theta_{2243}^{(n)} = \theta_{3422}^{(n)}, \quad \theta_{3343}^{(n)} = \theta_{3433}^{(n)}.
$$

In notation (3.8), the conservation law (3.4) can be represented as the matrix

$$
\left(
\begin{array}{c|ccccccccc:ccc:c}
* & u_{1,1}^{(n)} & u_{2,2}^{(n)} & u_{3,3}^{(n)} & u_{2,3}^{(n)} & u_{3,2}^{(n)} & u_{1,3}^{(n)} & u_{3,1}^{(n)} & u_{1,2}^{(n)} & u_{2,1}^{(n)} & u_{4,1}^{(n)} & u_{4,2}^{(n)} & u_{4,3}^{(n)} & u_5^{(n)} \\
\hline
\Theta_{11}^{(n)} & \theta_{1111}^{(n)} & \theta_{1122}^{(n)} & \theta_{1133}^{(n)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{1143}^{(n)} & \theta_{1155}^{(n)} \\
\Theta_{22}^{(n)} & \theta_{1122}^{(n)} & \theta_{2222}^{(n)} & \theta_{2233}^{(n)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{2243}^{(n)} & \theta_{2255}^{(n)} \\
\Theta_{33}^{(n)} & \theta_{1133}^{(n)} & \theta_{2233}^{(n)} & \theta_{3333}^{(n)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{3343}^{(n)} & \theta_{3355}^{(n)} \\
\Theta_{23}^{(n)} & 0 & 0 & 0 & \theta_{2323}^{(n)} & \theta_{2332}^{(n)} & 0 & 0 & 0 & 0 & 0 & \theta_{2342}^{(n)} & 0 & 0 \\
\Theta_{32}^{(n)} & 0 & 0 & 0 & \theta_{3223}^{(n)} & \theta_{2323}^{(n)} & 0 & 0 & 0 & 0 & 0 & \theta_{3242}^{(n)} & 0 & 0 \\
\Theta_{13}^{(n)} & 0 & 0 & 0 & 0 & 0 & \theta_{1313}^{(n)} & \theta_{1331}^{(n)} & 0 & 0 & \theta_{1341}^{(n)} & 0 & 0 & 0 \\
\Theta_{31}^{(n)} & 0 & 0 & 0 & 0 & 0 & \theta_{3113}^{(n)} & \theta_{1313}^{(n)} & 0 & 0 & \theta_{3141}^{(n)} & 0 & 0 & 0 \\
\Theta_{12}^{(n)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{1212}^{(n)} & \theta_{1221}^{(n)} & 0 & 0 & 0 & 0 \\
\Theta_{21}^{(n)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{2112}^{(n)} & \theta_{1212}^{(n)} & 0 & 0 & 0 & 0 \\
\hdashline
d_1^{(n)} & 0 & 0 & 0 & 0 & 0 & \theta_{1413}^{(n)} & \theta_{1431}^{(n)} & 0 & 0 & \theta_{1441}^{(n)} & 0 & 0 & 0 \\
d_2^{(n)} & 0 & 0 & 0 & \theta_{2423}^{(n)} & \theta_{2432}^{(n)} & 0 & 0 & 0 & 0 & 0 & \theta_{2442}^{(n)} & 0 & 0 \\
d_3^{(n)} & \theta_{3411}^{(n)} & \theta_{3422}^{(n)} & \theta_{3433}^{(n)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{3443}^{(n)} & \theta_{3555}^{(n)} \\
\hdashline
-\eta^{(n)} & \theta_{1155}^{(n)} & \theta_{2255}^{(n)} & \theta_{3355}^{(n)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{3555}^{(n)} & \theta_{5555}^{(n)}
\end{array}
\right).
\tag{3.9}
$$

We substitute expressions (3.4) with regard to representations (3.3) and the conservation law (3.9) into the system of Eqs. (3.1) and rewrite is in the form

$$
\begin{cases}
\mathbf{L}_{11}^{*\mathbf{f}}[u_1^{(n)}] + \theta_1^{(n)} u_{2,12}^{(n)} + \mathbf{L}_{13}^{*\mathbf{f}}[u_3^{(n)}] + \mathbf{L}_{14}^{*\mathbf{f}}[u_4^{(n)}] + \theta_{1155}^{(n)} u_{5,1}^{(n)} = 0, \\
\theta_1^{(n)} u_{1,12}^{(n)} + \mathbf{L}_{22}^{*\mathbf{f}}[u_2^{(n)}] + \mathbf{L}_{23}^{*\mathbf{f}}[u_3^{(n)}] + \mathbf{L}_{24}^{*\mathbf{f}}[u_4^{(n)}] + \theta_{2255}^{(n)} u_{5,2}^{(n)} = 0, \\
\mathbf{L}_{31}^{*\mathbf{f}}[u_1^{(n)}] + \mathbf{L}_{32}^{*\mathbf{f}}[u_2^{(n)}] + \mathbf{L}_{33}^{*\mathbf{f}}[u_3^{(n)}] + \mathbf{L}_{34}^{*\mathbf{f}}[u_4^{(n)}] + \mathbf{L}_{35}^{*\mathbf{f}}[u_5^{(n)}] = 0, \\
\mathbf{L}_{41}^{*\mathbf{f}}[u_1^{(n)}] + \mathbf{L}_{42}^{*\mathbf{f}}[u_2^{(n)}] + \mathbf{L}_{34}^{*\mathbf{f}}[u_3^{(n)}] + \mathbf{L}_{44}^{*\mathbf{f}}[u_4^{(n)}] + \mathbf{L}_{45}^{*\mathbf{f}}[u_5^{(n)}] = 0, \\
i\omega T_1^{(n)} [\theta_{1155}^{(n)} u_{1,1}^{(n)} + \theta_{2255}^{(n)} u_{2,2}^{(n)} + \theta_{3355}^{(n)} u_{3,3}^{(n)} + \theta_{3555}^{(n)} u_{4,3}^{(n)}] - \mathbf{L}_{55}^{*\mathbf{f}}[u_5^{(n)}] = 0.
\end{cases}
\tag{3.10}
$$

The problem is supplemented with the boundary conditions. On the surface $x_3 = h$, we have

$$\begin{aligned} &u_k^{(1)} = u_k^*(x_1, x_2), \quad (x_1, x_2) \in o_1, \quad \Theta_{3k}^{(1)} = f_k(x_1, x_2), \quad k = 1, 2, 3, \quad (x_1, x_2) \in o_2, \\ &u_5^{(1)} = \varphi^*(x_1, x_2), \quad (x_1, x_2) \in o_3, \quad d_3^{(1)} = f_4(x_1, x_2), \quad (x_1, x_2) \in o_4, \\ &u_5^{(1)} = T^*(x_1, x_2), \quad (x_1, x_2) \in o_5, \quad \lambda_{33}^{(1)} u_{5,3}^{(1)} = f_5(x_1, x_2), \quad (x_1, x_2) \in o_6. \end{aligned} \tag{3.11}$$

On the interface $x_3 = h_k$, $k = 2, \ldots, M$, we have

$$\Sigma_k^{(n)} = \Sigma_k^{(n+1)}, \quad u_k^{(n)} = u_k^{(n+1)} \quad (k = 1, \ldots, 5 \; n = 2, \ldots, M-1). \tag{3.12}$$

