

Acta Geophysica

vol. 59, no. 5, Oct. 2011, pp. 935-953 DOI: 10.2478/s11600-011-0034-0

Shale Volume Estimation Based on the Factor Analysis of Well-Logging Data

Norbert P. SZABÓ

University of Miskolc, Department of Geophysics, Miskolc, Hungary e-mail: norbert.szabo.phd@gmail.com

A b s t r a c t

In the paper factor analysis is applied to well-logging data in order to extract petrophysical information about sedimentary structures. Statistical processing of well logs used in hydrocarbon exploration results in a factor log, which correlates with shale volume of the formations. The so-called factor index is defined analogously with natural gamma ray index for describing a linear relationship between one special factor and shale content. Then a general formula valid for a longer depth interval is introduced to express a nonlinear relationship between the above quantities. The method can be considered as an independent source of shale volume estimation, which exploits information inherent in all types of well logs being sensitive to the presence of shale. For demonstration, two wellbore data sets originated from different areas of the Pannonian Basin of Central Europe are processed, after which the shale volume is computed and compared to estimations coming from independent inverse modeling.

Key words: factor analysis, maximum likelihood, factor index, factor log, shale volume.

1. INTRODUCTION

Petrophysical parameters can be derived from well-logging data by deterministic or statistical methods. Former procedures substitute data to explicit equations in order to determine non-measurable parameters separately. There are several different methods for the estimation of shale volume. The most common geophysical logs used for this purpose are natural gamma ray or spontaneous potential logs or the combination of the porosity logs such as neutron and density logs (Asquith and Krygowski 2004).

Statistics-based methods consist of mainly inversion techniques, which assume mathematical relationships between the original data and specified petrophysical parameters. The model parameters and their confidence intervals are estimated by processing data from different measurement probes in one inversion procedure. The optimal solution is obtained by fitting measured data to the theoretical ones calculated by probe response equations (Alberty and Hashmy 1984). Inversion applications solving the problem with point-by-point methods are well-documented (Mayer and Sibbit 1980, Ball *et al.* 1987, Baker Atlas 1996). Dobróka and Szabó (2005) introduced a different method using depthdependant response functions that are valid for a longer depth-interval and make it possible to derive petrophysical parameters for the same interval (instead of separated depth-points) by a joint inversion procedure. In the case when response equations do not exist, alternative statistical methods are applied in order to identify the connection between the data and the petrophysical model, *e.g.*, neural networks (Goncalves *et al.* 1995) and supervised expert systems (Peveraro and Lee 1988, Barstow 1984). Factor analysis represents a statistical approach, which is used to enhance the main information inherent in large-scale multidimensional data sets and extract non-measurable background variables. In this study it is assumed that the new variables derived by factor analysis may be connected to the petrophysical model. The basic principle of the theory of factor analysis can be found in the paper of Lawley and Maxwell (1962). At first, the method was used in psychology then it gained ground in many natural scientific fields. We can find a limited number of applications also in geophysics. Fraiha and Silva (1994) presented factor analysis as an ambiguity analysis method in gravity. Asfahani *et al.* (2005) used it for the interpretation of airborne magnetic and radiometric data for copper exploration purposes. Boguslavskii and Burmistrov (2009) studied the petrophysical properties of kimberlites and concluded on the composition and diamond content. In borehole geophysics, Urbancic and Bailey (1988) used it for gold detection. Kaźmierczuk and Jarzyna (2006) made principal component analysis on well-logging and geological data in order to evaluate lithology and saturation in a hydrocarbon field. In the present study, factor analysis is applied on wellbore data to find correlation between the extracted factors and petrophysical properties of rocks. Regression tests showed that there was a strong correlation between the factor scores and the specific volume of shale.

2. PETROPHYSICAL INTERPRETATION OF WELLBORE DATA

Well-logging measurements play an important role in exploration geophysics. The processing of well-logging data is applicable to determine essential petrophysical and geometrical properties of geological structures in the near vicinity of the borehole. Open-hole logging data contain information about porosity of rocks, water and hydrocarbon saturation in the pore space, specific volumes of shale and mineral constituents, and certain geometrical parameters, *e.g.*, the layer thicknesses of the formations. As a rule, other important quantities are derived from the interpreted parameters, *e.g.*, irreducible and movable hydrocarbon saturation and absolute permeability. In hydrocarbon exploration these quantities are especially important, because they underlie the calculation of reserves. Petrophysical parameters cannot be measured directly, but can be connected to well-logging data via theoretical probe response functions. In response functions, not only the above-mentioned quantities but mud-filtrate, pore-filling fluid and matrix characteristic values and textural parameters are also included. Normally a well-logging data set consists of lithology, porosity and saturation sensitive measurements. A typical combination of well logs used in hydrocarbon exploration is presented in Table 1.

