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A Mixed-Dimension Kinematic Estuarine Model

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ABSTRACT: A kinematic model, developed for use during conditions of low salinity when the upper reaches of an estuary are vertically mixed, divides the estuary into one- and two-dimensional regimes. The model has been applied to the Potomac by assuming that the estuary is one-dimensional above Maryland Point and two-dimensional seaward of Morgantown.

Introduction

The upper reaches of a partially mixed estuary contain a transitional zone, seaward of which the river can be considered the estuary proper, exhibiting an internal circulation with reverse bottom flow, and upstream of which the net flow is directed seaward at all depths. In the Potomac, this transition usually occurs upstream of Morgantown but below Douglas Point (Fig. 1). Such a transitional zone, containing water of low salinity, is favored as a spawning and nursery area by a large proportion of a river's pelagic population. In recent years, these zones have received increasing attention from man who uses their waters as a coolant for power stations.

Two-Dimensional Model

The two-dimensional model is an elaboration on a method demonstrated by Pritchard (1969). He showed that a partially mixed estuary could be modeled using a two-dimensional scheme which maintained the salt balance through a combination of horizontal and vertical advection plus vertical diffusion.

In addition to its net flow, a fluid parcel in an estuary experiences oscillatory horizontal motions due to the effects of the tides. The model under discussion is not required to reproduce such details of the tidal circulation and, in practice, the salinity data used as input to the model will not be collected for all stations at the same phase of the tide. Thus the longitudinal separation between grid points was chosen to be greater than the local tidal excursion which in the region of Douglas Point is of the order of 5.7 km (3.1 nautical miles) (Elliott 1975). The estuary was therefore segmented so that the minimum separation between sections was equal to 4 nautical miles.

Fig. 1 shows the segmentation scheme used. The distance between sections varied from 4 nautical miles near Douglas Point to a maximum separation of 10 nautical miles in the lower estuary. At each transect, the cross-sectional area was computed and tabulated as a function of depth for every 2 m interval. The run-off factors for each segment were derived from drainage area statistics (Seitz 1971). To estimate the fresh water inputs, the model made use of a ten day mean at Washington, D.C., weighted by the appropriate drainage areas.

Each segment was divided into an upper and lower layer, with a seaward directed flow in the upper layer and a return landward flow in the lower layer. The depth of the interface was chosen at each section independently; the interface was not required to be level. Selection of a particular depth was based on inspection of vertical salinity profiles plus knowledge of the vertical structure of the velocity field gained from current meter data. The interface depths were free parameters of the model and could be changed at will. The system is shown schematically in Fig. 2.

Given the vertical distribution of salinity and cross-sectional areas, the upper and lower mean salinities were computed at each section. The mean volume salinities were then computed from the sectional means as

$$\overline{Su_j} = \frac{2(Su_j\, Au_j\, \epsilon_j + Su_{j+1}\, Au_{j+1}(1-\epsilon_j))}{(Au_j + Au_{j+1})} \qquad (1)$$