At infinity, we have

$$x_3 \to -\infty \quad u_k^{(M)} \downarrow 0. \tag{3.13}$$

In formulas (3.10)–(3.12), we used the notation

$$\theta_1^{(n)} = \theta_{1122}^{(n)} + \theta_{1212}^{(n)}, \tag{3.14}$$

$$\mathbf{L}_{kk}^{*\mathbf{f}} = \mathbf{L}_{kk}^* + \frac{\partial \theta_{3kk3}^{(n)}}{\partial x_3} \frac{\partial}{\partial x_3} \quad (k = 1, 2, 3), \quad \mathbf{L}_{44}^{*\mathbf{f}} = \mathbf{L}_{44}^* + \frac{\partial \theta_{3443}^{(n)}}{\partial x_3} \frac{\partial}{\partial x_3} \quad \mathbf{L}_{55}^{*\mathbf{f}} = \mathbf{L}_{55}^* + \frac{\partial \lambda_{33}^{(n)}}{\partial x_3} \frac{\partial}{\partial x_3},$$

$$\mathbf{L}_{s3}^{*\mathbf{f}} = (\theta_{ss33}^{(n)} + \theta_{s3s3}^{(n)}) \frac{\partial^2}{\partial x_s \partial x_3} + \frac{\partial \theta_{s3s3}^{(n)}}{\partial x_3} \frac{\partial}{\partial x_s}, \quad \mathbf{L}_{s4}^{*\mathbf{f}} = (\theta_{ss43}^{(n)} + \theta_{3s4s}^{(n)}) \frac{\partial^2}{\partial x_s \partial x_3} + \frac{\partial \theta_{3s4s}^{(n)}}{\partial x_3} \frac{\partial}{\partial x_s},$$

$$\mathbf{L}_{3s}^{*\mathbf{f}} = (\theta_{ss33}^{(n)} + \theta_{s3s3}^{(n)}) \frac{\partial^2}{\partial x_s \partial x_3} + \frac{\partial \theta_{ss33}^{(n)}}{\partial x_3} \frac{\partial}{\partial x_s}, \quad \mathbf{L}_{4s}^{*\mathbf{f}} = (\theta_{ss43}^{(n)} + \theta_{3s4s}^{(n)}) \frac{\partial^2}{\partial x_s \partial x_3} + \frac{\partial \theta_{34ss}^{(n)}}{\partial x_3} \frac{\partial}{\partial x_s}, \quad s = 1, 2,$$

$$\mathbf{L}_{34}^{*\mathbf{f}} = \mathbf{L}_{34}^* + \frac{\partial \theta_{3343}^{(n)}}{\partial x_3} \frac{\partial}{\partial x_3}, \quad \mathbf{L}_{35}^{*\mathbf{f}} = \theta_{3355}^{(n)} \frac{\partial}{\partial x_3} + \frac{\partial \theta_{3355}^{(n)}}{\partial x_3}, \quad \mathbf{L}_{45}^{*\mathbf{f}} = \theta_{3555}^{(n)} \frac{\partial}{\partial x_3} + \frac{\partial \theta_{3555}^{(n)}}{\partial x_3},$$

$$\mathbf{L}_{kk}^* = \theta_{ikki}^{(n)} \frac{\partial^2}{\partial x_i^2} + \rho^{(n)} \omega^2 \quad (k, i = 1, 2, 3),$$

$$\mathbf{L}_{34}^* = \theta_{k34k}^{(n)} \frac{\partial^2}{\partial x_k^2}, \quad \mathbf{L}_{44}^* = \theta_{k44k}^{(n)} \frac{\partial^2}{\partial x_k^2}, \quad \mathbf{L}_{55}^* = \lambda_{kk}^{(n)} \frac{\partial^2}{\partial x_k^2} - i\omega T_1^{(n)} \theta_{5555}^{(n)} \quad (k = 1, 2, 3),$$

$$\Sigma_{\mathbf{e\tau}}^{(n)} = \{\Sigma_k^{(n)}\}_{k=1}^5, \quad \Sigma_p^{(n)} = \Theta_{3p}^{(n)} \quad (p = 1, 2, 3), \quad \Sigma_4^{(n)} = d_3^{(n)}, \quad \Sigma_5^{(n)} = -h_3^{(n)} = \lambda_{33}^{(n)} u_{5,3}^{(n)}. \tag{3.15}$$

Further, we pass to dimensionless normalized parameters [17, 18]:

$$x_i' = \frac{\omega^* x_i}{V_p^{(M)}}, \quad u_i^{(n)\prime} = \frac{\rho^{(M)} \omega^* V_p^{(M)}}{q_{11}^{(M)} T_0} u_i^{(n)}, \quad T^{(n)\prime} = \frac{T^{(n)}}{T_0}, \quad \varphi^{(n)\prime} = \frac{\varphi^{(n)}}{\varphi_0}, \quad \omega' = \frac{\omega}{\omega^*},$$

$$\omega^* = \frac{c_\varepsilon^{(M)} c_{11}^{(M)}}{\lambda_{11}^{(M)}}, \quad T_1^{(n)\prime} = \frac{T_1^{(n)}}{T_0}, \quad \lambda_{ij}^{(n)\prime} = \frac{\lambda_{ij}^{(n)}}{\lambda_{11}^{(M)}}, \quad \rho' = \frac{\rho}{\rho_0}, \quad c_\varepsilon' = \frac{c_\varepsilon}{c_\varepsilon^0},$$

$$\Theta_{ij}^{(n)\prime} = \frac{\Theta_{ij}^{(n)}}{q_{11}^{(M)} T_0}, \quad d_i^{(n)\prime} = \frac{c_{11}^{(M)}}{q_{11}^{(M)} T_0 e_{33}^{(M)}} d_i^{(n)}, \quad h_i^{(n)\prime} = \frac{V_p^{(M)}}{\omega^* T_0 \lambda_{11}^{(M)}} h_i^{(n)} \quad (i, j = 1, 2, 3),$$

$$\theta_{ijkl}^{(n)\prime} = \frac{\theta_{ijkl}^{(n)}}{c_{11}^{(M)}} \quad (k, l = 1, 2, 3), \quad \theta_{ik4p}^{(n)\prime} = \frac{\theta_{ik4p}^{(n)}}{e_{33}^{(M)}}, \quad \theta_{i4kp}^{(n)\prime} = \frac{\theta_{i4kp}^{(n)}}{e_{33}^{(M)}} \quad (i, k, p = 1, 2, 3),$$

$$\theta_{k44k}^{(n)\prime} = \frac{\theta_{k44k}^{(n)}}{e_{33}^{(M)}}, \quad \theta_{kk55}^{(n)\prime} = \frac{\theta_{kk55}^{(n)}}{e_{33}^{(M)}} \quad (k = 1, 2, 3), \quad \theta_{3555}^{(n)\prime} = \frac{c_{11}^{(M)}}{q_{11}^{(M)} e_{33}^{(M)}} p_3^{(n)}, \quad \theta_{5555}^{(n)\prime} = \frac{\rho^{(n)} c_\varepsilon^{(n)}}{T_0 \rho^{(M)} c_\varepsilon^{(M)}},$$

$$E = \frac{T_0 (q_{11}^{(M)})^2}{\rho^{(M)} c_\varepsilon^{(M)} c_{11}^{(M)}}, \quad E_p = \frac{\omega^* e_{33}^{(M)} \varphi_0}{q_{11}^{(M)} T_0 V_p^{(M)}}, \quad \eta = \frac{c_{11}^{(M)} \varepsilon_{33}^{(M)}}{(e_{33}^{(M)})^2}, \quad E_\eta = E_p \eta, \quad E_T^{(n)} = E T_1^{(n)\prime},$$

