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Relaxation Model of Dynamic Plastic Deformation of Materials

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Abstract—A version of the metal plasticity relaxation model based on a plasticity integral criterion with the characteristic relaxation time parameter is suggested. The dislocation concepts of metal plasticity together with the Maxwell model for a strongly viscous fluid are used to show that this characteristic relaxation time parameter can be interpreted in terms of dissipation and energy accumulation in the case of mobile dislocations. The coincidence of the values of characteristic plastic relaxation time obtained for various descriptions of the whisker deformation allows one to conclude that the characteristic relaxation time is a basic characteristic of the material dynamic properties.

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1. INTRODUCTION

Several cases of unexpected mechanical behavior of metals in deformation can readily be explained by taking their dynamic characteristics into account. One such well-known paradox is the sharp yield point phenomenon in metal whiskers [1, 2] when, in the case of quasistatic loading rates 10^{-4}–10^{-2} s^{-1}, the stresses attained in the material are dozens times greater than the value of its quasistatic yield point [3]. This effect can be explained in terms of the material "dislocation starvation" [4], when the amount of mobile defects is insufficient for ensuring the required plastic strain rate and the elastic stresses continue to grow. From the mechanical standpoint, this phenomenon expresses the fact that the material already experiences dynamic deformations for such "quasistatic" strain rates. Namely, for a material with such properties, these strain rates are already sufficient for exciting the dynamic deformation mode, which cannot be described on the basis of quasistatic concepts that there are critical flow stresses in the material. To describe the dynamic characteristics of the material, it is necessary to explicitly take into account the fact that the material cannot unboundedly experience plastic relaxations of the arising stresses and a certain time is required for the development of relaxation process inside the material. This can be attained by introducing an additional dynamic characteristic of the material itself, i.e., the characteristic plastic relaxation time parameter. Obviously, this parameter should be independent of the deformation process characteristics but should express the mechanical properties of the material itself in a wide range of loading rates.

2. VERSION OF PLASTICITY RELAXATION MODEL BASED ON INTEGRAL YIELD CRITERION

For the Voigt solid, the stress—strain dependence has the form

$$\sigma = 2G\varepsilon + \mu\dot{\varepsilon}, \tag{2.1}$$

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which can also be rewritten as $\dot{\varepsilon} + 2G\varepsilon/\mu = \sigma/\mu$. It is assumed that, along with elastic stresses in the dynamic strain mode, there is a certain additional viscous term proportional to the strain rate. Multiplying both sides of (2.1) by $2G\exp(2G/\mu)$, we obtain

$$2G\frac{d}{dt}\left[\varepsilon \cdot \exp\left(-\frac{t}{\tau}\right)\right] = \frac{\sigma(t)}{\tau}\exp\left(-\frac{t}{\tau}\right), \tag{2.2}$$

where the characteristic time $\tau = \mu/(2G)$ is introduced. By integrating (2.2) over time, we obtain

$$2G\varepsilon = \frac{1}{\tau}\int_0^t \sigma(s)\exp\left(-\frac{t-s}{\tau}\right) ds. \tag{2.3}$$

The left-hand side of this equation is the elastic stress in the quasistatic setting, where $\dot{\varepsilon} = 0$ in (2.1). Equation (2.2) can be interpreted in the framework of the "fading memory" concept [5] as follows: the contribution to the current state of the defect structure made by the load previously acting at times $s \ll t$ is much less than the contribution of the recently acting load. The current stress values must be replaced by its "already relaxed" value, and the stresses that earlier acted in the material enter the equation with constantly decreasing weight coefficients. In the general case, this implies the inequality in integral form

$$\int_0^t \sigma(s)K(t-s)\, ds \le \sigma_y^0, \tag{2.4}$$

where the kernel of the integral operator $K(t)$ is the memory decay function. By comparing (2.3) with (2.4), we see that the Voigt model is associated with the exponential memory decay

$$K(t) = \frac{1}{\tau}\exp\left(-\frac{t}{\tau}\right), \tag{2.5}$$

where, as in (2.3), the characteristic time has the form $\tau = \mu/(2G)$. As a simpler approximation to the exponential law (2.5), one can suggest to describe the decay by the step function [6]

$$K(t) = \begin{cases} \dfrac{1}{\tau}, & 0 \le t \le \tau, \\ 0, & t > \tau. \end{cases} \tag{2.6}$$

