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Solar Magnetic Activity: Topology and Prediction

Henrik LUNDSTEDT

Swedish Institute of Space Physics, Lund, Sweden
e-mail: henrik@lund.irf.se

Abstract

We ascribe the solar magnetic activity to the interplay between the plasma flow and the magnetic field. Observations by SOHO, Hinode and upcoming SDO are discussed. We then discuss the understanding and modeling of solar magnetic activity based on mathematical topological concepts. We present predictions using neural networks. Further we describe the outcome of the cycle 24 prediction panel. Finally, recommendations are given for making improved predictions.

Key words: solar magnetic activity, topology, prediction.

1. INTRODUCTION

Space weather is mainly driven by solar magnetic activity, and it is therefore very important to be able to predict it in a reliable way (Lundstedt 2005, 2006). However, we believe that predictions can be successful only if they are based on observations of physical quantities which are related to physical mechanisms. These predictions can become operational for space weather services of the International Space Environment Service (ISES), when real-time observations will be made available from the Solar Dynamics Observatory (SDO) which will produce such real-time data.

The physical effects of solar magnetic activity are described by observations and measurements of the solar magnetic field, based on the Zeeman and Hanle effect. The understanding is achieved through mathematics. In this article, topological methods are emphasized. Predictions are developed using methods based on probability and neural network.

2. INTERPRETATION AND DESCRIPTION OF SOLAR MAGNETIC ACTIVITY

We start by interpreting solar magnetic activity in terms of the interplay between the solar plasma flow vector (V) and the solar magnetic field vector (B). We describe solar magnetic activity first by using mathematical and then physical concepts. The concepts and relations are illustrated in Fig. 1, as a concept map (C-Map) (Messerotti 2002).

Description based on solar mathematical concepts

The plasma flow (V) and magnetic flux density (B) are described with vector fields which are purely mathematical entities. The relations between these entities are expressed in MHD differential equations. Energy storage and release, and dynamo mechanisms are described by with these equations. These equations are often too difficult to understand.

Topology enables us, however, to handle qualitative laws and determine qualitative, but provable behavior.

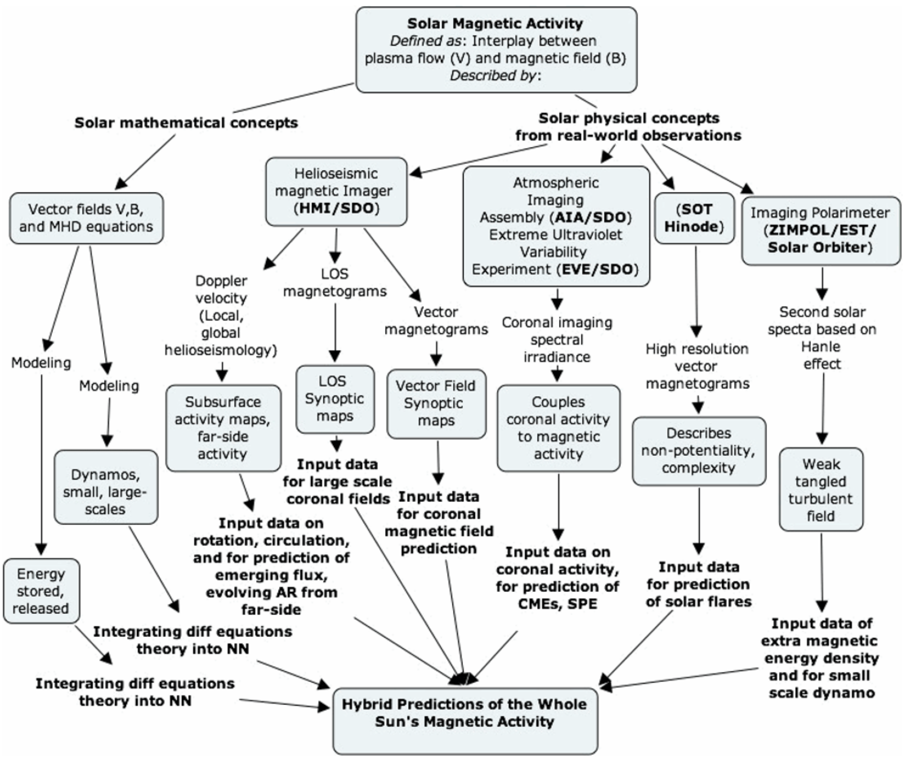

Fig. 1. A Concept-Map of solar magnetic activity and its predictions.