where E, E_p, and E_η are dimensionless normalizing multipliers. In the dimensionless parameters (from now on, the primes are omitted), the linearized equations (3.10) with the notation (3.14) taken into account become

$$\begin{cases}\mathbf{L}^{*\mathbf{f}}_{11}[u_1^{(n)}]+\theta_1^{(n)}u_{2,12}^{(n)}+\mathbf{L}^{*\mathbf{f}}_{13}[u_3^{(n)}]+E_p\mathbf{L}^{*\mathbf{f}}_{14}[u_4^{(n)}]+\theta_{1155}^{(n)}u_{5,1}^{(n)}=0,\\ \theta_1^{(n)}u_{1,12}^{(n)}+\mathbf{L}^{*\mathbf{f}}_{22}[u_2^{(n)}]+\mathbf{L}^{*\mathbf{f}}_{23}[u_3^{(n)}]+E_p\mathbf{L}^{*\mathbf{f}}_{24}[u_4^{(n)}]+\theta_{2255}^{(n)}u_{5,2}^{(n)}=0,\\ \mathbf{L}^{*\mathbf{f}}_{31}[u_1^{(n)}]+\mathbf{L}^{*\mathbf{f}}_{32}[u_2^{(n)}]+\mathbf{L}^{*\mathbf{f}}_{33}[u_3^{(n)}]+E_p\mathbf{L}^{*\mathbf{f}}_{34}[u_4^{(n)}]+\mathbf{L}^{*\mathbf{f}}_{35}[u_5^{(n)}]=0,\\ \mathbf{L}^{*\mathbf{f}}_{41}[u_1^{(n)}]+\mathbf{L}^{*\mathbf{f}}_{42}[u_2^{(n)}]+\mathbf{L}^{*\mathbf{f}}_{34}[u_3^{(n)}]+E_\eta\mathbf{L}^{*\mathbf{f}}_{44}[u_4^{(n)}]+\mathbf{L}^{*\mathbf{f}}_{45}[u_5^{(n)}]=0,\\ i\omega E_T^{(n)}[\theta_{1155}^{(n)}u_{1,1}^{(n)}+\theta_{2255}^{(n)}u_{2,2}^{(n)}+\theta_{3355}^{(n)}u_{3,3}^{(n)}+E_p\theta_{3555}^{(n)}u_{4,3}^{(n)}]-\mathbf{L}^{*\mathbf{f}}_{55}[u_5^{(n)}]=0.\end{cases}\tag{3.16}$$

The components of the extended vector $\mathbf{\Sigma}^{(n)}_{\mathbf{e}\boldsymbol{\tau}}$ (3.15) is represented in dimensionless form as

$$\begin{aligned}&\Sigma_1^{(n)}=\Theta_{31}^{(n)}=\theta_{3113}^{(n)}u_{1,3}+\theta_{1313}^{(n)}u_{3,1}^{(n)}+E_p\theta_{3141}^{(n)}u_{4,1}^{(n)},\\ &\Sigma_2^{(n)}=\Theta_{32}^{(n)}=\theta_{3223}^{(n)}u_{2,3}+\theta_{2323}^{(n)}u_{3,2}^{(n)}+E_p\theta_{3242}^{(n)}u_{4,2}^{(n)},\\ &\Sigma_3^{(n)}=\Theta_{33}^{(n)}=\theta_{1133}^{(n)}u_{1,1}+\theta_{2233}^{(n)}u_{2,2}^{(n)}+\theta_{3333}^{(n)}u_{3,3}^{(n)}+E_p\theta_{3343}^{(n)}u_{4,3}^{(n)}+\theta_{3355}^{(n)}u_5^{(n)},\\ &\Sigma_4^{(n)}=d_3^{(n)}=\theta_{3411}^{(n)}u_{1,1}+\theta_{3422}^{(n)}u_{2,2}^{(n)}+\theta_{3433}^{(n)}u_{3,3}^{(n)}+E_\eta\theta_{3443}^{(n)}u_{4,3}^{(n)}+\theta_{3555}^{(n)}u_5^{(n)},\\ &\Sigma_5^{(n)}=-h_3^{(n)}=\lambda_{33}^{(n)}u_{5,3}^{(n)}.\end{aligned}\tag{3.17}$$

4. GREEN FUNCTION OF A PRESTRESSED THERMOELECTROELASTIC HALF-SPACE WITH INHOMOGENEOUS COATING

Applying the Fourier transform in the coordinates x_1 and x_2 (α_1 and α_2 are the transformation parameters) to problem (3.16), (3.11)–(3.13), we write it in the form

$$\begin{cases}\mathbf{L}^{\mathbf{\Lambda f}}_{11}[U_1^{(n)}]-\alpha_1\alpha_2\theta_1^{(n)}U_2^{(n)}+\mathbf{L}^{\mathbf{\Lambda f}}_{13}[U_3^{(n)}]+E_p\mathbf{L}^{\mathbf{\Lambda f}}_{14}[U_4^{(n)}]-i\alpha_1\theta_{1155}^{(n)}U_5^{(n)}=0,\\ -\alpha_1\alpha_2\theta_1^{(n)}U_1^{(n)}+\mathbf{L}^{\mathbf{\Lambda f}}_{22}[U_2^{(n)}]+\mathbf{L}^{\mathbf{\Lambda f}}_{23}[U_3^{(n)}]+E_p\mathbf{L}^{\mathbf{\Lambda f}}_{24}[U_4^{(n)}]-i\alpha_2\theta_{2255}^{(n)}U_5^{(n)}=0,\\ \mathbf{L}^{\mathbf{\Lambda f}}_{31}[U_1^{(n)}]+\mathbf{L}^{\mathbf{\Lambda f}}_{32}[U_2^{(n)}]+\mathbf{L}^{\mathbf{\Lambda f}}_{33}[U_3^{(n)}]+E_p\mathbf{L}^{\mathbf{\Lambda f}}_{34}[U_4^{(n)}]+\mathbf{L}^{\mathbf{\Lambda f}}_{35}[U_5^{(n)}]=0,\\ \mathbf{L}^{\mathbf{\Lambda f}}_{41}[U_1^{(n)}]+\mathbf{L}^{\mathbf{\Lambda f}}_{42}[U_2^{(n)}]+\mathbf{L}^{\mathbf{\Lambda f}}_{34}[U_3^{(n)}]+E_\eta\mathbf{L}^{\mathbf{\Lambda f}}_{44}[U_4^{(n)}]+\mathbf{L}^{\mathbf{\Lambda f}}_{45}[U_5^{(n)}]=0,\\ \omega E_T^{(n)}[\alpha_1\theta_{1155}^{(n)}U_1^{(n)}+\alpha_2\theta_{2255}^{(n)}U_2^{(n)}+i\theta_{3355}^{(n)}U_3^{(n)\prime}+i\theta_{3555}^{(n)}E_pU_4^{(n)\prime}]-\mathbf{L}^{\mathbf{\Lambda f}}_{55}[U_5^{(n)}]=0,\end{cases}\tag{4.1}$$