Shale volume is treated as a basic parameter in well-log analysis. This quantity has got a strong influence on most types of well logs. By definition, shale volume expresses the ratio of the volume of clay and other fine grain particles (mainly silt) to the total volume of rock. The clay can be distributed

Table 1

Well log types frequently used in hydrocarbon exploration and their specification

in the formations in three forms. They appear as dispersed particles in the pore space or thin laminae within the sequence of layers or minerals embedded in the matrix structure of rock. Sedimentary formations contain different amount of shale, so the theoretical value of shale volume falls between 0 and 1 (or 0- 100%). If the unit volume of rock is divided into three parts, *i.e.*, pore space, shale and matrix of rock, the material balance equation specifies that

$$
\Phi + V_{\rm sh} + \sum_{i=1}^{n} V_{\text{ma},i} = 1 , \qquad (1)
$$

where Φ denotes porosity, V_{sh} is shale volume, V_{ma} is the specific volume of the i -th matrix component and n is the number of matrix constituents, *i.e.*, minerals excluding shale particles. The presence of shale affects the size of effective porosity, the quantity of movable hydrocarbon saturation and permeability, which are the most important parameters in reservoir classification. Because of the relatively high influence on the measurements, the interpretation of welllogging data requires response equations corrected for the shale effect. Shale corrected response equations can be written in the general form as

$$
d_k = \Phi d_{k,f} + V_{\rm sh} d_{k,\rm sh} + \sum_{i=1}^{n} V_{\rm ma,i} d_{k,\rm ma,i} , \qquad (2)
$$

were d_k represents the k-th measured variable, $d_{k,f}$ is the value of k-th parameter of the fluid, $d_{k,sh}$ and $d_{k,m}$ are the k-th parameters of shale and matrix, respectively. The most of the observed physical variables follow this linear type of equation excluding the specific resistivity. Shale volume can be derived either from eq. (2) independently or by inversion when all of the response equations are integrated into one interpretation procedure.

The most frequently used deterministic method for the shale volume estimation is based on the data measured by the gamma ray probe. In the first step, the gamma ray index is calculated:

$$
i_{GR} = \frac{GR - GR_{\min}}{GR_{\max} - GR_{\min}} ,
$$
 (3)

where GR denotes the gamma ray reading of the given depth-point, GR_{min} and GR_{max} are the gamma ray values of the clean formation and shale, respectively (Asquith and Krygowski 2004). For a first order approximation of shale volume

$$
V_{\rm sh} = i_{\rm GR} \tag{4}
$$

can be used, which usually overestimates the shale content of rocks. For obtaining a more realistic estimation, non-linear relationships are usually used. Larionov (1969) introduced the following formulae for shale volume calculation in sedimentary sequences:

$$
V_{\rm sh} = \begin{cases} 0.083 \left(2^{3.7i_{\rm GR}} - 1\right) & \text{Tertiary or younger rocks} \\ 0.33 \left(2^{2i_{\rm GR}} - 1\right) & \text{older rocks} \end{cases} \tag{5}
$$

In this study, factor analysis was applied in order to infer shale volume from well-logging data and an empirical relationship was established, which seemed to be valid for a large area.

3. FACTOR ANALYSIS OF WELL LOGS

Factor analysis was applied for studying a possible connection between the factor scores and petrophysical parameters. Consider the arrangement of different type of well-logging data as input for factor analysis in the matrix form

$$
\mathbf{D} = \begin{pmatrix} \mathbf{SP}_1 & \mathbf{GR}_1 & \cdots & \mathbf{RD}_1 \\ \mathbf{SP}_2 & \mathbf{GR}_2 & \cdots & \mathbf{RD}_2 \\ \vdots & \vdots & & \vdots \\ \mathbf{SP}_i & \mathbf{GR}_i & \cdots & \mathbf{RD}_i \\ \vdots & \vdots & & \vdots \\ \mathbf{SP}_N & \mathbf{GR}_N & \cdots & \mathbf{RD}_N \end{pmatrix},
$$
 (6)

where the D_{ik} element at the *i*-th row and *k*-th column of matrix **D** represents the datum observed by the k -th probe in the i -th depth point. The size of datamatrix \mathbf{D} is N-by-M, where N is the total number of depth points in the logged interval and M is the number of measurement types (*i.e.*, original variables).

