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Dynamic Deformation of Soft Soil Media: Experimental Studies and Mathematical Modeling

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Abstract—A complex experimental-theoretical approach to studying the problem of high-rate strain of soft soil media is presented. This approach combines the following contemporary methods of dynamical tests: the modified Hopkinson–Kolsky method applied to medium specimens contained in holders and the method of plane wave shock experiments. The following dynamic characteristics of sand soils are obtained: shock adiabatic curves, bulk compressibility curves, and shear resistance curves. The obtained experimental data are used to study the high-rate strain process in the system of a split pressure bar, and the constitutive relations of Grigoryan's mathematical model of soft soil medium are verified by comparing the results of computational and natural test experiments of impact and penetration.

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1. INTRODUCTION

The dynamic properties of soft soils have been studied by many authors [1–7], but the range of pressures (< 50 MPa) and strain rates ($< 10^2$ s^{-1}) considered in these studies is insufficiently high.

Only isolated attempts were made to study the dynamic compressibility in the intermediate range of loads (50–500 MPa) and strain rates (10^2–10^4 s^{-1}). These attempts are related to the modified Kolsky method with the use of split Hopkinson pressure bars (SHPB) and a confining holder. But the possibility of using this method [8–12] to study the compressibility and plastic properties of soil media is restricted by the elastic limit of the pressure bar and holder materials, and the range of considered loads does not exceed 0.5 GPa.

Higher values of load amplitudes are attained under shock wave loading [13–19]. For the pressure ranges up to several gigapascals, there are no experimental data of the p–ρ form because of severe methodological difficulties in measurements of two components of the stress tensor. Plane wave shock or explosion experiments used in the field permit reliably determining only the shock adiabatic (SA) curve relating the longitudinal component of the stress tensor σ_z to the uniaxial strain ε_z or the density ρ. In such experiments, the strength effects were often neglected, and it was assumed that SA σ_z–ε_z curve coincides with the hydrostatic p–ρ compression curve (the hydrodynamic approximation).

Thus, at present there are no sufficiently developed efficient methods for studying the physical-mechanical properties of soil media in a wide pressure range. It seems very promising to develop a complex experimental-theoretical approach [18, 19] to the study of the properties of soils under dynamic

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loading and plane wave tests in combination with the data obtained by the modified SHPB method with the use of well-known soil models [1, 2].

2. EXPERIMENTAL STUDIES OF SHOCK COMPRESSIBILITY OF SOIL MEDIA

The dynamic compressibility of soft soils can be studied experimentally in a wide range of strain rates by the following two complementary methods. The modified Kolsky method combined with the use of the split Hopkinson pressure bar (SHPB) is used for the strain rates $\sim 10^3\,\mathrm{s}^{-1}$ and loads up to 500 MPa, and the plane wave shock experiment is used for higher strain rates (greater than $10^4\,\mathrm{s}^{-1}$ and loads greater than 500 MPa). In both methods, the same type of stress–strain states of the specimen takes place, i.e., the one-dimensional strain ε_z and the bulk stress state with different stress tensor components σ_z and σ_r. This approach [20, 21] permits one one to obtain data for the dynamic compressibility in the pressure range from dozens of megapascals to several gigapascals.

The traditional Kolsky method was modified to perform dynamic tests of weakly connected media (soft soils). In the tests, it was proposed to place a soil specimen in a rigid holder between the end surfaces of pressure bars; the holder prevents the radial deformations of the specimen. Under these conditions, the radial deformation of the soil specimen can be neglected compared with the longitudinal deformation. Thus, it can be assumed that the strain state of the specimen is one-dimensional ($\varepsilon_z = 0$), and the stress state is a bulk state. The further modifications of this method were aimed at determining the radial component of the stress tensor by measuring the circular strains of the rigid holder by strain transducers. Such approaches allow one to determine the stress tensor components σ_z and σ_r and the shear strength $\tau = \sigma_z - \sigma_r$ of soil.

The modified SHPB method, which was earlier described and analyzed in detail [8–10], allows one, in a single experiment, to determine the principal components of the stress tensor and obtain the uniaxial strain σ_z–ε_z curves, the bulk compressibility p–ρ curves, the τ–p dependence of the shear resistance on the pressure, and the lateral earth pressure coefficient $\xi = \sigma_r/\sigma_z$.