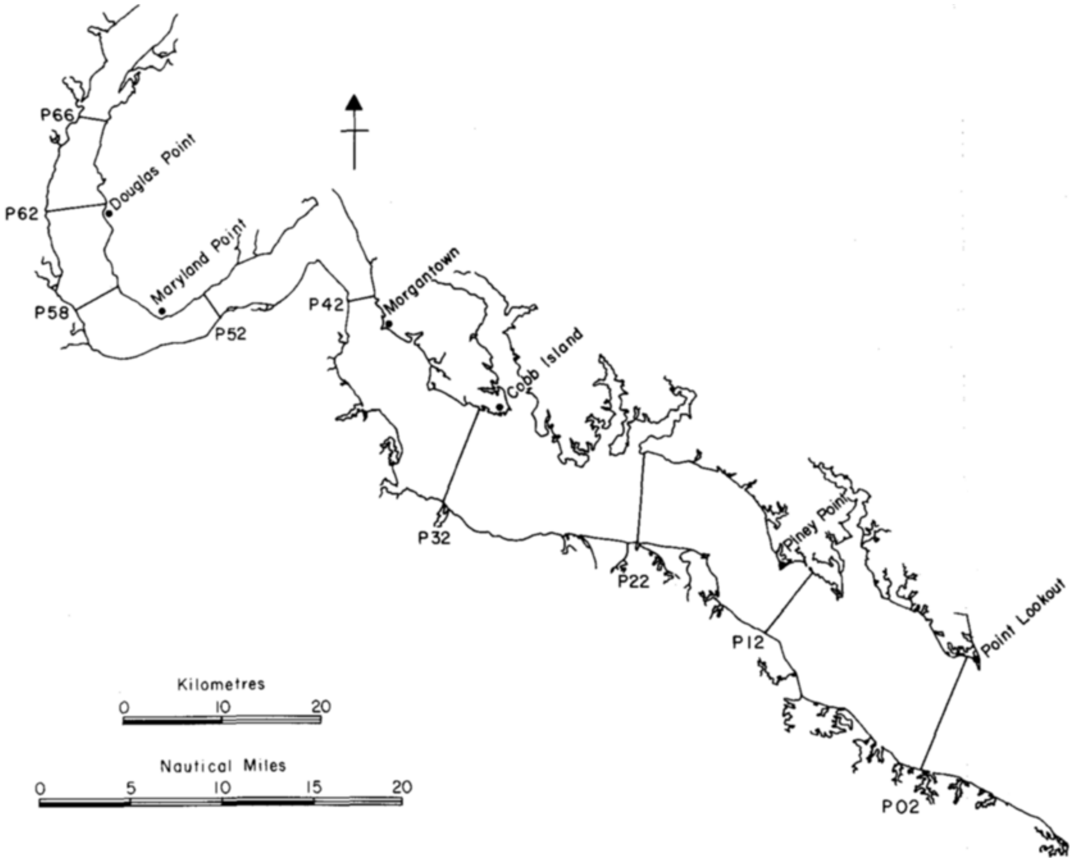
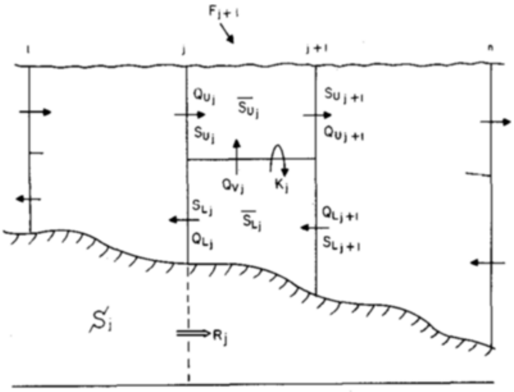


Fig. 1. Segmentation of the Potomac estuary.



and

$$\overline{Sl_j} = \frac{2(Sl_j\,Al_j\,\epsilon_j + Sl_{j+1}(1-\epsilon_j))}{(Al_j + Al_{j+1})} \qquad (2)$$

where the upper and lower sectional mean salinities have been weighted by the cross-sectional areas and horizontal factors ϵ_j. [A discussion of scaling factors such as the ϵ_j can be found in Wilson (1970) and Ward and Espey (1971)]

By simultaneously solving the salt balance equation and the continuity equation for the total volume upstream of section j, the horizontal fluxes Qu_j and Ql_j can be found from the basin equations as:

$$Qu_j = \frac{\dfrac{\partial \$_j}{\partial t} + Sl_j\,(R_j - \dfrac{\partial V_j}{\partial t})}{(Sl_j - Su_j)} \qquad (3)$$

and

$$Ql_j = \frac{\dfrac{\partial \$_j}{\partial t} + Su_j\,(R_j - \dfrac{\partial V_j}{\partial t})}{(Sl_j - Su_j)} \qquad (4)$$

In these equations, $\$_j$ and V_j are the total salt and volume, respectively, upstream of section j. The vertical advective flux can then be found from continuity and the vertical exchange coefficient found by applying salt balance to the upper portion of segment j. (Using an exchange coefficient, the diffusive flux is given by $K\ \Delta S$ where ΔS is the salinity difference across the face. Thus K is equivalent to $kA(\Delta x)^{-1}$ where k is a diffusivity, A is the area of the face and Δx is the length scale for estimating ΔS.)

In a similar manner, a pollutant balance can be written for the upper and lower layer of each segment; this differs from the salt balance only in the inclusion of a possible source term within each layer. For an estuary segmented by n transects, this would yield 2(n−1) equations in 2n unknowns. Two boundary conditions are imposed to close the system; it is usual to assume that the water entering the estuary in the upper layer at the head and in the lower layer at the mouth should be of a known concentration. The steady state concentrations can then be obtained by matrix inversion.