Description based on solar physical concepts

Real-world description is obtained through observations and measurements of physical quantities. In Fig. 1 we illustrate observations using instruments onboard the Solar Dynamics Observatory, the Solar Optical Telescope (SOT) onboard Hinode and the Zurich Imaging Polarimeter (ZIMPOL) planned for Solar Orbiter.

The Helioseismic Magnetic Imager (HMI) onboard SDO will be used to obtain subsurface maps, data on far-side activity, line-of-site (LOS) synoptic maps and vector magnetogram (VF) maps. The Atmospheric Imaging Assembly (AIA) and the Extreme Ultraviolet Variability Experiment (EVE), both onboard SDO, will make the connection to coronal activity. SOT onboard Hinode produces high resolution vector magnetograms. With ZIMPOL, a second solar spectrum has been discovered (Stenflo 2004). The different polarized structures in the second solar spectrum are affected through the Hanle effect and by a hidden magnetic field. This hidden (to the Zeeman effect) weak and tangled turbulent field carries more magnetic-energy density than earlier thought (Stenflo 2004).

3. MODELING USING TOPOLOGICAL METHODS

Coronal phenomena such as solar flares and coronal mass ejections are the coronal response to the photospheric dynamics (Longcope 2005). This response can be described and understood with topology. The solar cycle and the modulation of it may also be described and understood topologically by studying dynamical systems.

Topology

The objects of topology are topological spaces. Let *X* be a set. A topology on *X* is a collection T of subsets of *X* called the open sets, such that:

- any (finite or infinite) union of open sets is open,
- any finite intersection of open sets is open,
- both *X* and the empty set *O* are open.

An open set is a set for which every point in the set has a neighborhood lying in the set. A subset *A* is dense in a space *X* if and only if *A* intersects every nonempty open set in *X*. The set *X* together with a topology T is called a topological space. Topology has evolved in many directions. Most interesting to us are the following subdivisions:

- Point set topology – studies points and sets in topological spaces, and continuous functions between topological spaces.

- Algebraic topology – studies how to "count and add" topological features.
- Differential topology – studies the interactions between topology and calculus.

We say that two maps f_0, f_1 : $X \rightarrow Y$ are homotopic, if there is a continuous function $H : X \times Y \rightarrow Y$, $s \in I$ such that:

- $H(x, 0) = \lambda_0(s)$ for all $x \in X$,
- $H(x, 1) = \lambda_1(s)$ for all $x \in X$.

The map H is called a homotopy from f_0 to f_1. The concept of homotopy is then used to develop the winding number, i.e., of a loop $\gamma : I \longrightarrow \mathbb{R}^2 - \{P\}$ around a point P is simply the number of times the loop γ (of radius r) winds around the point P. The index of V at P is defined as $Index_P\ (V) = W(V \circ \gamma_r, 0)$, where 0 denotes the origin in $\mathbb{R}^2$.

The Poincaré-Index Theorem says: Suppose that U is an open set in $\mathbb{R}^2$, that D is a closed disk in U, and that V is a vector field on U having only isolated zeros, all of which are contained in D. Let γ denotes the path $\gamma(s) = (x_0, y_0) + [r \cos(2\pi s), r \sin(2\pi s)]$, where (x_0, y_0) is the center of D and r is the radius of D. Then the winding number, $W(V \circ \gamma, 0) = \Sigma_{P \in Z}\ Index_P\ (V)$ where P is an isolated zero of the vector field.

Integrals of vector fields (magnetic fields) can then be calculated from the winding number and indices.

Topological chaos and dynamic systems: A dynamical system is said to be chaotic (Devaney chaotic) if there exists at least one dense orbit and a set of periodic orbits is dense. Topological invariants of periodic orbits can identify the strange attractor and the stretching and squeezing mechanisms. The linking number (Pohl 1968), developed by Gauss, is such an invariant. The goal is to determine the stretching and squeezing mechanisms which occur repeatedly to build up a strange attractor.

4. SHORT-TERM PREDICTION: DAYS, MONTHS AHEAD

Coronal phenomena such as solar flares and coronal mass ejections are the coronal response to the photospheric and subsurface motions. This response can be modelled with topology.