The main concepts of the experimental method for determining the dynamic soil characteristics with the description of the wave processes which occur in the SHPB system were described in detail in [12]. We recall that the load on the holder on the side of the earth, which is characterized by the stress component σ_r, is related to the circular strain ε_θ, which is registered on the outer surface of the holder, by the known analytic solution of the problem of stress–strain state of a thick-walled tube segment under the action of constant internal pressure in the case of elastic deformations of the holder. Earlier [12], the maximum radial stresses were determined under which the deformation of the holders made of the aluminum alloy D16T and steel 30KhGSA is elastic.

Figure 1 illustrates the averaged data obtained in nine tests with sand of air humidity and density $1.5\,\mathrm{g/cm^3}$ by the modified Kolsky method at various load levels. The markers show the shock compressibility (a) and shear resistance (b) curves in GPa, and the solid lines are the approximating dependencies.

To determine the pressure in a specimen in plane wave experiments, it is necessary to measure two components of the stress, which, in principle, is possible by using two-component pressure transducers, for example, manganin transducers. But methodologically, it is very difficult to perform this, because soil media are inhomogeneous and the operation life of the measuring transducers is very short, because they are "short-circuited" by soil particles.

The experiments aimed at determining the shock compressibility of soils are based on the reflection method [22]. A plate is thrown on the specimen through a screen-plate. The thicknesses of the impactor plate, the screen plate, and the specimen are chosen so that the unloading waves from the free surfaces do not distort the picture of one-dimensional deformation in the compression wave. It is assumed that the impactor and screen material SA curves and the initial density of the investigated material are known.

The experimental plant for plane wave shock experiments consists of a single-stage gas gun and a chamber with a delivery device where the assembly with a soil specimen is placed [15]. The impact velocity varies in the range of 100–500 m/s and is measured by electrical contact transducers whose error does not exceed 2%. The speed of the compression wave propagation in the specimen is determined by two dielectric pressure transducers located on the specimen surface. As the plane wave passes through the specimen with transducers, electric signals are generated on the transducer plates, and these signals are registered by the digital storage oscilloscope.

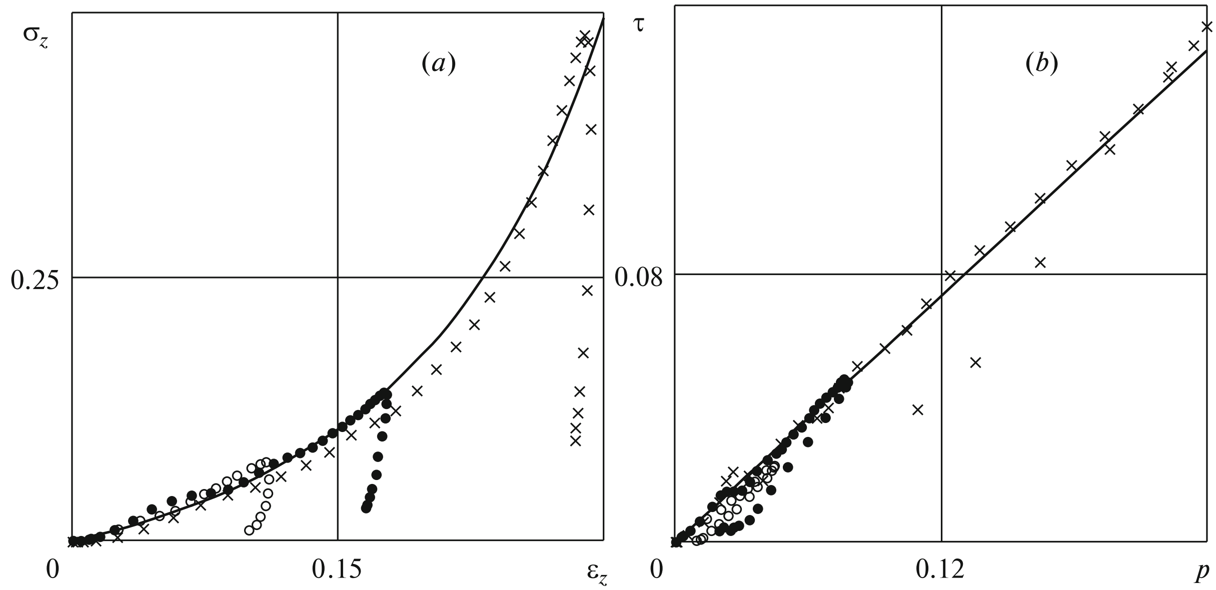

Fig. 1.