$x_3=h:$

$$\begin{cases}\theta_{3113}^{(n)}U_1^{(n)\prime}-i\alpha_1\theta_{1313}^{(n)}U_3^{(n)}-i\alpha_1E_p\theta_{3141}^{(n)}U_4^{(n)}=F_1,\\ \theta_{3223}^{(n)}U_2^{(n)\prime}-i\alpha_2\theta_{2323}^{(n)}U_3^{(n)}-i\alpha_2E_p\theta_{3242}^{(n)}U_4^{(n)}=F_2,\\ -i\alpha_1\theta_{1133}^{(n)}U_1^{(n)}-i\alpha_2\theta_{2233}^{(n)}U_2^{(n)}+\theta_{3333}^{(n)}U_3^{(n)\prime}+E_p\theta_{3343}^{(n)}U_4^{(n)\prime}+\theta_{3355}^{(n)}U_5^{(n)}=F_3,\\ -i\alpha_1\theta_{3411}^{(n)}U_1^{(n)}-i\alpha_2\theta_{3422}^{(n)}U_2^{(n)}+\theta_{3433}^{(n)}U_3^{(n)\prime}+E_\eta\theta_{3443}^{(n)}U_4^{(n)\prime}+\theta_{3555}^{(n)}U_5^{(n)}=F_4,\\ \lambda_{33}^{(n)}U_5^{(n)\prime}=F_5,\end{cases}\tag{4.2}$$

$x_3=h_k\quad(k=2,\dots,M):$

$$\Sigma_k^{\Lambda(n)}=\Sigma_k^{\Lambda(n+1)},\quad U_k^{(n)}=U_k^{(n+1)}\quad(k=1,\dots,5,\ n=2,\dots,M-1),\tag{4.3}$$

$$x_3\to-\infty\quad U_k^{(M)}\downarrow 0.\tag{4.4}$$

Here $U_k^{(n)}$, $\Sigma_k^{\Lambda(n)}$, and F_k $(k=1,\dots,5)$ are the Fourier transforms of the components of extended vectors $\mathbf{u}^{(n)}_{\mathbf{e}\boldsymbol{\tau}}$, stresses $\Sigma^{(n)}_{\mathbf{e}\boldsymbol{\tau}}$ (3.17), and the prescribed load $\mathbf{f}_{\mathbf{e}\boldsymbol{\tau}}$. We used the following notation in (4.1):

$$\mathbf{L}^{\mathbf{\Lambda}}_{kk}=\theta_{3kk3}^{(n)}\frac{\partial^2}{\partial x_3^2}-\alpha_s^2\theta_{skks}^{(n)}+\rho^{(n)}\omega^2\quad(k=1,2,3),\quad \mathbf{L}^{\mathbf{\Lambda}}_{34}=\theta_{3343}^{(n)}\frac{\partial^2}{\partial x_3^2}-\alpha_s^2\theta_{s34s}^{(n)},$$

$$\mathbf{L}^{\mathbf{\Lambda}}_{44}=\theta_{3443}^{(n)}\frac{\partial^2}{\partial x_3^2}-\alpha_s^2\theta_{s44s}^{(n)},\quad \mathbf{L}^{\mathbf{\Lambda}}_{55}=\lambda_{33}^{(n)}\frac{\partial^2}{\partial x_3^2}-\alpha_s^2\lambda_{ss}^{(n)}-i\omega T_1^{(n)}\theta_{5555}^{(n)},\quad s=1,2,$$

$$\mathbf{L}_{kk}^{\mathbf{\Lambda f}} = \mathbf{L}_{kk}^{\mathbf{\Lambda}} + \theta_{3kk3}^{(n)\prime}\frac{\partial}{\partial x_3} \quad (k=1,2,3), \quad \mathbf{L}_{44}^{\mathbf{\Lambda f}} = \mathbf{L}_{44}^{\mathbf{\Lambda}} + \theta_{3443}^{(n)\prime}\frac{\partial}{\partial x_3}, \quad \mathbf{L}_{55}^{\mathbf{\Lambda f}} = \mathbf{L}_{55}^{\mathbf{\Lambda}} + \lambda_{33}^{(n)\prime}\frac{\partial}{\partial x_3},$$

$$\mathbf{L}_{s3}^{\mathbf{\Lambda f}} = -i\alpha_s(\theta_{ss33}^{(n)} + \theta_{s3s3}^{(n)})\frac{\partial}{\partial x_3} + \theta_{s3s3}^{(n)\prime}\frac{\partial}{\partial x_s}, \quad \mathbf{L}_{s4}^{\mathbf{\Lambda f}} = -i\alpha_s(\theta_{ss43}^{(n)} + \theta_{3s4s}^{(n)})\frac{\partial}{\partial x_3} + \theta_{3s4s}^{(n)\prime}\frac{\partial}{\partial x_s},$$

$$\mathbf{L}_{3s}^{\mathbf{\Lambda f}} = -i\alpha_s(\theta_{ss33}^{(n)}+\theta_{s3s3}^{(n)})\frac{\partial}{\partial x_3} + \theta_{ss33}^{(n)\prime}\frac{\partial}{\partial x_s}, \quad \mathbf{L}_{4s}^{\mathbf{\Lambda f}} = -i\alpha_s(\theta_{ss43}^{(n)}+\theta_{3s4s}^{(n)})\frac{\partial}{\partial x_3} + \theta_{34ss}^{(n)\prime}\frac{\partial}{\partial x_s}, \quad s=1,2,$$

$$\mathbf{L}_{34}^{\mathbf{\lambda f}} = \mathbf{L}_{34}^{\mathbf{\Lambda}} + \theta_{3343}^{(n)\prime}\frac{\partial}{\partial x_3}, \quad \mathbf{L}_{35}^{\mathbf{\lambda f}} = \theta_{3355}^{(n)}\frac{\partial}{\partial x_3} + \theta_{3355}^{(n)\prime}, \quad \mathbf{L}_{45}^{\mathbf{\lambda f}} = \theta_{3555}^{(n)}\frac{\partial}{\partial x_3} + \theta_{3555}^{(n)\prime}.$$

Here and hereafter, the prime denotes the derivative with respect to x_3.

To solve problem (4.1)–(4.4), we use the approach developed in [24, 25]. For this, we introduce the new variables

$$\mathbf{Y}^{(n)} = \begin{pmatrix} \mathbf{Y}_{\mathbf{\Sigma}}^n \\ \mathbf{Y}_{\mathbf{u}}^n \end{pmatrix}, \quad \mathbf{Y}_{\mathbf{\Sigma}}^n = \|\Sigma_k^{\Lambda(n)}\|_{k=1}^5, \quad \mathbf{Y}_{\mathbf{u}}^n = \|U_k^{(n)}\|_{k=5}^{10}. \tag{4.5}$$

In the new variables, system (4.1) becomes

$$\mathbf{Y}^{(n)\prime} = \mathbf{M}^{(n)}(\alpha_1,\alpha_2,x_3)\mathbf{Y}^{(n)}, \tag{4.6}$$

