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Robust distributed model predictive consensus of discrete-time multi-agent systems: a self-triggered approach*

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Abstract: This study investigates the consensus problem of a nonlinear discrete-time multi-agent system (MAS) under bounded additive disturbances. We propose a self-triggered robust distributed model predictive control consensus algorithm. A new cost function is constructed and MAS is coupled through this function. Based on the proposed cost function, a self-triggered mechanism is adopted to reduce the communication load. Furthermore, to overcome additive disturbances, a local minimum–maximum optimization problem under the worst-case scenario is solved iteratively by the model predictive controller of each agent. Sufficient conditions are provided to guarantee the iterative feasibility of the algorithm and the consensus of the closed-loop MAS. For each agent, we provide a concrete form of compatibility constraint and a consensus error terminal region. Numerical examples are provided to illustrate the effectiveness and correctness of the proposed algorithm.

Key words: Consensus; Self-triggered control; Distributed model predictive control
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1 Introduction

As an advanced control method, model predictive control (MPC) has advantages of handling system constraints in an explicit form (Mayne et al., 2000) and implementing optimal control. Therefore, MPC has attracted extensive attention from researchers in the control field (Magni et al., 2003; Li and Shi, 2014; Rosolia et al., 2017). In addition, some large-scale discrete-time multi-agent systems (MASs) have emerged, such as multi-region power systems (Mohamed et al., 2011) and wireless sensor networks (Xi et al., 2010). Inspired by the distributed control system (DCS) with MPC, there are several studies of distributed MPC (DMPC) in the literature, e.g., Summers and Lygeros (2012), Al-Gherwi et al. (2013), and Zheng et al. (2013). Compared with the traditional centralized MPC through a single controller, DMPC is more appealing due to its flexible control structure and high performance.

Recently, DMPC for MAS has become a hot research area. One common research direction is the consensus of MAS using DMPC schemes. A wide variety of solution strategies have been proposed to ensure consensus (Müller et al., 2012; Zhan and Li, 2013; Cheng et al., 2015; Li and Yan, 2015). The consensus of MAS is required to design a distributed control protocol, which uses neighboring information to reach a state agreement with respect to each agent. For linear MAS, Li and Yan (2015) developed a distributed receding horizon control (RHC) protocol, which first explicitly expresses neighbor information versus implicit description (Zhan and Li, 2013). In

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addition, more detailed results to ensure consensus were included in Li and Yan (2015) for MAS with linear-time invariant and one-dimensional dynamics. Li et al. (2016) focused on how to design an information exchange mechanism for consensus. In particular, the authors analyzed detailed consensus conditions for finite and infinite horizon cases. They also indicated that consensus performance is related to the network topology. For nonlinear MAS, a relatively general cooperative control DMPC framework was reported by Müller et al. (2012), where a non-iterative solution method was adopted for each agent; i.e., optimization problems are solved only once at each sampling instant, and this results in fewer communication requirements. In Gao et al. (2017), consensus for second-order nonlinear systems with a dynamic reference was considered. Three critical components, the terminal cost, terminal region, and auxiliary controller, were denoted in a more understandable way. Moreover, Gao et al. (2017) proposed a time-varying compatibility constraint to ensure the convergence of the closed-loop system. Note that external disturbances were not considered in the aforementioned papers (Müller et al., 2012; Zhan and Li, 2013; Cheng et al., 2015; Li and Yan, 2015; Li et al., 2016; Gao et al., 2017). In a practical environment, systems are inevitably affected by ubiquitous uncertainties. Therefore, considering bounded additive disturbances, in this study, we propose a robust DMPC consensus strategy for a class of nonlinear MAS.