The model of factor analysis can be defined as

$$
\mathbf{D} = \mathbf{F}\mathbf{L}^T + \mathbf{E} \,, \tag{7}
$$

where **F** denotes the N -by-a matrix of factor scores, **L** is the M -by-a matrix of factor loadings and **E** is the N -by- M matrix of residuals (T is the transpose symbol). The number of factors should be less than that of the original variables $(a \lt M)$. The *j*-th column of **F** represents the values of the *j*-th new variable (*i.e.*, factor) computed for different depth points. The data set of one extracted factor can be considered as a new well log and named as factor log. The matrix **L** represents the weights of the original variables on the derived factors. The factor analysis model can be written as

$$
\mathbf{R} = \mathbf{L}\mathbf{L}^T + \mathbf{\Psi} \,, \tag{8}
$$

where $\mathbf{R} = \mathbf{D}^T \mathbf{D}/N$ is the correlation matrix of the standardized original variables, and $\Psi = \mathbf{E}^T \mathbf{E}/N$ is the diagonal matrix of specific variances, which is independent of the common factors. The determination of factor loadings leads to searching the eigenvalues of $\mathbf{R}-\Psi$ matrix in eq. (8), for which a noniterative approximate solution was suggested by Jöreskog (2007). The factor scores can be estimated by the maximum likelihood method, where the following log-likelihood function is optimized

$$
\Omega = -\frac{1}{2} \left(\mathbf{D} - \mathbf{FL}^T \right)^T \mathbf{\Psi}^{-1} \left(\mathbf{D} - \mathbf{FL}^T \right) = \max . \tag{9}
$$

The computation of factor scores is based on the fulfillment of the $\partial \Omega / \partial F = 0$ condition. Assuming linearity, an unbiased estimation was suggested by Bartlett (1937)

$$
\mathbf{F} = \left(\mathbf{L}^T \mathbf{\Psi}^{-1} \mathbf{L}\right)^{-1} \mathbf{L}^T \mathbf{\Psi}^{-1} \mathbf{D} . \tag{10}
$$

For the better interpretation of factors, an orthogonal transformation of factor loadings is usually applied (Lawley and Maxwell 1962). In this research, the factors were rotated by means of the *varimax* algorithm (Kaiser 1958).

A fundamental assumption for using the maximum likelihood method is that data in eq. (6) are required to follow M-dimensional Gaussian distribution. The normality of data distributions can be verified by some empirical statistics. The skewness of data measured by the k-th logging instrument can be defined as

$$
\mu^{(k)} = \frac{\frac{1}{N} \sum_{i=1}^{N} (D_{ik} - \bar{D}_k)^3}{\left(\frac{1}{N} \sum_{i=1}^{N} (D_{ik} - \bar{D}_k)^2\right)^{\frac{3}{2}}},
$$
\n(11)

which is the ratio of the third central moment and the cube of the standard deviation (\bar{D}_k is the mean of data measured by the k-th probe). If $\mu \approx 0$ the probability density function of the observed variable is symmetrical and the data follow normal distribution. For the same verification, the kurtosis can also be used

$$
\gamma^{(k)} = \frac{\frac{1}{N} \sum_{i=1}^{N} (D_{ik} - \bar{D}_k)^4}{\left(\frac{1}{N} \sum_{i=1}^{N} (D_{ik} - \bar{D}_k)^2\right)^2} - 3 ,
$$
\n(12)

which is the ratio of the fourth central moment and the square of the variance. This quantity measures the peakedness of the probability density function, which for the case of $\gamma \approx 0$ is Gaussian type. For the linear case, Pearson's correlation coefficient, r , can be applied for characterizing the dependence between two variables, but in case of non-linear relationships, Spearman's rank correlation coefficient, ρ , can be preferably used (Isaaks and Srivastava 1989).