$$\mathbf{M}^{(n)} = \begin{pmatrix}
0 & 0 & m_{13}^{(n)} & m_{14}^{(n)} & 0 & m_{16}^{(n)} & m_{17}^{(n)} & 0 & 0 & m_{1,10}^{(n)} \\
0 & 0 & m_{23}^{(n)} & m_{24}^{(n)} & 0 & m_{26}^{(n)} & m_{27}^{(n)} & 0 & 0 & m_{2,10}^{(n)} \\
m_{31}^{(n)} & m_{32}^{(n)} & 0 & 0 & 0 & 0 & 0 & m_{38}^{(n)} & m_{39}^{(n)} & 0 \\
m_{41}^{(n)} & m_{42}^{(n)} & 0 & 0 & 0 & 0 & 0 & m_{48}^{(n)} & m_{49}^{(n)} & 0 \\
0 & 0 & m_{53}^{(n)} & m_{54}^{(n)} & 0 & m_{56}^{(n)} & m_{57}^{(n)} & 0 & 0 & m_{5,10}^{(n)} \\
m_{61}^{(n)} & 0 & 0 & 0 & 0 & 0 & 0 & m_{68}^{(n)} & m_{69}^{(n)} & 0 \\
0 & m_{72}^{(n)} & 0 & 0 & 0 & 0 & 0 & m_{68}^{(n)} & m_{69}^{(n)} & 0 \\
0 & 0 & m_{83}^{(n)} & m_{84}^{(n)} & 0 & m_{86}^{(n)} & m_{87}^{(n)} & 0 & 0 & m_{8,10}^{(n)} \\
0 & 0 & m_{93}^{(n)} & m_{94}^{(n)} & 0 & m_{96}^{(n)} & m_{97}^{(n)} & 0 & 0 & m_{9,10}^{(n)} \\
0 & 0 & 0 & 0 & m_{10,5}^{(n)} & 0 & 0 & 0 & 0 & 0
\end{pmatrix}, \tag{4.7}$$

$$m_{k3}^{(n)} = -i\alpha_k r_{ke}^{(n)} r_0^{-(n)}, \quad m_{k4}^{(n)} = -i\alpha_k r_k^{(n)} E_p r_0^{-(n)},$$

$$m_{k,5+k}^{(n)} = -\alpha_k^2(E_p(\theta_{kk43}^{(n)})^2\theta_{3333}^{(n)} - 2E_p r_k^{(n)}\theta_{kk43}^{(n)} - E_\eta\theta_{3443}^{(n)}(\theta_{kk33}^{(n)})^2)r_0^{-(n)} + P_k^{(n)},$$

$$m_{k,10}^{(n)} = i\alpha_k(\theta_{3355}^{(n)} r_{ke}^{(n)} + E_p\theta_{3555}^{(n)} r_k^{(n)} + \theta_{kk55}^{(n)})r_0^{-(n)} \quad (k=1,2),$$

$$m_{17}^{(n)} = \alpha_1\alpha_2(E_p r_1^{(n)}\theta_{2243}^{(n)} + \theta_1^{(n)} r_0^{(n)} + \theta_{2233}^{(n)} r_{1e}^{(n)})r_0^{-(n)}, \quad m_{26}^{(n)} = m_{17}^{(n)},$$

$$m_{3k}^{(n)} = i\alpha_k\theta_{k3k3}^{(n)}\theta_{3kk3}^{-(n)} \quad (k=1,2), \quad m_{38}^{(n)} = -\sum_{k=1}^{2}\alpha_k^2(\theta_{k3k3}^{(n)})^2\theta_{3kk3}^{-(n)} + P_3^{(n)},$$

$$m_{39}^{(n)} = -\sum_{k=1}^{2}\alpha_k^2 E_p\theta_{k3k3}^{(n)}\theta_{3k4k}^{(n)}\theta_{3kk3}^{-(n)} + E_p P_4^{(n)},$$

$$m_{49}^{(n)} = -\sum_{k=1}^{2}\alpha_k^2 E_p(\theta_{3k4k}^{(n)})^2\theta_{3kk3}^{-(n)} + E_\eta P_5^{(n)},$$

$$m_{4k}^{(n)} = i\alpha_k\theta_{3k4k}^{(n)}\theta_{3kk3}^{-(n)} \quad (k=1,2), \quad m_{48}^{(n)} = m_{39}^{(n)}(E_p)^{-1},$$

$$m_{53}^{(n)} = -i\omega E_T^{(n)} r_{3e}^{(n)} r_0^{-(n)}, \quad m_{54}^{(n)} = -i\omega E_p E_T^{(n)} r_3^{(n)} r_0^{-(n)},$$

$$m_{56}^{(n)} = \omega E_T^{(n)} \alpha_1 r_0^{-(n)} (\theta_{3355}^{(n)} r_{1e}^{(n)} + E_p \theta_{3555}^{(n)} r_1^{(n)} + \theta_{1155}^{(n)} r_0^{(n)}),$$
$$m_{57}^{(n)} = \omega E_T^{(n)} \alpha_2 r_0^{-(n)} (\theta_{3355}^{(n)} r_{2e}^{(n)} + E_p \theta_{3555}^{(n)} r_2^{(n)} + \theta_{2255}^{(n)} r_0^{(n)}),$$
$$m_{5,10}^{(n)} = i\omega E_T^{(n)} r_0^{-(n)} (\theta_{3355}^{(n)} r_{3e}^{(n)} + E_p \theta_{3555}^{(n)} r_3^{(n)}) + P_6^{(n)},$$
$$m_{61}^{(n)} = (\theta_{3113}^{(n)})^{-1}, \quad m_{68}^{(n)} = m_{31}^{(n)}, \quad m_{69}^{(n)} = E_p m_{41}^{(n)},$$
$$m_{72}^{(n)} = (\theta_{3223}^{(n)})^{-1}, \quad m_{78}^{(n)} = m_{32}^{(n)}, \quad m_{79}^{(n)} = E_p m_{42}^{(n)},$$
$$m_{83}^{(n)} = -E_\eta \theta_{3443}^{(n)} r_0^{-(n)}, \quad m_{84}^{(n)} = E_p \theta_{3343}^{(n)} r_0^{-(n)},$$
$$m_{86}^{(n)} = m_{13}^{(n)}, \quad m_{87}^{(n)} = m_{23}^{(n)}, \quad m_{8,10}^{(n)} = r_{3e}^{(n)} r_0^{-(n)},$$
$$m_{93}^{(n)} = m_{84}^{(n)} (E_p)^{-1}, \quad m_{94}^{(n)} = -\theta_{3333}^{(n)} r_0^{-(n)}, \quad m_{96}^{(n)} = m_{14}^{(n)} (E_p)^{-1},$$
$$m_{97}^{(n)} = m_{24}^{(n)} (E_p)^{-1}, \quad m_{9,10}^{(n)} = r_3^{(n)} r_0^{-(n)}, \quad m_{10,5}^{(n)} = \lambda_{33}^{-(n)} = (\lambda_{33}^{(n)})^{-1},$$
$$P_k^{(n)} = \alpha_i^2 \theta_{ikki}^{(n)} - \rho^{(n)} \omega^2 \quad (k = 1, 2, 3),$$
$$P_4^{(n)} = \alpha_i^2 \theta_{i34i}^{(n)}, \quad P_5^{(n)} = \alpha_i^2 \theta_{i44i}^{(n)},$$
$$P_6^{(n)} = \alpha_i^2 \lambda_{ii}^{(n)} + i\omega T_1^{(n)} \theta_{5555}^{(n)} \quad (i = 1, 2),$$
$$r_0^{(n)} = E_p (\theta_{3343}^{(n)})^2 - E_\eta \theta_{3333} \theta_{3443}^{(n)}, \quad r_s^{(n)} = \theta_{ss43}^{(n)} \theta_{3333}^{(n)} - \theta_{ss33}^{(n)} \theta_{3343}^{(n)} \quad (s = 1, 2),$$
$$r_3^{(n)} = \theta_{3555}^{(n)} \theta_{3333}^{(n)} - \theta_{3355}^{(n)} \theta_{3343}^{(n)},$$
$$r_{se}^{(n)} = E_\eta \theta_{ss33}^{(n)} \theta_{3343}^{(n)} - E_p \theta_{ss43}^{(n)} \theta_{3343}^{(n)} \quad (s = 1, 2), \quad r_{3e}^{(n)} = E_\eta \theta_{3355}^{(n)} \theta_{3443}^{(n)} - \theta_{3555}^{(n)} \theta_{3343}^{(n)}.$$

System (4.6) in notation (4.5), (4.7) is a system of first-order ordinary differential equations with variable coefficients. This system can be solved by using several numerical methods. In the present paper, we solve it by the Runge–Kutta method modified by Merson, which permits reconstructing the solution with a prescribed accuracy. We expand the function $Y_k^{(n)}$ as

$$Y_k^{(n)} = \sum_{p=1}^{10} c_{p+g}(\alpha_1, \alpha_2) y_{kp}^{(n)}(\alpha_1, \alpha_2, x_3), \quad k = 1, 2, \ldots, 10, \quad g = 10(n-1), \quad n = 1, 2, \ldots, M-1 \quad (4.8)$$

in linearly independent solutions $y_{kp}^{(n)}(\alpha_1, \alpha_2, x_3)$ of the Cauchy problem for Eq. (4.6) with the initial conditions $y_{kp}^{(n)}(\alpha_1, \alpha_2, 0) = \delta_{kp}$.