Solar flares and subsurface flows

A significant correlation between strong plasma downflows based on local helioseismology and high magnetic activity, indicated by strong solar flares, was found in Jensen *et al.* (2004) (Fig. 2). A neural network was trained to predict an event of at least one major solar flare based on maps of subsurface

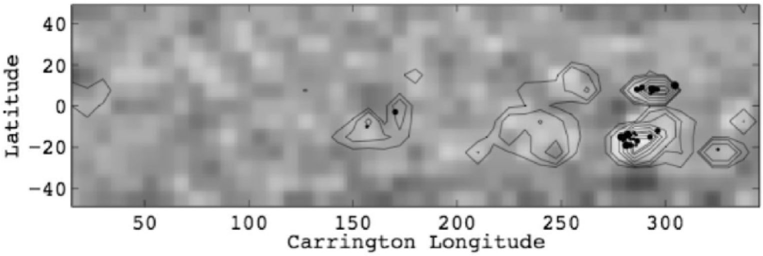

Fig. 2. The synoptic map shows the divergence of the observed flows at a depth of 4.6 Mm, based on MDI, SOHO data. Bright regions represent inflow and dark outflow. The contour lines show the magnitude of magnetic field strength. The size of the dots indicates flare from small (C), medium (M) and large (X). The synoptic map shows Carrington rotation 2009, i.e., during the Halloween event in October 2003.

flows. The predictions were promising, despite the lack of a large input dataset (Jensen *et al.* 2004). Such large dataset and near-real time maps will become available from both Global Oscillation Network Group (GONG) and will come shortly from SDO.

Solar flares and vector magnetic fields

It has been known for a long time that solar flare activity is related to the non-potentiality and complexity of the solar magnetic activity. Cui *et al.* (2006) describe the non-potentiality and complexity by introducing three quantities: maximum horizontal gradient, length of neutral line, and the number of topologically singular points (based on Poincaré-Index Theorem). They found an interesting relation between the solar flare productivity and the above mentioned quantities.

The relationship can be fitted with a sigmoid function and herewith modeled with a neural network. The three quantities can be derived from vector magnetograms. Operational forecasts will be possible with real-time vector magnetograms, which will be available from SDO.

5. MID-TERM PREDICTIONS: YEARS, A CYCLE AHEAD

Solar synoptic maps provide an important visualization of global patterns. Maps are available of sub-surface flows, photospheric and coronal magnetic fields. In Lundstedt (2007) we averaged longitudinally synoptic maps from Wilcox Solar Observatory (WSO) at Stanford. The data cover three cycles from 1976 up to the present. The averaged map is shown in Fig. 3. Many interesting topological features are visible: the variation of the butterfly diagrams, the transport of flux to the poles, and the asymmetry for the both hemispheres.

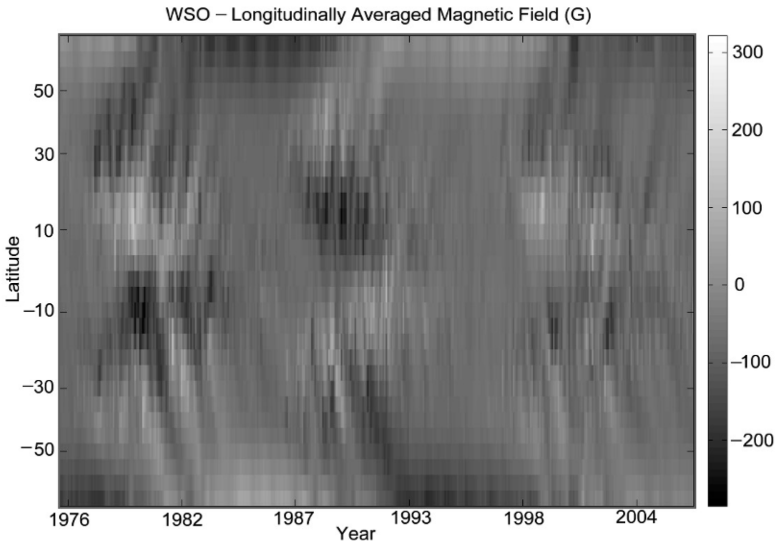

Fig. 3. Longitudinally averaged synoptic magnetic fields, based on WSO data.

Neural networks have been trained, based on data from the longitudinally averaged synoptic map, to predict the total magnetic flux Carrington rotations ahead. A correlation coefficient of 0.82 was reached between the predicted and observed values two years ahead (Lundstedt 2007). Similar studies are planned using SDO data.