4. DERIVATION OF SHALE VOLUME FROM FACTOR SCORES

Factor analysis of wellbore data sets originated from different boreholes showed a strong correlation between the first factor (*i.e.*, the first column of matrix **F**) and shale content (see Figs. 2, 6, and 9 in Section 5). Assuming linear connection between the first factor, F_1 , and shale volume, $V_{\rm sh}$, an analogous formula to gamma ray index defined in eq. (3) can be set. The so-called i_F factor index in the given depth point is

$$
i_{F_1} = \frac{F_1 - F_{1,\min}}{F_{1,\max} - F_{1,\min}} ,
$$
\n(13)

where F_1 is the factor score computed in the point, $F_{1,\text{min}}$ and $F_{1,\text{max}}$ are the minimum and maximum value of the factor log, respectively. As a first approximation, the shale volume is assumed to be written as

$$
V_{\rm sh} = i_{F_1} \tag{14}
$$

A non-linear relationship between the two quantities gives a more precise description, which is valid for the entire range of the independent variable. According to regression tests on field data it was experienced that shale volume can be computed from factor scores by the following exponential relationship

$$
V_{\rm sh} = ae^{-bF_1} \tag{15}
$$

where a and b are properly chosen areal coefficients. For comparing shale volume estimations derived from different methods a data misfit can be computed

$$
\delta = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(V_{\text{sh},i}^{(inv)} - V_{\text{sh},i}^{(F_1)} \right)^2},\tag{16}
$$

where letter i in lower indexes indicates the i -th shale volume, which was estimated by inversion and factor analysis of wellbore data, respectively. The factor analysis of standardized original variables results in negative and positive factor scores (see Fig. 2 in Section 5). For the sake of comparability, factor scores were found necessary to be scaled. The transformation of factor scores into an arbitrary interval can be performed by using the following formula

$$
F_1' = F_{1,\min}' + i_{F_1} \left(F_{1,\max}' - F_{1,\min}' \right) , \qquad (17)
$$

where $F'_{1,\text{min}}$ and $F'_{1,\text{max}}$ are the desired lower and upper limits of the new factor F'_1 , respectively. Since shale volume ranges between 0 and 100%, the same interval for the factor scores was chosen (see Figs. 6 and 9 in Section 5).

5. FIELD EXAMPLE FOR LINEAR CASE

In the Pannonian Basin, a thick Tertiary sedimentary sequence overlays the Mesozoic, Paleozoic and Precambrian basement. Most of sedimentary reservoirs in the area are high or medium porosity sandstones interbedded with clay, silt, marl, and other different kinds of layers. The oil- and gas-bearing formations situated mostly between 1000 and 3000 m in depth represent a wide variety of structural, stratigraphical and combined traps in the province (Dolton 2006).

Factor analysis was tested on a data set observed in a hydrocarbon exploratory borehole (Well-1). The sequence of strata consisted of shaly-sandy layers saturated with water and gas. Measured logs can be seen in Fig. 1, which served as statistical samples of the original variables for the factor analysis. The processed depth interval was 150 m in length by 0.1 m sampling intervals (13 500 data). The average of correlation coefficients between the measured variables (*i.e.*, well-logging data) was 0.10. The number of original variables was reduced to two uncorrelated factors by using eq. (10). The number of factors was specified previously by experience, since two factors had explained more than 90% of the variance of original variables. The values of factor loadings can be seen in Table 2. Log types being sensitive mainly to lithology such

Fig. 1. Well-1 borehole logs of 150 m interval as original data for factor analysis. CAL [inch] is caliper log, SP $[mV]$ is spontaneous potential log, GR $[API]$ is natural gamma ray log, CN [%] is compensated neutron log, DEN [g/cm³] is density log, AT [µs/ft] is acoustic travel time log, RMLL [Ohm-m] is microlaterolog, RS [Ohm-m] is shallow resistivity log, and RD [Ohm-m] is deep resistivity log (by courtesy of MOL Hungarian Oil and Gas Company).

Table 2

Factor loadings derived from the well-logging data set measured in Well-1 (150 m case)

Fig. 2. Factor log as the result of the factor analysis of well-logging data set measured in Well-1 (on the left); linear relationship between the factor scores and shale content and its Pearson's correlation coefficient, r (on the right).