The thicknesses of the impactor plate, the screen plate, and the specimen are chosen so that the unloading waves from the free surfaces can not distort the picture of one-dimensional deformation in the compression wave. The measurement of the impact velocity V and the speed D of the compression wave propagation in the specimen together with the known adiabatic curves of the impactor and the screen plate allow one to determine the SA point of the considered medium [11]. Tests performed at various impact velocities allow one to obtain a series of SA points for the material, and this series is well described by the linear dependence [13, 14, 21]

$$D = A + BV. \tag{2.1}$$

Here the value of the constant A is approximately equal to the value of the speed of the plane compression wave propagation in soil under small pressures; the value of B characterizes the limit compressibility of soil.

Dependence (2.1) and the Hugoniot conditions imply the well-known relation between the stress σ_z (it is assumed that $\sigma_z > 0$) and the bulk strain $\varepsilon_z = \theta$ [13]

$$\sigma_z(\theta) \equiv \frac{\rho_0 A^2 \theta}{(1 - B\theta)^2}, \quad \theta = 1 - \frac{\rho_0}{\rho}. \tag{2.2}$$

As was previously mentioned, the use of two complementary methods with the same type of the stress–strain state allows one to construct unified σ_z–ε_z curves under the uniaxial strain conditions in a wide range of loading parameters. But the mathematical models of deformation (equations of state) with a relationship between the pressure p and the density ρ are widely used in many practically important problems of soil media dynamics.

A simple method is proposed to determine the p–ε_z or p–ρ dependence in the range of loads up to several GPa on the basis of the obtained shock adiabatic σ_z–ε_z curve and the τ–p dependence determined by the Kolsky method. Namely, for loads greater than 500 MPa, the shock adiabatic curve is determined in the plane wave experiments; in the range of loads up to 500 MPa, the test results are used in the Kolsky method to determine the τ–p shear resistance–pressure dependence, which is approximated by the linear function

$$\tau = C + \tan\varphi \cdot p. \tag{2.3}$$

Under the assumption that the dependence (2.3) preserves its linearity and in the case of pressures up to several gigapascals, the pressure p is determined from the stress axial component σ_z with SA by the formula

$$p = \sigma_z - \frac{4\tau}{3} = \frac{\sigma_z - 4C/3}{1 + 4\tan\varphi/3}. \tag{2.4}$$

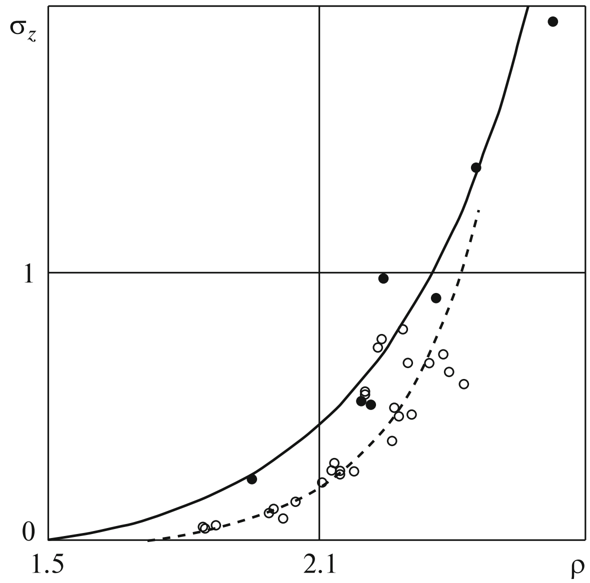

Fig. 2.

Thus, for $C \approx 0$, the pressure dependence on the bulk strain can be described by the formula

$$p = \frac{\rho_0 a^2 \theta}{(1 - b\theta)^2}. \tag{2.5}$$

In the case of the linear dependence (2.3) of the shear resistance on the pressure, the parameters a and b in (2.5) are related to the SA parameters by $a = A/\sqrt{1 + 4\tan\varphi/3}$ and $b = B$.

In Fig. 2, the dark and light points present the results of plane wave [20, 21] and reversed [17] experiments in the form of stress (GPa) dependence on the density; the solid and dashed lines present the approximating dependencies of the form (2.2) with the constants $\rho_0 = 1.5\,\mathrm{g/cm^3}$, $A = 511\,\mathrm{m/s}$, $B = 1.72$ and $\rho_0 = 1.72\,\mathrm{g/cm^3}$, $A = 460\,\mathrm{m/s}$, $B = 2.35$, respectively. The unified shock compressibility curve (solid line) of sand soil with the initial density $1.5\,\mathrm{g/cm^3}$ (also see Fig.1 a) agrees well with the earlier obtained results [14] for sand of nearly the same density. We also note that the SA curves of sand with various initial densities approach each other for pressures greater than 1 GPa.