Then Eq. (2.6) implies the metal integral yield criterion in the form [6, 7]

$$\frac{1}{\tau}\int_{t-\tau}^t \left[\frac{\Sigma(s)}{\sigma_y^0}\right]^\alpha ds \le 1. \tag{2.7}$$

Here $\Sigma(t)$ is the function describing the time dependence of the stress, τ is the characteristic stress relaxation time, σ_y^0 is the static yield point, and α is the stress sensitivity coefficient. Under the assumption of elastic deformation $\Sigma(t) = 2G\varepsilon(t)$, this criterion allows one to compute the time t_* of appearance of the macroscopic yield corresponding to the time at which (2.7) becomes an equality. The phenomenological approach based on the concept of incubation yield time [7] also allows one to take into account the plastic deformation evolution and describe the sharp yield point effect. Consider the simplest version of such a model. We assume that the deformation in the specimen increases linearly as $\varepsilon(t) = \dot{\varepsilon}tH(t)$, where $H(t)$ is the Heaviside function. We introduce the dimensionless relaxation

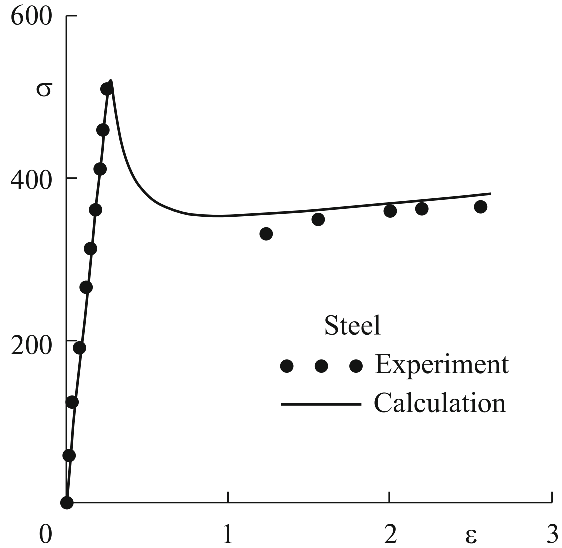

Fig. 1.

function $0 < \gamma(t) \leq 1$ by the condition

$$\gamma(t) = \begin{cases} 1, & \text{for } \dfrac{1}{\tau}\displaystyle\int\limits_{t-\tau}^{t}\left[\frac{\Sigma(s)}{\sigma_Y}\right]^{\alpha} ds \leq 1, \\ \dfrac{1}{\left\{\dfrac{1}{\tau}\displaystyle\int\limits_{t-\tau}^{t}\left[\dfrac{\Sigma(s)}{\sigma_y^0}\right]^{\alpha} ds \leq 1\right\}^{1/\alpha}}, & \text{for } \dfrac{1}{\tau}\displaystyle\int\limits_{t-\tau}^{t}\left[\frac{\Sigma(s)}{\sigma_Y}\right]^{\alpha} ds > 1. \end{cases} \tag{2.8}$$

Here $\Sigma(t) = 2G\dot{\varepsilon}tH(t)$ is the function coinciding with the stress in the specimen at the stage of elastic deformation, i.e., before the time t_* of appearance of the macroscopic plastic flow; this time is calculated by criterion (2.7). We assume that the following condition is satisfied at the subsequent times $t \geq t_*$ corresponding to the plastic deformation of the material:

$$\frac{1}{\tau}\int\limits_{t-\tau}^{t}\left[\frac{\gamma(t)\Sigma(s)}{\sigma_y}\right]^{\alpha} ds = 1. \tag{2.9}$$