Fig. 3. Shale volume logs estimated by petrophysical interpretation (on the left) and factor analysis using $V_{\rm sh} = i_{F1}$ formula of well-logging data set measured in Well-1 (150 m case) (on the right).

as GR and SP got the highest weights related to Factor 1. This factor was identified as a good shale indicator. In Figure 2, the log of Factor 1 and the linear regression model of shale content as a function of factor scores can be seen, separately. The correlation coefficient was 0.96, which represented a straight and very strong relation between the two variables. In Figure 3, the shale volume logs estimated by separate petrophysical interpretation and factor analysis based on eq. (14) can be compared. The amount of fitting between the two curves was 5.4% according to eq. (16), which verified that the independent solutions were physically the same.

6. FIELD EXAMPLE FOR NON-LINEAR CASE

In the previous section, results were given for a relatively short (150 m) depth interval. A more detailed study required the processing of larger amount of data. In Figure 4, well logs of Well-1 can be seen, where the total number of data was 54 009 in the logged interval of 600 m. It was mentioned earlier that the maxi-

Fig. 4. Well-1 borehole logs of 600 m interval as original data for factor analysis. CAL [inch] is caliper log, SP [mV] is spontaneous potential log, GR [API] is natural gamma ray log, CN [%] is compensated neutron log, DEN [g/cm³] is density log, AT [µs/ft] is acoustic travel time log, RMLL [Ohm-m] is microlaterolog, RS [Ohm-m] is shallow resistivity log and RD [Ohm-m] is deep resistivity log (by courtesy of MOL Hungarian Oil and Gas Company).

mum likelihood estimation is optimal when data follow Gaussian distribution. This condition was satisfied sufficiently in case of this big sample. Among the applied well logs, the CN, SP and AT were the closest to normal distribution. As an example, the histogram of compensated neutron data is shown in Fig. 5. In the figure, the normal probability plot of neutron data can also be seen, where the points of the sample lie close to a straight line. The skewness of neutron data was -0.4 and the kurtosis was -0.17 computed by eqs. (11) and (12). The RD and CAL log having a small number of outliers were the farthest from normal distribution. The mean of correlation coefficients computed between pairs of well-logging data was 0.08, and that of the factors was zero after the factor analysis. In Table 3, the extracted factor loadings can be seen. Comparing Table 2 to Table 3, it can be noticed that in case of GR and SP logs the magnitude and sign were the same, related to Factor 1. The neutron and resistivity data as samples of original variables had bigger weights on the factor than in the previous case. For comparing the results obtained from different wellbores, the factors were rotated by the varimax criterion and scaled. The new interval of factor scores (0-100) was computed by eq. (17). In Figure 6 the log of the first (scaled) factor and the exponential relationship between the factor scores and

Fig. 5. Histogram of the neutron porosity measured in Well-1 and its normal probability plot (on the left); the histogram of the natural gamma ray intensity measured in Well-2 and its normal probability plot (on the right).

shale contents based on eq. (15) can be seen. The rank correlation coefficient between Factor 1 and shale volume was 0.99 (when assuming linear connection the Pearson's correlation coefficient was 0.82). Based on regression tests on additional data sets from different boreholes of the area, the exponent b was fixed as 0.037. Afterwards, the non-linear regression analysis of Well-1 resulted in a model specified with $a = 2.67$. In Figure 7, shale volume logs estimated by separate inversion and factor analysis can be seen. The data misfit based on eq. (16) was 8.2%, which was caused by the dispersion of data around the model as well as the different performance of the two independent procedures.

For testing the method by comparison, another well-logging data set originated from a different area of the Great Hungarian Plain was used. In Well-2, 113 861 data from a length of 1045 m logged interval were utilized (see Fig. 8). The correlation coefficient between original data was 0.06. In case of this sample, GR, CN, PE, and DEN logs were the closest to the Gaussian distribution.

Table 3

Factor loadings derived from the well-logging data set measured in Well-1 (600 m case)

Fig. 6. Scaled factor log as the result of the factor analysis of well-logging data set measured in Well-1 (on the left); non-linear (exponential) relationship between the factor scores and shale content, and its Spearmen's rank correlation coefficient, ρ (on the right).

Fig. 7. Shale volume logs estimated by petrophysical interpretation (on the left) and factor analysis using non-linear relationship between factor scores and shale content (on the right) of well-logging data set measured in Well-1 (600 m case).