3. MODEL REPRESENTATION OF SOFT SOIL MEDIA

The mathematical model [1] of the dynamics of soft soil media can be written in the cylindrical coordinate system roz (where oz is the symmetry axis) as the system of equations

$$\begin{aligned}
&\frac{dr}{dt} + r(v_{r,r} + v_{z,z}) = -\frac{rv_r}{r},\\
&r\frac{dv_r}{dt} - s_{rr,r} - s_{rz,z} = \frac{2s_{rr} - s_{zz}}{r},\\
&r\frac{dv_z}{dt} - s_{rz,r} - s_{zz,z} = \frac{s_{rz}}{r},\\
&D_J s_{ij} + H(J_2 - f_2^2(p)/3)s_{ij} = Ge_{ij} \quad (i, j = r, z),\\
&\frac{d\rho_*}{dt} = \frac{d\rho}{dt} H(\rho - \rho_*) H\left(\frac{d\rho}{dt}\right),
\end{aligned} \tag{3.1}$$

$$p = f_1(\rho, \rho_*) H(\rho^* - \rho) H(\rho_0 - \rho), \tag{3.2}$$

where the following notation is used: ρ_0, ρ, and ρ_* are the initial, current and maximum density attained in the loading process, v_i, σ_{ij}, s_{ij}, and e_{ij} are the respective components of the velocity vector, Cauchy stress tensor, and deviators of the stress and strain rate tensors ($i, j = r, z$), p is the pressure, D_J is the

Yaumann derivative, d/dt is the total derivative with respect to time, G is the shear modulus of the soil medium, $J_2 \equiv 0.5s_{ij}s_{ij}$ is the second invariant of the stress tensor in the von Mises–Schleicher plasticity condition, and sums are taken over repeated indices. The symbol after comma denotes differentiation with respect to the corresponding variable.

Let us specify the form of the functions f_1 and f_2 in Grigoryan's model (3.1), (3.2) of a soil medium. The shear resistance of the medium is determined by the fractional-rational dependence of the yield point on the pressure

$$f_2(p) \equiv \sigma_0 + \frac{kp}{1 + kp/(\sigma_m - \sigma_0)}, \tag{3.3}$$

where σ_0 and σ_m are the adhesion and the limit value of the yield point and the coefficient k characterizes the internal friction of soil.

Under high pressures, the shear resistance is often neglected if the so-called hydrodynamic approximation $f_2(p) \equiv \sigma_0$ is used; under relatively small pressures, the linear dependence is used,

$$f_2(p) \equiv \sigma_0 + kp, \quad \sigma_0 = 2C, \quad k = 2\tan\varphi. \tag{3.4}$$

Along with the irreversible shear strains, irreversible bulk strains are also typical of soft soils because of their high porosity. The initial stage of soil deformation is assumed to be linearly elastic. The parameters of the loading curve segment under pressures exceeding the elasticity limit were determined in Section 2, and with (2.2), (2.5), and (3.4) taken into account, we obtain $f_1(\theta) \equiv \rho_0 a^2\theta/(1 - b\theta)^2$, $a^2 = A^2/(1 + 2k/3)$, and $b = B$. The relations between the pressure and the bulk strain or the density in soil are generally assumed to be distinct for loading and unloading [16, 19, 23]. A procedure for constructing the composite dynamic diagram of compressibility and the unloading curves of soft soil was proposed earlier [16].

As is well known, one of the main hypotheses in the Kolsky method is the assumption that the specimen SSS is homogeneous. This assumption was verified by numerical computations by the using the packet of applied programs "Dynamics-2" [24]. The computations were performed for the bars and the holder produced of D16T alloy with Young modulus $E = 70$ GPa, Poisson's ratio $= 0.3$, and density $\rho = 2.7$ g/cm^3. For sand, the density was $\rho_0 = 1.5$ g/cm. We obtained the following constants of the shock adiabatic curve (2.1): $A = 511$ m/s and $B = 1.72$; the shear modulus was $G = 150$ MPa; and the constants of the dependence (3.4) of the yield point on the pressure were $k = 1.2$ and $\sigma_0 = 0$.