The above analysis is appropriate for a two-layered estuary in which longitudinal diffusion can be neglected. Difficulties arise when the salinity tends to vertical homogeneity making expressions (3) and (4) infinite. Such conditions occur in the upper Potomac estuary during typical spring conditions (Fig. 3).

The Mixed Model

During the fall salinity maximum, current measurements suggest that a weak two-layered flow may extend as far upstream as 60 nautical miles (110 km) from the river mouth (Elliott 1975). However, during most of the year and especially during the spring, an

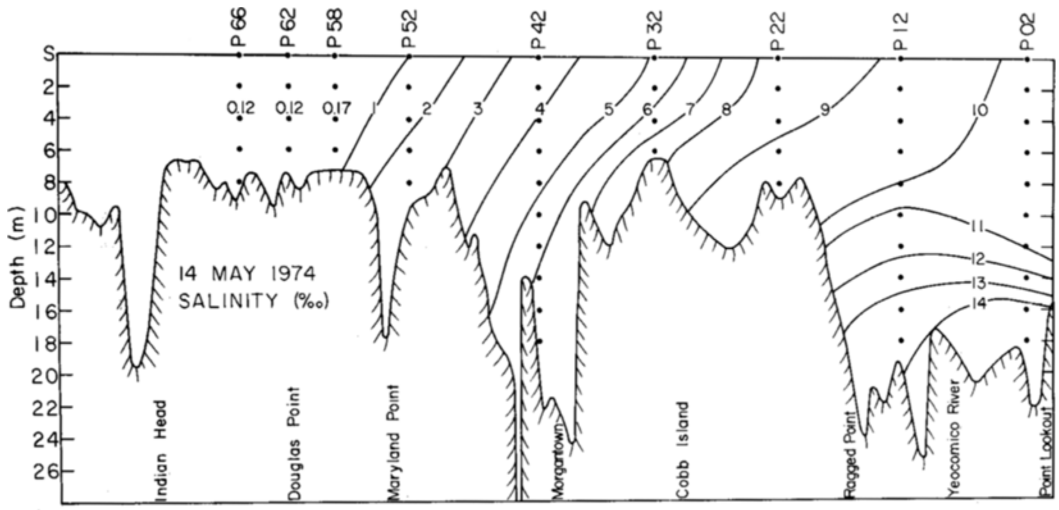


Fig. 3. Salinity section, May 1974.

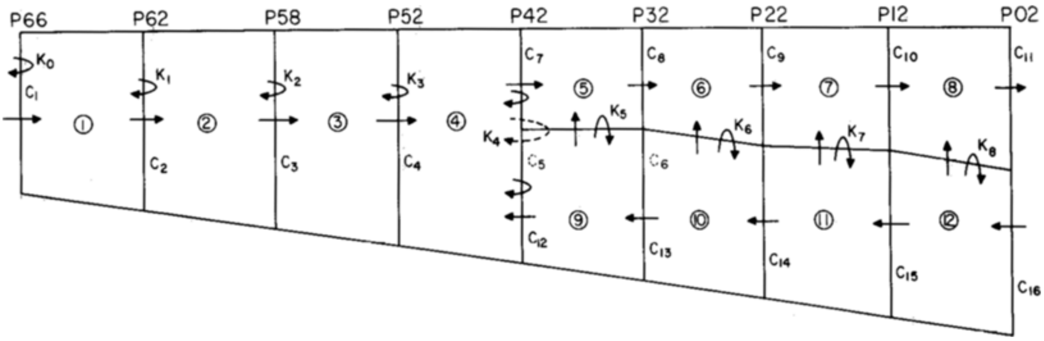


Fig. 4. Schematic representation of the mixed model.

estuarine circulation is unlikely to be well developed above Maryland Point. Therefore, a model was developed that was one-dimensional down to Maryland Point but was two-dimensional below Morgantown. This is shown schematically in Fig. 4.

A one-dimensional salt balance, when applied to the estuary upstream of section j, gives

$$\frac{\partial \$_j}{\partial t} = K_{j-1}(\bar{S}_j - \bar{S}_{j-1}) - Qh_j S_j$$

which can be solved for K_{j-1}, the horizontal exchange coefficient at section j. S_j and $\bar{S}_j$ are one-dimensional sectional and volume averaged salinities.