For GR the skewness was −0.13 and the kurtosis was −0.36. RD was the farthest from normal distribution again. The loadings of both uncorrelated factors can be found in Table 4. The factors were rotated and scaled as in case of Well-1. The magnitudes and signs of factor loadings for these two distant wells were the same, which confirmed the feasibility of the method. Beside the constant b value (0.037), $a = 2.85$ was given. In Figure 9 the same model approximates to the data than was used in Fig. 6, by the same rank correlation coefficient 0.99 (when assuming linear connection it was 0.9). The misfit between the curves of shale volumes obtained by inversion and factor analysis separately was 5.8% (see Fig. 10). By averaging-out the values of coefficient a given by field studies done up to the present, the author suggests the

$$
V_{\rm sh} = 2.76 \, e^{-0.037 F_1} \tag{18}
$$

Fig. 8. Well-2 borehole logs of 1045 m interval as original data for factor analysis. SP $[mV]$ is spontaneous potential log, GR $[API]$ is natural gamma ray log, CN $[\%]$ is compensated neutron log, DEN [g/cm³] is density log, PE [barn/e] is photoelectric absorption index, and RD [Ohm-m] is deep resistivity log (by courtesy of MOL Hungarian Oil and Gas Company).

Table 4

Well log	Factor 1	Factor 2
SP	-0.88	0.01
GR	0.88	-0.21
RD	-0.79	-0.01
CN	0.14	-0.83
DEN	0.72	0.69
PE	0.79	0.36

Factor loadings derived from the well-logging data set measured in Well-2 (1045 m case)

Fig. 9. Scaled factor log as the result of the factor analysis of well-logging data set measured in Well-2 (on the left); non-linear (exponential) relationship between the factor scores and shale content, and its Spearmen's rank correlation coefficient, ρ (on the right).

Fig. 10. Shale volume logs estimated by petrophysical interpretation (1st track for the interval of 800-1300 m and 3rd track for the interval of 1300-1845 m) and factor analysis using non-linear relationship between factor scores and shale content (2nd track for the interval of 800-1300 m and 4th track for the interval of 1300-1845 m) of well-logging data set measured in Well-2.

formula for the estimation of shale volume in the region of the Great Hungarian Plain. The method has been tried out both on water and hydrocarbon-bearing sedimentary sequences, but in case of complex reservoirs, *i.e.*, metamorphic or volcanic rocks have not been tested. In that case, not only shale but several other mineral components constitute the matrix of rock. It is assumed that factors might be correlated to other petrophysical properties, too.

7. CONCLUSIONS

Beside the quantity of information, professional practice lays ever-increasing claim also to the quality of the interpretation results. This purpose requires advanced well-log analysis methods. Beside deterministic and inversion procedures, new statistical methods can also be used for extracting useful information from the well-logging data set. It is inferred that factor analysis is applicable to extract the shale content as basic lithological information from wellbore data. In this stage of research, we assume a non-linear connection between the first factor and shale volume in sedimentary geological environments. This relation proves to be straight representing a very strong correlation between the two variables. The method gives consistent results both in water and hydrocarbon reservoirs. Data sets from different geological areas are needed for further research. On the other hand, data sets following non-Gaussian statistics will require a robust procedure.

A c k n o w l e d g e m e n t s. The described work was carried out as part of the TÁMOP-4.2.1.B-10/2/KONV-2010-0001 project in the framework of the New Hungarian Development Plan. The realization of this project is supported by the European Union, co-financed by the European Social Fund. The author is also grateful for the support of the research team of the Department of Geophysics, University of Miskolc and the MOL Hungarian Oil and Gas Company for providing field data.