The actual stresses in the deformed specimen for $t \geq t_*$ are determined by the relation

$$\sigma(t) = 2g(t)\varepsilon(t), \tag{2.10}$$

where $g(t) = G^{1-\beta}(t)$ and β is a scalar parameter ($0 \leq \beta \leq 1$) which controls the strengthening degree. (The case $\beta = 0$ corresponds to the absence of strengthening.) Figure 1 illustrates an example of calculations of the deformation curve in the case where the deformation linearly increases with time on the basis of (2.7)–(2.10) for a hypothetical material with deformation strengthening approximately corresponding to soft steel [8], ($G = 100$ MPa, $\sigma_Y = 200$ MPa, $\beta = 0.18$, $\tau = 0.4$ s, and $\dot{\varepsilon} = 8.5 \times 10^{-3}\,\text{s}^{-1}$). The crystal strain rate is $\dot{\varepsilon} = 8.5 \times 10^{-3}\,\text{s}^{-1}$, which corresponds to the case of sufficiently slow "quasistatic" loading. Nevertheless, in the first seconds of the deformation, the stresses behaves nonmonotonically and their maximum value is by an order of magnitude greater than the value of the static yield point for this material. The experimental points are taken from [8] (from now on, the stresses on the ordinate axis are measured in MPa). The simulation of deformation of copper and cadmium whiskers in a setting similar to the experiments [9, 10] on the basis of (2.7)–(2.10) shows that the experimental data correspond to $\tau = 11$ for copper and $\tau = 9$ for cadmium, and this is several orders of magnitude greater than the relaxation times obtained for base metals [5–7].

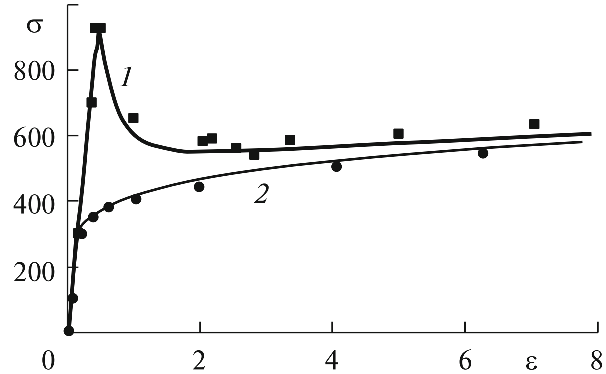

Fig. 2.

Figure 2 illustrates the simulation of the sharp yield point effect with the subsequent deformation strengthening according to the model (2.7)–(2.10) in the case of deformation of steel specimens ($G = 78\,\mathrm{GPa}$, $\sigma_Y = 310\,\mathrm{MPa}$, $\alpha = 1$). The points correspond to the experimental data given in [11]. In this material, the sharp yield point effect, which is typical of whiskers, is already not observed at quasistatic strain rates $t \times 10^2\,\mathrm{s}^{-1}$ (curve *2*), but this effect is clearly observed in the case of dynamic deformation at the rates $10^3\,\mathrm{s}^{-1}$ (curve *1*). The points correspond to the recently obtained experimental data [11]. It is remarkable that in this case the obtained value of relaxation time scale is already equal only to 14 microseconds.

3. KINETIC REPRESENTATION OF THE RELAXATION PROCESS IN PLASTIC DEFORMATION

Another possible approach to the description of dynamic deformation with the characteristic plastic relaxation time parameter taken into account is the Maxwell model for a strongly viscous liquid [12]. In the framework of this model, it is assumed that there is a stress relaxation time τ such that the material experiences elastic deformation at smaller times and viscoplastic deformation at greater times. The equation of variations in the shear stresses σ_τ can be written as [12]

$$\frac{d\sigma_\tau}{dt} = 2G\frac{d\varepsilon}{dt} - \frac{\sigma_{\mathrm{eff}}}{\tau}H(\sigma_{\mathrm{eff}}), \tag{3.1}$$

where G is the shear modulus of the material, $\sigma_{\mathrm{eff}} - \sigma_\tau - \sigma_y$ is the acting effective stress such that the shear stress relaxation is possible for its positive value, σ_y is the barrier stress which it is necessary to overcome for dislocations to be able to slide, and $H(x)$ is the Heaviside function. By integrating (3.1) with the constant strain $\dot{\varepsilon} = \mathrm{const}$ and the initial condition $\sigma_\tau(0) = \sigma_y$, we obtain the following time dependence of the stresses [12]:

$$\sigma_\tau(t) = \sigma_y + 2G\tau\dot{\varepsilon}\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]. \tag{3.2}$$

In the case of a steady-state flow, the expression (3.2) determines the level of maximal stresses attained in the material, and it can be rewritten as

$$\sigma_\tau^{\max} = \sigma_y + 2G\tau\dot{\varepsilon}. \tag{3.3}$$

The maximum shear stresses (3.3) are proportional to the yield point of the material. It follows from (3.3) that, in the case of small characteristic times and strain rates, we have a constant value of the yield point, which can vary only due to the strain strengthening. On the other hand, at high strain rates or at large characteristic relaxation times, the second term in (3.3) becomes significant and the dependence of the yield point on the strain rate is rather complicated.