We represent the solution for the half-space with regard to (3.5) in the form

$$\begin{aligned} U_p^{(M)}(\alpha_1, \alpha_2, x_3) &= -i\alpha_p \sum_{k=1}^{5} f_{pk}^{(M)} c_{k+g} e^{\sigma_k^{(M)} x_3}, \quad p = 1, 2, \quad g = 10(M-1), \\ U_p^{(M)}(\alpha_1, \alpha_2, x_3) &= \sum_{k=1}^{5} f_{pk}^{(M)} c_{k+g} e^{\sigma_k^{(M)} x_3}, \quad p = 3, 4, 5. \end{aligned} \quad (4.9)$$

Here $\sigma_k^{(n)}$ are the roots of the characteristic equation ($n = M$)

$$\det \mathbf{M}_{\boldsymbol{\sigma}}^{(n)} = 0,$$

$$\mathbf{M}_{\boldsymbol{\sigma}}^{(n)}(r) = \begin{pmatrix} A_{11}^{(n)} & -\alpha_2^2 \theta_1^{(n)} & r\theta_2^{(n)} & r\psi_1^{(n)} E_p & \theta_{1155}^{(n)} \\ -\alpha_1^2 \theta_1^{(n)} & A_{22}^{(n)} & r\theta_3^{(n)} & r\psi_2^{(n)} E_p & \theta_{2255}^{(n)} \\ -\alpha_1^2 r\theta_2^{(n)} & -\alpha_2^2 r\theta_3^{(n)} & A_{33}^{(n)} & A_{34}^{(n)} E_p & r\theta_{3355}^{(n)} \\ -\alpha_1^2 r\psi_1^{(n)} & -\alpha_2^2 r\psi_2^{(n)} & A_{34}^{(n)} & A_{44}^{(n)} E_\eta & r\theta_{3555}^{(n)} \\ -\alpha_1^2 i\omega E_T^{(n)} \theta_{1155}^{(n)} & -\alpha_2^2 i\omega E_T^{(n)} \theta_{2255}^{(n)} & i\omega E_T^{(n)} r\theta_{3355}^{(n)} & i\omega E_T^{(n)} E_\eta r\theta_{3555}^{(n)} & -A_{55}^{(n)} \end{pmatrix} \quad (4.10)$$

$$A_{kk}^{(n)} = \theta_{3kk3}^{(n)}(\sigma_k^{(n)})^2 - \alpha_s^2\theta_{skks}^{(n)} + \rho^{(n)}\omega^2, \quad k=1,2,3, \quad A_{34}^{(n)} = \theta_{3343}^{(n)}(\sigma_2^{(n)})^2 - \alpha_s^2\theta_{s34s}^{(n)},$$
$$A_{44}^{(n)} = \theta_{3443}^{(n)}(\sigma_k^{(n)})^2 - \alpha_s^2\theta_{s44s}^{(n)}, \quad A_{55}^{(n)} = \lambda_{33}^{(n)}(\sigma_k^{(n)})^2 - \alpha_s^2\lambda_{ss}^{(n)} - i\omega T_1^{(n)}\theta_{5555}^{(n)}, \quad s=1,2, \tag{4.11}$$
$$\theta_2^{(n)} = \theta_{1133}^{(n)} + \theta_{1313}^{(n)}, \quad \theta_3^{(n)} = \theta_{2233}^{(n)} + \theta_{2323}^{(n)}, \quad \psi_1^{(n)} = \theta_{1143}^{(n)} + \theta_{3141}^{(n)}, \quad \psi_2^{(n)} = \theta_{2243}^{(n)} + \theta_{3242}^{(n)}.$$

The coefficients $f_{pk}^{(n)}$ $(p,k=1,\dots,5)$ satisfy the homogeneous system of equations with the matrix $\mathbf{M}_{\boldsymbol{\sigma}}^{(n)}(\sigma_k^{(n)})$ (4.10). The unknown variables c_k are determined by substituting expressions (4.8) into the boundary conditions (4.2) and (4.3) which results in the linear system of algebraic equations

$$\mathbf{AC} = \mathbf{F}. \tag{4.12}$$

Here $\mathbf{C} =\uparrow \{c_p\}_{p=1}^{10(M-1)+5}$ is the vector of unknowns, $\mathbf{F} =\uparrow \{\mathbf{F}_{\mathbf{e}\boldsymbol{\tau},\mathbf{F}_0}\}$, $\mathbf{F}_{\mathbf{e}\boldsymbol{\tau}}$ is the Fourier transform of the vector of prescribed load, and $\mathbf{F}_0$ is a vector whose dimension is determined by the problem geometry. The matrix $\mathbf{A}$ can be represented as

$$\mathbf{A} = \begin{pmatrix} \mathbf{B}^1(h_1) & 0 \\ \mathbf{A}^1(h_{2,\dots,M}) & \mathbf{B}^M(h_M) \end{pmatrix}, \tag{4.13}$$

where $\mathbf{B}^1(h_1)$ and $\mathbf{B}^M(h_M)$ are rectangular 5×10 and 10×5 matrices, respectively. The matrices $\mathbf{A}$ and $\mathbf{A}^1(h_{2,\dots,M})$ are square matrices whose dimension depends on the geometric parameters of the problem and is determined by the formulas $[5(2M-1)]$ and $[10(M-1)]$, respectively. The entries of the matrix (4.13) have the form

$$\mathbf{B}^1(h_1) = \|y_{kp}^{(1)}(\alpha_1,\alpha_2,h_1)\|_{k=1,\dots,5}^{p=1,\dots,10}, \tag{4.14}$$

$$\mathbf{B}^M(h_M) = \begin{pmatrix} \mathbf{l}^M \\ \mathbf{f}^M \end{pmatrix}, \quad \mathbf{l}^M = \|-l_{ij}^{(M)}\|_{i,j=1}^5, \quad \mathbf{f}^M = \|-f_{ij}^{(M)}\|_{i,j=1}^5, \tag{4.15}$$