R e f e r e n c e s

- Alberty, M.W., and K.H. Hashmy (1984), Application of ULTRA to log analysis. **In:** *25th Annual Logging Symposium SPWLA, 10-13 June 1984, New Orleans, USA*, Society of Petrophysicists and Well Log Analysts, Houston, paper 1984-Z.
- Asfahani, J., M. Aissa, and R. Al-Hent (2005), Statistical factor analysis of aerial spectrometric data, Al-Awabed area, Syria: a useful guide for phosphate and uranium exploration, *Appl. Radiat. Isotopes* **62**, 649-661, DOI: 10.1016/j.apradiso.2004.08.050.
- Asquith, G.B., and D.A. Krygowski (2004), *Basic Well Log Analysis*, 2nd ed., AAPG Methods in Exploration Series, No. 16, The American Association of Petroleum Geologists, Tulsa.
- Baker Atlas (1996), OPTIMA, eXpress reference manual, Baker Atlas, Western Atlas Int. Inc., Houston.
- Ball, S.M., D.M. Chace, and W.H. Fertl (1987), The Well Data System (WDS): An advanced formation evaluation concept in a microcomputer environment. **In:** *Proc. SPE Eastern Regional Meeting, 21-23 October 1987*, *Pittsburgh, USA*, Society of Petroleum Engineers Inc., Richardson, paper 17034, 61-85, DOI: 10.2118/17034-MS.
- Barstow, D.R. (1984), Artificial Intelligence at Schlumberger, *AI Magazine* **5**, 4, 80-82.
- Bartlett, M.S. (1937), The statistical conception of mental factors, *Brit. J. Psychol. – Gen. Sect.* **28**, 1, 97-104, DOI: 10.1111/j.2044-8295.1937.tb00863.x.
- Boguslavskii, M.A., and A.A. Burmistrov (2009), Petrophysical properties of kimberlites from the Komsomolsky pipe and their relationship to its composition, formation conditions, and diamond content, *Mosc. Univ. Geol. Bull.* **64**, 6, 354-363, DOI: 10.3103/S0145875209060040.
- Dobróka, M., and N.P. Szabó (2005), Combined global/linear inversion of welllogging data in layer-wise homogeneous and inhomogeneous media, *Acta Geod. Geophys. Hung.* **40**, 2, 203-214, DOI: 10.1556/AGeod.40.2005.2.7.
- Dolton, G.L. (2006), Pannonian Basin Province, Central Europe (Province 4808) Petroleum geology, total petroleum systems, and petroleum resource assessment, *USGS Bull.* 2204-B, 1-47.
- Fraiha, S.G.C., and J.B.C. Silva (1994), Factor analysis of ambiguity in geophysics, *Geophysics* **59**, 7, 1083-1091, DOI: 10.1190/1.1443664.
- Goncalves, C.A., P.K. Harvey, and M.A. Lovell (1995), Application of a multilayer neural network and statistical techniques in formation characterization. **In:** *36th Annual Logging Symposium SPWLA, 26-29 June 1995, Paris, France*, Society of Petrophysicists and Well Log Analysts, Houston, paper 1995-FF.
- Isaaks, E.H., and R.M. Srivastava (1989), *An Introduction to Applied Geostatistics*, Oxford University Press, Oxford, 561 pp.
- Jöreskog, K.G. (2007), Factor analysis and its extensions. **In:** R. Cudeck and R.C. MacCallum (eds.), *Factor Analysis at 100. Historical Developments and Future Directions*, Lawrence Erlbaum Associates Inc. Publishers, New Jersey, 47-77.
- Kaiser, H.F. (1958), The varimax criterion for analytical rotation in factor analysis, *Psychometrika* **23**, 3, 187-200, DOI: 10.1007/BF02289233.
- Kaźmierczuk, M., and J. Jarzyna (2006), Improvement of lithology and saturation determined from well logging using statistical methods, *Acta Geophys.* **54**, 4, 378-398, DOI: 10.2478/s11600-006-0030-y.
- Larionov, V.V. (1969), *Radiometry of Boreholes*, NEDRA, Moscow (in Russian).
- Lawley, D.N., and A.E. Maxwell (1962), Factor analysis as a statistical method, *J. Roy. Stat. Soc. D – Stat*. **12**, 3, 209-229.
- Mayer, C., and A. Sibbit (1980), GLOBAL, a new approach to computer-processed log interpretation. **In:** *Proc. 55th SPE Annual Technical Conference and Exhibition, 21-24 September 1980, Dallas, USA*, American Institute of Mining, Mettalurgical, and Petroleum Engineers, Dallas, paper 9341, 1-14, DOI: 10.2118/9341-MS.
- Peveraro, R.C.A., and J.A. Lee (1988), HESPER: An expert system for petrophysical formation evaluation. **In:** *SPE European Petroleum Conference, 16-19 October 1988, London, UK*, Society of Petroleum Engineers, Richardson, 361-370, DOI: 10.2118/18375-MS.
- Urbancic, T.I., and R.C. Bailey (1988), Statistical techniques applied to borehole geophysical data in gold exploration, *Geophys. Prospect.* **36**, 7, 752-771, DOI: 10.1111/j.1365-2478.1988.tb02191.x.

Received 4 March 2011 Received in revised form 23 April 2011 Accepted 13 May 2011