Ever since the sunspot cycle was discovered by H. Schwabe in 1843, using only 17 years of data, scientists have described solar activity cycles by the sunspot number.

The Cycle 24 Prediction Panel

NOAA/NASA/ISES sponsored a panel, consisting of 11 participants, which tried to reach a consensus on the next sunspot cycle, Cycle 24. Over 40 different predictions were examined, based on climatology methods, spectral analysis, neural networks, precursor methods and dynamo models. Predictions range from very weak to very strong: Svalgaard *et al.* (2005) predicted Cycle 24 to be the weakest in 100 years based on the polar field strength. Dikpati *et al*. (2006) predict a strong Cycle 24 based on applying a dynamo model. It is interesting that Choudhuri *et al.* (2007), also using a dynamo model, concluded that Cycle 24 will be weak.

On April 25, 2007 the panel announced their first predictions: The Solar Minimum will occur in March 2008 (±6 months), which marks the end of

Cycle 23 and start of Cycle 24. The length of Cycle 23 will then be 11.75 years, i.e., longer than the average of 11 years.

The Cycle 24 will peak at a sunspot number of 140(±20) in October 2011 or it will peak at a sunspot number of 90(±10) in August 2012. An average solar cycle peaks at 114 and therefore the next cycle will neither be extreme nor average.

The panel is split down the middle on whether it will be bigger or smaller than average. The panel will re-evaluate conditions on the Sun every 3 months and update this prediction annually, or as things change.

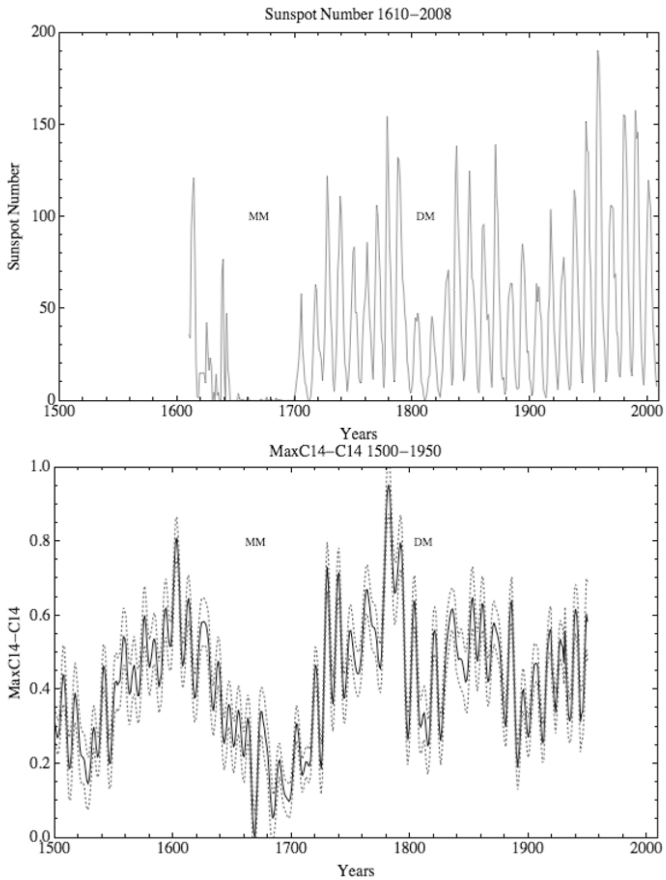

Fig. 4. The upper panel shows the group sunspot number R_G 1610-1995 and the sunspot number R_z 1995-2005. The lower panel shows maximum ^{14}C production rate value – ^{14}C production rate 1500-1950. MM stands for Maunder Minimum, DM for Dalton Minimum. The two one sigma curves are also plotted.

Sunspot number as an indicator of solar magnetic activity

The sunspot number is used as an indicator of long-term solar magnetic activity. At most, the sunspot number covers only 23 sunspot cycles. These cycles largely differ both in amplitude and length. During the so-called Maunder Minimum (MM) 1645-1715, almost no sunspots were observed, though the ^{14}C production showed about 11-year variations during the MM (Fig. 4) (Lundstedt *et al.* 2006).