The section which had both two-layered flow and horizontal diffusion required the use of modified basin equations. These were obtained by assuming that the conservation of the total mass and salt, upstream of the section, was maintained by the combination of two-layered advection plus horizontal diffusion. After simultaneously solving these two conservation equations for the situation shown in Fig. 5, modified basin equations were obtained in the form

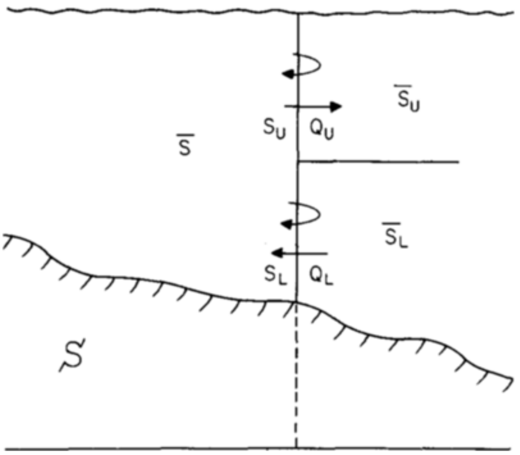


Fig. 5. Two layered flow with horizontal diffusion.

$$Qu = \left[\frac{\partial \$}{\partial t} + \left(R - \frac{\partial V}{\partial t}\right) Sl - K\frac{Au}{Ah}(\bar{S}u - \bar{S}) - K\frac{Al}{Ah}(\bar{S}l - \bar{S})\right] / (Sl - Su) \qquad (5)$$

and

$$Ql = \left[\frac{\partial \$}{\partial t} + \left(R - \frac{\partial V}{\partial t}\right) Su - K\frac{Au}{Ah}(\bar{S}u - \bar{S}) - K\frac{Al}{Ah}(\bar{S}l - \bar{S})\right] / (Sl - Su) \qquad (6)$$

Where $\$$ is the total salt upstream of the section, V is the upstream volume and R is the river discharge. Thus Qu and Ql could be computed if K, the horizontal exchange coefficient, was known.

K was found by first converting the upstream exchange coefficients, determined by a one-dimensional salt balance, into diffusivities. The values of diffusivity were then

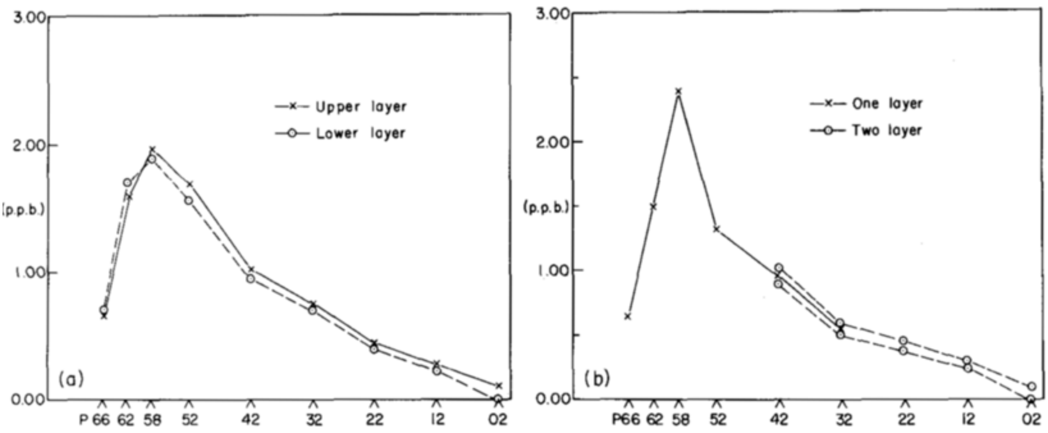


Fig. 6. Comparison between the models for a unit source; (a) two layered, (b) mixed model.

extrapolated downstream and the extrapolated diffusivity converted back into an exchange coefficient and used in (5) and (6). Setting up a pollutant balance for each box gave 12 equations for the 16 unknowns, c_j. Boundary conditions, specifying that the water entering the system be of known concentration, supplied two further equations. Equivalence relationships were used to give the final two equations. Where the models merged, the one-dimensional concentrations were required to equal the sectional means of the two-dimensional values as shown in Fig. 4. This ensured that the models matched where they overlapped.