We use Eq. (3.1) to estimate the characteristic time of plastic relaxation due to the dislocation sliding. Consider the dislocation model of crystal plasticity, which states that the plastic strain is caused by displacements of separate defects of the crystal structure [3]. At times exceeding the acceleration time of dislocations (of the order of 10^{-10} s), we can assume that their motion is steady-state and $\dot{\sigma} = 0$.

Then, under the condition that the high external stresses are significantly greater than the barrier stresses $\sigma_\tau \gg \sigma_y$, it follows from (3.2) that

$$\sigma_\tau = 2G\tau\dot{\varepsilon}. \tag{3.4}$$

In [13], it was shown that, by substituting (3.4) into the Orovan relation [3] and by using the dislocation dynamic equation [14–16], we obtain the following expression for the characteristic stress relaxation time:

$$\tau_{\rm D} = \frac{B_f}{Gb^2\rho_{\rm D}} \sim \frac{B_f}{E_{\rm D}}, \tag{3.5}$$

where $E_{\rm D}$ is the total elastic energy of dislocation lines per unit volume and $B_f = (v_D/2)\rho b^2$ characterizes the rate of scattering of the dislocation kinetic energy [16]; $v_D \sim 10^{13}\,\mathrm{s}^{-1}$ is the nearly Debye frequency whose inverse value gives the time of scattering of the kinetic energy of mobile dislocations and ρ is the material density. The time (3.5) is inversely proportional to the dislocation density and can vary in sufficiently broad ranges. At the initial deformation stages, which are important for the sharp yield point effect, the relaxation time is completely determined by the density of dislocations which are present in the material before the deformation.

From the standpoint of the above-mentions models of an elastoviscoplastic body, there are no fundamental differences between the shear strength in the case of "quasistatic" strain and in the case of "dynamic" strain. Everything is determined by the values of the parameters describing the internal structure and the defect substructure of the material. The transition between the "static" and "dynamic" modes occurs according to (3.3) at the strain rates $\dot{\varepsilon}_{\rm tr} \sim y_b/(2G\tau) \equiv y_b/\mu$, where the coefficient $\mu = 2G\tau$ characterizing the dynamic viscosity depends on the elastic properties and the defect substructure of the metal. For various defect concentrations in the material, this strain rate can take practically any values.

The defect structure and the mechanical properties of the material vary in the course of plastic deformation. The characteristic time variation is structurally determined by variations in the dislocation density in the material which, in the absence of deformation strength ($\sigma_y(\varepsilon)$ which holds at the initial stages of deformation in problems of the sharp yield point effect), can be represented as

$$\rho_{\rm D}(\varepsilon_{\rm pl}) = \rho_{\rm D}^{\max} + (\rho_{\rm D}^0 - \rho_{\rm D}^{\max})\exp(-\varepsilon_{\rm pl}k_\alpha). \tag{3.6}$$

By substituting (3.6) into (3.5), for the characteristic relaxation time we have

$$\tau(\varepsilon_{\rm pl}) = \frac{B_f}{Gb^2\rho_{\rm D}^{\max} + (\rho_{\rm D}^0 - \rho_{\rm D}^{\max})\exp(-\varepsilon_{\rm pl}k_\alpha)}. \tag{3.7}$$

Finally, by substituting (3.7) into (3.3), we obtain an equation for calculating the dependence of maximum stresses (attained in the material in the process of its plastic flow, i.e., the dynamic yield point) on the degree of deformation.