$$l_{sk}^{(M)} = \sigma_k^{(M)}\theta_{3ss3}^{(M)}f_{sk}^{(M)} + \theta_{s3s3}^{(M)}f_{3k}^{(M)} + E_p\theta_{3s4s}^{(M)}f_{4k}^{(M)} \quad (s=1,2), \quad l_{5k}^{(M)} = \sigma_k^{(M)}\lambda_{33}^{(M)}f_{5k}^{(M)},$$
$$l_{3k}^{(M)} = \sigma_k^{(M)}\theta_{3333}^{(M)}f_{3k}^{(M)} - \alpha_s^2\theta_{ss33}^{(M)}f_{sk}^{(M)} + \sigma_k^{(M)}E_p\theta_{3343}^{(M)}f_{4k}^{(M)} + \theta_{3355}^{(M)}f_{5k}^{(M)},$$
$$l_{4k}^{(M)} = \sigma_k^{(M)}\theta_{3433}^{(M)}f_{3k}^{(M)} - \alpha_s^2\theta_{34ss}^{(M)}f_{sk}^{(M)} + \sigma_k^{(M)}E_\eta\theta_{3443}^{(M)}f_{4k}^{(M)} + \theta_{3355}^{(M)}f_{5k}^{(M)} \quad (s=1,2),$$

$$\mathbf{A}^1(h_{2,\dots,M}) = \begin{pmatrix} \mathbf{B}^1(h_2) & \mathbf{P}^2(h_2) & 0 & 0 & \vdots & 0 & 0 \\ 0 & \mathbf{B}^2(h_3) & \mathbf{P}^3(h_3) & 0 & \vdots & 0 & 0 \\ 0 & 0 & \mathbf{B}^3(h_4) & \mathbf{P}^4(h_4) & \vdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \vdots & \mathbf{B}^{M-2}(h_{M-1}) & \mathbf{P}^{M-1}(h_{M-1}) \\ 0 & 0 & 0 & 0 & \vdots & 0 & \mathbf{P}^{M-1}(h_{M-1}) \end{pmatrix}. \tag{4.16}$$

In representation (4.16), $\mathbf{B}^n(h_k)$ and $\mathbf{P}^k(h_l) = -\mathbf{B}^k(h_l)$ are 10×10 matrices. The upper index corresponds to the layer number, the argument, to the interface between layers. The matrices $\mathbf{B}^n(h_k)$ in general form with regard to the notation (4.14) and (4.15) are determined by the formula

$$\mathbf{B}^1(h_1) = \|y_{kp}^{(1)}(\alpha_1,\alpha_2,h_1)\|_{k,p=1}^{10}, \quad \mathbf{B}^1(h_k) = \|y_{kp}^{(n)}(\alpha_1,\alpha_2,h_k)\|_{k,p=1}^{10}, \quad n=2,3,\dots,M-1. \tag{4.17}$$

The dispersion equation of the problem has the form

$$\Delta_0 = \det\mathbf{A} = 0.$$

Applying the inverse Fourier transform to expressions (4.8), (4.16), and (4.9) with regard to (4.10)–(4.12), we obtain the solution of the boundary value problem (3.1)–(3.5) in the form

$$\mathbf{u}^{(n)}_{\mathbf{e}\boldsymbol{\tau}}(x_1, x_2, x_3) = \frac{1}{4\pi^2} \iint\limits_{\Omega} \mathbf{k}^{(n)}_{\mathbf{e}\boldsymbol{\tau}}(x_1 - \xi, x_2 - \eta, x_3)\mathbf{f}_{\mathbf{e}\boldsymbol{\tau}}(\xi, \eta)\, d\xi\, d\eta, \tag{4.18}$$

$$\mathbf{k}^{(n)}_{\mathbf{e}\boldsymbol{\tau}}(s, t, x_3) = \int\limits_{\Gamma_1} \int\limits_{\Gamma_2} \mathbf{K}^{(n)}_{\mathbf{e}\boldsymbol{\tau}}(\alpha_1, \alpha_2, x_3) e^{-i(\alpha_1 s + \alpha_2 t)} d\alpha_1\, d\alpha_2, \tag{4.19}$$

$$\mathbf{K}^{(n)}_{\mathbf{e}\boldsymbol{\tau}}(\alpha_1, \alpha_2, x_3) = \|K^{(n)}_{lj}\|^5_{l,j=1}. \tag{4.20}$$

The components of the matrix $\mathbf{K}^{(n))}_{\mathbf{e}\boldsymbol{\tau}}$ are defined by the formulas ($p = 10(n-1)$, $n = 1, 2, \ldots, M-1$):

$$K^{(n)}_{lj} = \frac{1}{\Delta_0} \sum_{k=1}^{10} \Delta_{jk+p} y^{(n)}_{l+5k}(\alpha_1, \alpha_2, x_3), \quad l, j = 1, \ldots, 5, \tag{4.21}$$

for the half-space

$$K^{(M)}_{lj} = \frac{1}{\Delta_0} \sum_{k=1}^{5} f^{(M)}_{lk} \Delta_{jk+10(M-1)} e^{\sigma^{(M)}_k x_3}, \quad l, j = 1, \ldots, 5, \tag{4.22}$$

where Δ_0 and Δ_{ns} are the determinant and the algebraic complement of the corresponding element of the matrix $\mathbf{A}$ whose entries are given by formulas (4.14)–(4.17).

The integral representation (4.18) with the Green function (4.19)–(4.22) determine the medium point displacement under the action of a load prescribed on its surface. The contours Γ_1 and Γ_2 pass in the domain of the integrand analyticity and are chosen according to the rules given in [30].

CONCLUSION

When the properties of new artificial materials are studied, the working properties and the strength characteristics of pieces operating under the constant action of various factors and manufactured from contemporary high-technology materials are estimated, and materials with prescribed properties are designed, special demands are imposed on the modeling of media and processes arising in such materials experiencing various external actions. In this case, it is necessary to combine the adequate consideration of the material properties and their variations depending on the character of the initial mechanical and thermal actions and the possibility of constructing sufficiently simple and effective solutions.

The numerical-analytical method proposed in this paper to construct the Green function of complex, including functionally graded, media permits sufficiently completely taking into account not only the difference between the structure elements but also the character of the initial stress states of functionally graded elements. It should be noted that, composing the functional dependence in inhomogeneous components, one can consider variations in the properties of either one “reference” material or two materials. The nonlinear relations of mechanics of thermoelectroelasticity were linearized to consider the influence of the initial actions on the material property variations in the framework of the theory of small strain imposed on finite strains. Preserving the second-order terms with respect to strains, electric field, and the temperature deviation in the thermodynamic potential permits obtaining simpler and more convenient formulas of linearized constitutive relations and equations of motion of the medium on the one hand, and taking into account the influence of nonlinear effects of the action of the temperature and mechanical strains on the variations in the initial properties of the material on the other hand.

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REFERENCES

1. I. V. Anan’ev and V. A. Babeshko, “Punch Oscillations on a Layer with Depth-Depending Characteristics,” Izv. Akad. Nauk SSSR. Mekh. Tverd. Tela, No. 1, 64–69 (1978) [Mech. Solids (Engl. Transl.)].