The results of numerical computations of the deformation processes in the SHPB system are illustrated in Fig. 3. The upper group of curves illustrates the dependencies of the axial stresses (MPa) on time (μs) on the lateral surface (solid line) and the specimen center (dashed line) compared with the experimental data (dark points). The figure also shows the boundaries of the confidence interval, which were determined with reliability 0.94 in six experiments.

A good agreement of the results is observed at both stages of loading and unloading. The lower group of curves corresponds to the calculated values of the radial stresses on the lateral surface (solid line) and at the center of the specimen (dashed line). The light points in this figure shown the experimental data obtained by measuring the circular strains of the holder.

Thus, the results illustrated in Fig. 3 testify that the specimen stress state is homogenous within the limits of the experimental data spread. The difference between the results of numerical experiment from the results of the natural experiment does not exceed the error of the natural experiment, which testifies that the determination of the soil model parameters is correct in the considered range of loads.

4. MODEL VERIFICATION IN PENETRATION PROBLEMS

Consider the problem of penetration of a cylindrical impactor with plane end surface into sand soil with a constant velocity. System (3.1), (3.2) of dynamic deformation equations for the soil medium is supplemented with initial and boundary conditions. On the impactor head part contacting with the soil medium, we pose the condition that the impactor is "impenetrable" in the normal direction and in the direction of particle sliding along the tangent with dry friction according to the mixed model of friction. On the soil and impactor free surfaces, the normal and tangent stresses are set to be zero, because the soil flow around the cylindrical part adjacent to the impactor end surface is of detached character. The outer boundaries of the soil computation regions are assumed to be rigid and correspond to the holder

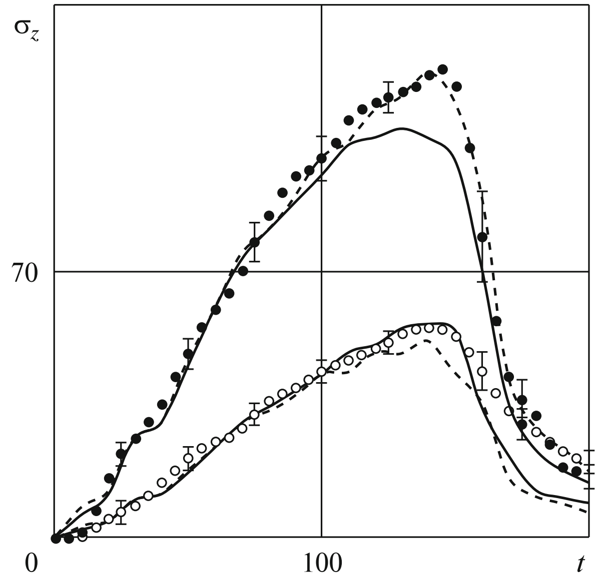

Fig. 3.

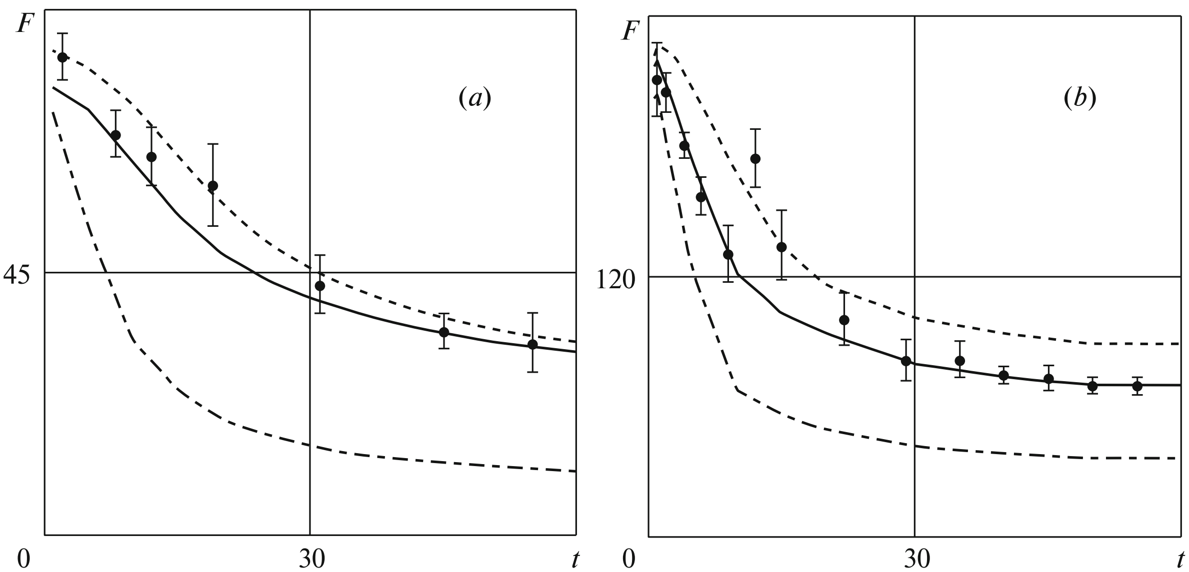

Fig. 4.