The total strain of the material is the sum of the plastic and elastic parts each of which varies in the course of deformation. The value of the plastic deformation is determined by integrating the Orovan relation [3] and generally depends on the dislocation velocity and on the their density [14–16], but in the case of whicker deformation, the density of mobile dislocations varies by more than six orders of magnitude while the variations in their velocity are insignificant. Therefore, to estimate the fraction of plastic deformation, we can assume that it is proportional to the dislocation density and use the approximate relation $\varepsilon_{\rm pl} = (\rho_{\rm D}/\rho_{\rm D}^{\rm flow})\varepsilon$, where $\rho_{\rm D}^{\rm flow} \sim 10^{13}\,\mathrm{m}^{-2}$ is the dislocation density corresponding to the purely plastic deformation of the material.

The results of calculations according to this model were compared with experimental data [8–11]. For the constants we used the tabulated parameters [17] and the constants of the dislocation plasticity model [14–16]. At the initial state of deformation, the shear stresses increase elastically as $\sigma_\tau^{\rm el}(\varepsilon) = 2G\varepsilon$. The plastic stress relaxation begins when the condition $\sigma_\tau^{\rm el} \geq$ is satisfied, and these stresses must be recalculated with the relaxation mechanisms of plasticity taken into account. If the amount of dislocations in the material is small, then the elastic stresses still continue to grow for a certain time until the dislocation density becomes sufficient for the stress relaxation rate to exceed the rate of growth of elastic stresses. For example, for the strain rate $10^{-2}\,\mathrm{s}^{-1}$, we have the time $\tau \sim 0.1$ s and $2G\tau\dot{\varepsilon} \sim 100$ MPa already for the dislocation density $\rho_{\rm D} \sim 10^5\,\mathrm{m}^{-2}$ which is quite comparable with the values of σ_y; on the

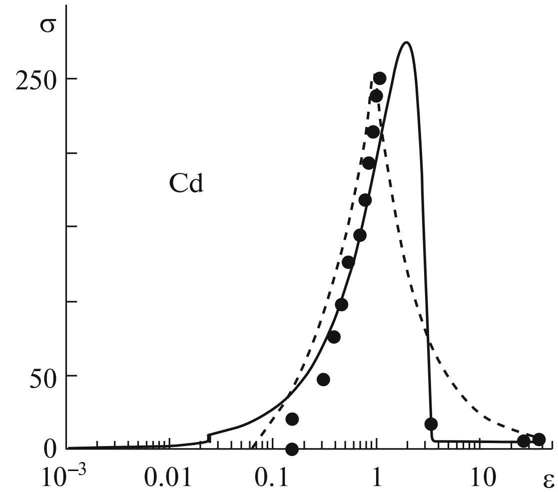

Fig. 3.

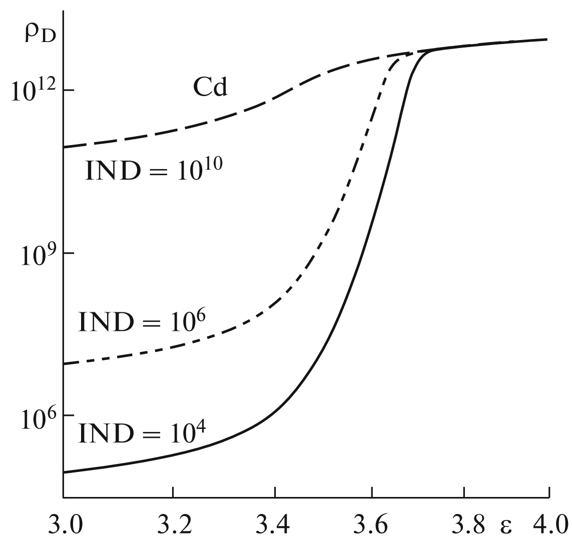

Fig. 4.

other hand, this correction decreases to hundredth fractions of megapascal and becomes negligibly small as the dislocation density increases to normal values $\rho_{\mathrm{D}} > 10^{10}\,\mathrm{m}^{-2}$.