Extreme solar radio bursts during sunspot minimum

Also that showing the sunspot number might not give us the general picture of the solar magnetic activity is the following event close to the sunspot minimum. Activity increased to high levels on 5 and 6 December 2006 as Region 930 produced three major flares: an X9/2N at 05/1035 UTC associated with Types II (estimated velocity 860 km/s) and IV radio sweeps and a 12000 sfu Tenflare, an M6/SF at 06/0823 UTC associated with a Type IV radio sweep and 340 sfu Tenflare, and an X6/3B at 06/1847 UTC (Fig. 5) (Bothmer and Zhukov 2007). This very unexpected activity even observed (Strong and Saba 2007) raises the question: Are we looking at the Solar Cycle in a completely wrong way?

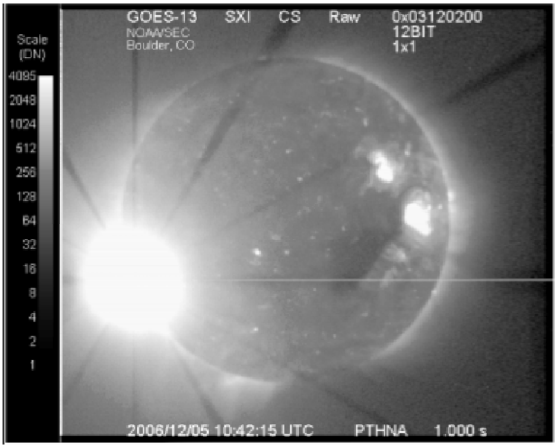

Fig. 5. The Xray flare associated with the extreme radioburst on 5 December 2006. Courtesy: NOAA GOES-13.

6. LONG-TERM PREDICTIONS: MODULATION OF CYCLES

The modulation of the solar magnetic activity cycle can be modeled with third order Lorenz type equations (Tobias *et al.* 1995). By changing the angular velocity the state can change from cyclic, aperiodic to a chaotic state.

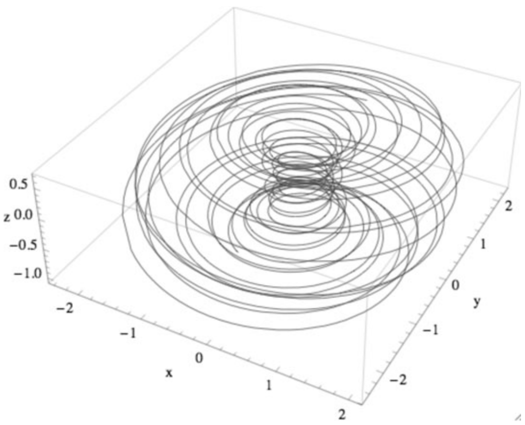

Fig. 6. The aperiodic state of a Lorenzian dynamical system. *X* represents the toroidal magnetic field, *Y* the poloidal field and *Z* the hydrodynamics.

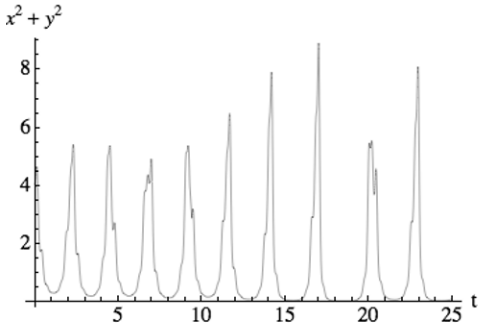

Fig. 7. The aperiodic state described by the sum of the squared toroidal and squared poloidal magnetic field.

The aperiodic state is illustrated in Fig. 6. In Figure 7 the squared toroidal field plus the squared poloidal field is plotted. The similarity with variation of the sunspot number is evident.

Points of sets can be orbits (trajectories of dynamic systems). Topological invariants of periodic orbits can then identify the strange attractor and the stretching and squeezing mechanisms. A study of that is in progress.

7. DISCUSSION AND CONCLUSIONS

To improve the understanding and predictions of solar magnetic activity we need observations of the plasma flow (V) and magnetic field (B) below and on the surface and in the corona. Real-time synoptic maps can adequately visualize the activity.

To predict explosive events, such as solar flares, we need real-time vector magnetic field observations at all scales. The difficulties in predicting the so-called Schwabe's sunspot cycle, using both data and theory-driven models (Bushby and Tobias 2007), might point out new aspects that lead to rethink about solar cycles.

Observations of V and B by Hinode, SDO, Solar Orbiter and other upcoming missions will play a very important role in improving our knowledge. Modeling and predicting using topological methods will further improve our knowledge and predictions.

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