geometry used in the experiment [17]. At the initial time, the stresses and the velocity of soil particles are zero. The impactor is assumed to be rigid and moving at a constant velocity equal to the impact velocity.

The sand soil is a dry mixture of quartz sand of natural composition with initial density $\rho_0 = 1.72 \mathrm{g/cm}^3$. The SA parameters $A = 460\,\mathrm{m/s}$ and $B = 2.3$ of this soil were determined earlier in reversed experiments [17] for the impact velocities $V = 50{-}450\,\mathrm{m/s}$.

The markers in Fig. 4 show the experimentally obtained dependencies of the force (kN) of resistance to the impactor penetration with plane end surface into sand versus time (μs) for the impact velocities $180 \pm 10\,\mathrm{m/s}$ (a) and $335 \pm 15\,\mathrm{ms}$ (b). The solid, dashed, and dash-pointed lines correspond to the computation results obtained by using the nonlinear and linear dependencies (3.3) and (3.4) of the yield point on the pressure and in the hydrodynamic approximation ($f_2 \equiv 0$).

To compare the computation results with experimental data, the maxima of the resistance force in Fig. 4 were matched at $t \approx 0$. As was previously noted [19, 23], the computation results obtained by using the linear dependence of the yield point on the pressure satisfactorily agree with the experimental

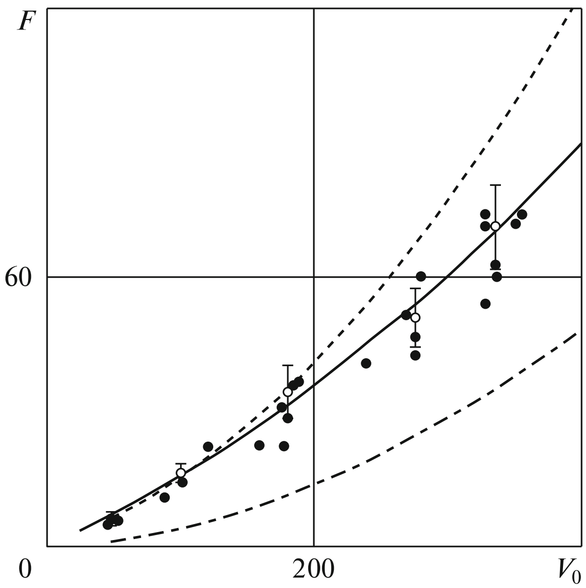

Fig. 5.

data only for the impact velocities less than 200 m/s. The differences between the results of numerical modeling obtained by using the nonlinear dependence of the yield point on the pressure which was obtained as a result of the identification [18, 19] and the experimental data lie within the natural data spread. The hydrodynamic approximation turns out to be unsatisfactory in this range of impact velocities.

Figure 5 illustrates the dependence of the force (kN) of resistance to the impactor penetration with plane end surface into sand at the quasistationary stage on the impactor velocity (m/s). The dark markers correspond to experimental results, the light markers correspond to the impact velocities $V_0 = 48, 100, 180, 275, 335$ m/s and were obtained as averaged over five tests, and the intervals show the error with the confidence probability equal to 0.95. The lines correspond to the results of numerical computations (the same notation as in Fig. 5).

5. CONCLUSION

The paper deals with the complex experimental-theoretical approach to solving the problem of high-rate strain of soft soil media, which combines the contemporary method of dynamical tests, the study of the impact deformation processes and obtaining the dynamical properties of soils on this basis, the choice of contemporary mathematical models and their constitutive relations which adequately describe the basic effects of high-rate strain, the identification of the constitutive relations by using experimentally obtained data, and their verification by comparing the results of computational and nature test experiments on impact and penetration. The common experimental and computational studies permit significantly increasing the reliability of the obtained results, which opens broad prospects in complex studies of nonlinear effects and laws of dynamical deformation of soft soil media.

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