The calculations describing the deformation of cadmium and copper specimens were performed, the comparison with experimental data [8] was used to determine the values of the initial dislocation density (IND), and the values of characteristic times of the material deformation were calculated by (3.5). In the case of copper deformation, we obtained $\rho_{\mathrm{D}}^{0(Cu)} \sim 10^2\,\mathrm{m}^{-2}$ and $\tau_{\mathrm{D}}^{0(Cd)} \sim 10\,\mathrm{s}$; in experiments with cadmium, we obtained $\rho_{\mathrm{D}}^{0(Cu)} \sim 10^4\,\mathrm{m}^{-2}$ and $\tau_{\mathrm{D}}^{0(Cd)} \sim 4\,\mathrm{s}$.

Figure 3 illustrates three deformation modes at the rate $2.2 \times 10^{-2}\,\mathrm{s}^{-1}$, which are significantly different. The solid curve presents the results of calculations by the Maxwell model with the dislocation kinetics taken into account, where the characteristic time is four seconds. The dash-dotted line corresponds to calculations by the model (2.7)–(2.10), where the characteristic time is nine seconds. The purely elastic mode is realized until the value of the static yield point σ_y is attained, and then the plastic flow begins, but since the amount of dislocations in the material is insufficient, the stresses continue to grow in the almost elastic mode. In this case, the characteristic relaxation time is assumed to be constant, and it corresponds to the initial dislocation density. The graphs of dislocation density dependence on the strain, which are given in Fig. 4 for various values of the initial dislocation density

(IND), show that the dislocation density grows rather slowly until the value of the of order of $10^6\,\mathrm{m}^{-2}$ is attained; after this it increases by six orders of magnitude while the degree of deformation varies only by 0.1%–0.2%. At this strain rate ($2.2 \times 10^{-2}\,\mathrm{s}^{-1}$), this occurs in time $t \sim \varepsilon/\dot{\varepsilon}$, which is approximately 0.1 s. In this case, the "width" of the sharp yield point does not exceed 4% of the total deformation, which takes nearly 2 s of time. All these times are significantly less than the obtained characteristic relaxation time of the material. Figure 4 also demonstrates that the significant influence of plasticity begins as the deformation exceeds 3%. Comparing Fig. 3 with Fig. 4, we see that all first experimental points are actually in the region of purely elastic deformation. In particular, this justifies our neglect of the deformation strengthening phenomenon when modeling the sharp yield point effect. Figure 3 shows that, in the case of deformations exceeding 2%, the flow stress sharply decreases to the value of the static yield point which is attained at deformations nearly equal to 4%. In this region, a significant increase in the dislocation density leads to an appropriated decrease in the relaxation time (3.5). The kinetic equation for the dislocation density (3.6) is important for describing the stress drop only at this stage and does not play any significant role in the remaining part of the whisker deformation region.

The relaxation times obtained by the structure approach (3.5) agree well with the times obtained on the basis of the phenomenological plasticity model determined by the criterion (2.7), (2.9). This coincidence of the relaxation times calculated by various approaches allows one to assume that the expression (3.5) correctly describes the relationship between the material internal structure characteristics and the macroscopic parameter of characteristic times of plastic deformation and can be used to determine them. On the other hand, this also shows that the dynamic approach is rather general and the relaxation processes determined by characteristic plastic relaxation times play a fundamental role in the description of deformation processes. In this sense, any specific realization of the relaxation mechanism in the framework of the chosen plasticity model is of minor importance.

4. CONCLUSION

Thus, the dynamic deformation mode is determined not by external factors but by the relation between the strain rate and the material ability to relaxation of elastic stresses arising in it. The value of the characteristic plastic relaxation time determines the relaxation properties of the material and is one of its basic dynamic characteristics. This time is independent of the strain rate and is determined at the microlevel by the dynamic properties of the basic carriers of plastic deformation and by their amount per unit volume. The coincidence of the values of characteristic time obtained by various approaches confirms the claim that its nature is universal. At relatively small strains, the mechanical properties are determined by a single value of the characteristic time corresponding to the initial dislocation density. At strong plastic deformations, it is necessary to take into account the irreversible changes in the defect structure of the material, which can be described by simple kinetic equations for the dislocation density variations [13–15]. Macroscopically, this can be expressed as a decrease in the characteristic time of plastic deformation with time. In the case of deformation of metal whickers, the characteristic relaxation time turns out to be very large; i.e., it is several orders of magnitude greater than the relaxation times typical of usual specimens of metals with large dislocation densities [5].

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