2. I. I. Vorovich and V. A. Babeshko, *Dynamic Mixed Problems of Elasticity for Nonclassical Domains* (Nauka, Moscow, 1979) [in Russian].
3. I. V. Anan'ev, V. V. Kalinchuk, and I. B. Poliakova, "On Wave Excitation by a Vibrating Stamp in a Medium with Inhomogeneous Initial Stresses," Prikl. Mat. Mekh. **47** (3), 483–489 (1983) [J. Appl. Math. Mech. (Engl. Transl.) **47** (3), 408–413 (1983)].
4. V. A. Babeshko, E. V. Glushkov, and Zh. F. Zinchenko, *Dynamics of Inhomogeneous Linearly Elastic Media* (Nauka, Moscow, 1989) [in Russian].
5. V. V. Kalinchuk, I. V. Lysenko, and I. B. Polyakova, "Singularities of the Interaction of a Vibrating Stamp with an Inhomogeneous Heavy Base," Prikl. Mat. Mekh. **53** (2), 301–308 (1989) [J. Appl. Math. Mech. (Engl. Transl.) **53** (2), 235–241 (1989)].
6. X. Cao, F. Jin, and I. Jean, "Calculation of Propagation Properties of Lamb Waves in a Functionally Graded Material (FGM) Plate by Power Series Technique," NDT&E Int. **44**, 84–92 (2011).
7. G. R. Liu, K. Y. Dai, X. Han, and T. Ohyoshi, "Dispersion and Characteristics of Waves in Functionally Graded Piezoelectric Plates," J. Sound Vibr. **268**, 131–147 (2003).
8. J. Du, X. Jin, J. Wang, and K. Xian, "Love Wave Propagation in Functionally Graded Piezoelectric Material Layer," Ultrasonics **46**, 13–22 (2007).
9. Z.-H. Qian, F. Jin, T. Lu, and K. Kishimoto, "Transverse Surface Waves in a Layered Structure with a Functionally Graded Piezoelectric Substrate and a Hard Dielectric Layer," Ultrasonics **49**, 283–297 (2009).
10. B. Collet, M. Destrade, and G. A. Maugin, "Bleustein–Gulyaev Waves in Some Functionally Graded Materials," Europ. J. Mech. A. Solids **25** (5), 695–706 (2006).
11. G. N. Praveen and J. N. Reddy, "Nonlinear Transient Thermoelastic Analysis of Functionally Graded Ceramic-Metal Plates," Int. J. Solids Struct. **35**, 4457–4476 (1998).
12. Y. Obata and N. Nosa, "Optimum Material Design for Functionally Gradient Material Plate," Arch. Appl. Mech. **66**, 581–589 (1996).
13. J. N. Reddy, "Analysis of Functionally Graded Plates," Int. J. Numer. Mech. Engng **47**, 663–684 (2000).
14. J. N. Reddy and R. A. Arciniega, "Free Vibration Analysis of Functionally Graded Ceramic-Metal Plates," in *Analysis and Design of Plated Structures*, Vol. 2: *Dynamics* (Springer, New York, 2006), pp. 293–321.
15. J. N. Reddy and R. A. Arciniega, "Mechanical and Thermal Buckling of Functionally Graded Ceramic-Metal Plates," in *Analysis and Design of Plated Structures*, Vol. 1: *Stability* (Springer, New York, 2006).
16. R. D. Mindlin, "On the Equations of Motion of Piezoelectric Crystals," in *Problems of Continuum Mechanics* (SIAM, Philadelphia, 1961), pp. 282–290.
17. R. D. Mindlin, "Equations of High Frequency Vibrations of Thermo-Piezoelectric Plate," Int. J. Solids Struct. **10**, 625–637 (1974).
18. W. Nowacki, "Some General Theorems of Thermo-Piezoelectricity," J. Thermal Stresses **1** (2), 171–182 (1978).
19. W. Nowacki, "Foundations of Linear Piezoelectricity," in *Electromagnetic Interaction in Elastic Solids*, Chapter 1 (Springer, New York, 1979).
20. D. S. Chandrasekharaiah, "A Temperature Rate Dependent Theory of Thermo-Piezoelectricity," J. Thermal Stresses **7**, 293–306 (1984).
21. D. S. Chandrasekharaiah, "A Generalized Linear Thermoelasticity Theory of Piezoelectric Media," Acta Mech. **71**, 39–49 (1988).
22. J. N. Sharma and M. Pal, "Propagation of Lamb Waves in a Transversely Isotropic Piezothermoelastic Plate," J. Sound Vibr. **270**, 587–610 (2004).
23. J. N. Sharma, M. Pal, and D. Chand, "Propagation Characteristics of Rayleigh Waves in Transversely Isotropic Piezothermoelastic Materials," J. Sound Vibr. **284**, 227–248 (2005).
24. J. N. Sharma and V. Walia, "Effect of Rotation on Rayleigh Waves in Piezothermoelastic Half-Space," Int. J. Solids Struct. **44**, 1060–1072 (2007).
25. T. I. Belyankova, V. V. Kalinchuk, and G. Yu. Suvorova, "A Dynamic Contact Problem for a Thermoelastic Prestressed Layer," Prikl. Mat. Mekh. **76** (5), 811–822 (2012) [J. Appl. Math. Mech. (Engl. Transl.) **76** (5), 537–546 (2012)].
26. V. V. Kalinchuk, G. Yu. Suvorova, and T. I. Belyankova, "Green Functions of a Thermoelastic Prestresses Layer," Vestnik Yuzhn. Nauchn. Tsentra RAN **8** (3), 14–21 (2012).
27. T. I. Belyankova, V. V. Kalinchuk, and D. N. Sheidakov, "Equations of Dynamics of Prestressed Electrothermoelastic Medium," Vestnik Yuzhn. Nauchn. Tsentra RAN **7** (2), 3–10 (2011).
28. V. V. Kalinchuk and T. I. Belyankova, "On Dynamics of a Medium with Properties Continuously Varying in Depth," Izv. Vyssh. Uchebn. Zaved. Sev.-Kav. Region. Estest. Nauki. Special issue, 46–49 (2004).
29. V. V. Kalinchuk and T. I. Belyankova, *Dynamics of the Surface of Inhomogeneous Media* (Fizmatgiz, Moscow, 2009) [in Russian].
30. N. F. Tiersten, "On the Nonlinear Equations of Thermoelectroelasticity," Int. J. Engng Sci. **9**, 587–604 (1971).
31. A. I. Lur'ie, *Nonlinear Theory of Elasticity* (Nauka, Moscow, 1980) [in Russian].

32. G. A. Maugin and A. Berezovski, "Material Formulation of Finite-Strain Thermoelasticity and Applications," J. Thermal Stresses **22**, 421 (2000).
33. V. V. Kalinchuk and T. I. Belyankova, *Dynamic Contact Problems for Prestressed Electroelastic Media* (Fizmatgiz, Moscow, 2006) [in Russian].
34. J. Nye, *Physical Properties of Crystals: Their Representation by Tensors and Matrices* (Clarendon, Oxford, 1957; Mir, Moscow, 1967).
35. V. A. Babeshko, O. M. Babeshko, and O. V. Evdokimova, "On the Problem of Estimating the State of Coated Materials," Dokl. Ross. Akad. Nauk **409** (4), 481–485 (2006) [Dokl. Phys. (Engl. Transl.) **51** (8), 423–428 (2006)].
36. V. A. Babeshko, S. V. Ratner, and P. V. Syromyatnikov, "On Mixed Problems for Thermoelectroelastic Media with Discontinuous Boundary Conditions," Dokl. Ross. Akad. Nauk **412** (6), 753–758 (2007) [Dokl. Phys. (Engl. Transl.) **52** (2), 90–95 (2007)].
37. V. A. Babeshko, S. V. Ratner, and P. V. Syromyatnikov, "On Mixed Problems for Multilayer Anisotropic Composites," Vestnik Yuzhn. Nauchn. Tsentra RAN **2** (4), 5–11 (2006).
38. V. A. Babeshko, O. M. Babeshko, and O. V. Evdokimova, "Addressing the Problem of Investigating Coated Materials," Dokl. Ross. Akad. Nauk **410** (1), 49–52 (2006) [Dokl. Phys. (Engl. Transl.) **51** (9), 509–512 (2006)].