Verification

During the fall of 1973, a dye release was made in the vicinity of Douglas Point (P62, Fig. 1). Dye concentrations were sampled at selected transects and tidally and sectionally averaged concentrations computed.

High salinities throughout the estuary permitted the use of both a full two-layered model and the mixed-dimension model. Initially, the upstream boundary concentrations were set to zero but this resulted in predicted concentrations lower than those observed. Better agreement was obtained by setting the upstream value equal to the mean observed boundary concentration. Also, since the source was located at the boundary between adjacent segments, the models were further tuned by varying the proportion of the source that went into the adjoining boxes until the best fit was obtained. Both models then gave good predictions of the observed concentrations. Fig. 6 shows a comparison of the two models, using the fall data to predict concentrations due to a unit source of 1 gm.s^{-1}. Both models were in good agreement in the lower estuary; the mixed model predicted higher concentrations in the vicinity of the source.

The mixed model was developed for use during spring conditions of low salinity when the full two-layered model could not be used. An application of the mixed model during such conditions is shown in Fig. 7. Using observed salinity data, the model was run to determine the advective fluxes and the exchange coefficients. All pollutant sources were set to zero, and the upper and lower sectional mean salinities at the mouth were supplied as boundary conditions. The concentration at the head of the estuary was not specified. As a result of this, the predicted concentrations should correspond to the observed salinities; Fig. 7 shows the agreement. As an estimate of horizontal diffusivity, the value determined at P52 was 2.4 × 10^6 cm^2.s^{-1}.

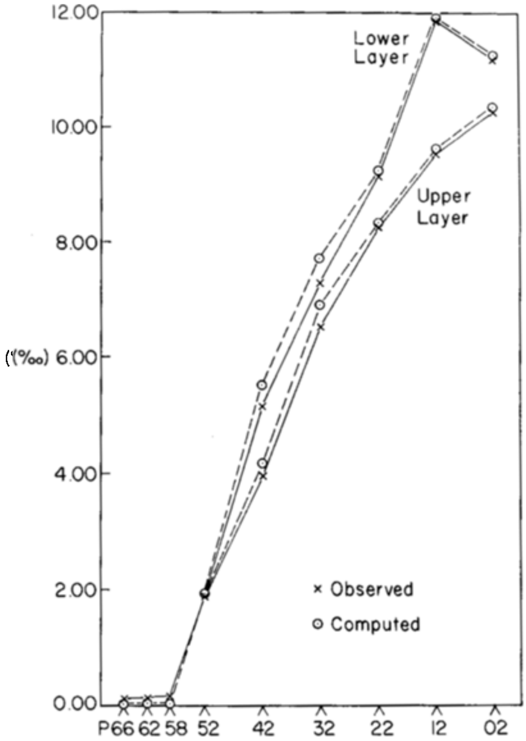


Fig. 7. Observed and computed salinities, May 1974, mixed model.

Summary

A mixed model that is one-dimensional toward the head of an estuary and which becomes two-dimensional further downstream appears to reproduce features of estuarine mixing, especially during the spring conditions of low salinity. Such a model may have application to biological problems which involve the transport of fish eggs and larvae.

ACKNOWLEDGMENTS

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APPENDIX

Notation

Su_j	Upper mean salinity, section *j*.
Sl_j	Lower mean salinity, section *j*.
Qu_j	Upper horizontal flux, section *j*.
Ql_j	Lower horizontal flux, section *j*.
Qv_j	Vertical flux, segment *j*.
K_j	Exchange coefficient, segment *j*.
k_j	Eddy diffusivity, segment *j*.
F_j	Fresh water run-off, segment *j*.
R_j	Net flux, section *j*.
$\$_j$	Total integrated salt upstream of section *j*.
Au_j	Upper cross-sectional area, section *j*.
ϵ_j	Horizontal factor, segment *j*.
$\bar{S}u_j$	Upper mean volume salinity, segment *j*.
$\bar{S}l_j$	Lower mean volume salinity, segment *j*.
V_j	Total integrated volume upstream of section *j*.
Qh_j	One dimensional horizontal flux, section *j*.
Ah_j	One dimensional sectional area, section *j*.