Note that the vast majority of existing DMPC algorithms require control execution at each sampling instant (Su et al., 2019a). Inevitably, a great deal of computing and communication consumption is generated in this process. Simultaneously, due to the limitations of the actual network, a large amount of communication may induce a certain degree of deterioration on the controlled system. Event-triggered control is an effective energy-saving strategy that can achieve aperiodic control for a small average sampling rate (Zou YY et al., 2017). Currently, event-triggered control is applied widely in various fields. Theoretical and practical results can be seen in previous publications (Ferrara et al., 2012; Lehmann et al., 2013; Zou WC and Xiang, 2019). Recently, Zou WC et al. (2020a, 2020b) applied the event-triggered scheme to consensus-tracking control and containment control of MAS, and Zeno behavior was avoided by designing appropriate triggered rules. However, an event-triggered mechanism needs an additional detection part to continuously obtain the current state of the actual system, which is undesirable for some systems with high sampling costs. Thus, self-triggered control was proposed using previously predicted states to pre-determine the next triggering instant (Heemels et al., 2012). This avoids the disadvantage of the high-frequency sampling problem of event-triggered control. A self-triggered MPC algorithm for a single nonlinear system was explored in several studies (Hashimoto et al., 2017; Liu et al., 2018; Su et al., 2019b), which implemented a self-triggered control to MPC and stabilized the closed-loop system. In Hashimoto et al. (2017), Liu et al. (2018), and Su et al. (2019b), the control input and self-triggered strategy were designed via an optimization problem, whereas the results obtained were asynchronously determined. In Li et al. (2018), an independent variable reflecting the cost of communication was incorporated into the system's cost function to synchronously trade off the desired triggering and control behavior. It may require fewer conservative design parameters compared with several other studies (Hashimoto et al., 2017; Liu et al., 2018; Su et al., 2019b). For a linear MAS, Zhan et al. (2019a) and Mi et al. (2020) co-designed a self-triggered mechanism and DMPC to achieve a coordinated value, which efficiently reduced the communication load. However, a few studies combined the self-triggered mechanism and DMPC algorithm to solve the consensus problem of nonlinear uncertain MAS. Thus, we propose a self-triggered robust DMPC consensus algorithm, which considers both control costs and communication load. Our study partially extends the results of Liu et al. (2018) to the case of distributed control of MAS.

The main contributions of this paper are twofold:

1. We introduce a self-triggered strategy via optimization, which relieves the heavy communication burden. The maximum triggering interval is user-defined and is no more than the predicted horizon. Considering bounded additive disturbances, for each discrete-time nonlinear agent, we use a min–max robust DMPC which explicitly includes uncertainty realizations as optimized decision variables in the entire optimization control problem, which is more intuitive compared with robust DMPC using a nominal model (Zhan et al., 2019b).

2. Sufficient conditions are presented to guarantee the feasibility of the optimization algorithm and the consensus over the considered MAS, where we employ the invariant set theory to implement the input-to-state practical stability (ISpS) framework with respect to the consensus error. Moreover, based on the latest neighboring information, we provide a specific form of compatibility constraints and consensus error terminal regions.

Notations: $\mathbb{R}$ and $\mathbb{N}$ stand for the sets of real numbers and non-negative integers, respectively. $\mathbb{R}^n$ denotes a set of n-dimensional real column vectors. Given an arbitrary column vector, $\|\cdot\|$ represents its Euclidean norm. Let $\mathbb{R}_{\geq c_1}$, $\mathbb{R}_{(c_1,c_2]}$, $\mathbb{N}_{\geq c_1}$, and $\mathbb{N}_{(c_1,c_2]}$ denote $\{t\in\mathbb{R}\,|\,t\geq c_1\}$, $\{t\in\mathbb{R}\,|\,c_1<t\leq c_2\}$, $\{t\in\mathbb{N}\,|\;t\geq c_1\}$, and $\{t\in\mathbb{N}\,|\,c_1<t\leq c_2\}$, respectively. A scalar continuous function $\alpha:\mathbb{R}_{\geq 0}\to\mathbb{R}_{\geq 0}$ is a class of K if it is strictly increasing and $\alpha(0)=0$. If a function belongs to a class K and meets $\alpha(s)\to+\infty$ as $s\to+\infty$, it is called a class of K_∞. If for every fixed $t\in\mathbb{R}_{\geq 0}$, $\beta(\cdot,t)$ is a class of K, and for every fixed $s\in\mathbb{R}_{\geq 0}$, $\beta(s,\cdot)$ is decreasing and $\lim_{t\to\infty}\beta(s,t)=0$, then $\beta(\cdot,\cdot)$ is a class of KL.

2 Preliminaries and problem formulation

We study a group of perturbed nonlinear MASs consisting of M agents. The communication topology can be represented by a directed graph $G=\{V, E, A\}$, where the vertex set is $V=\{1, 2, \ldots, M\}$, the edge set is $E\subseteq\{(i, j): i, j\in V, i\neq j\}$, and the adjacency matrix is $A\in\mathbb{R}^{M\times M}$ with $A=[a_{ij}]$. In particular, if agent i can receive the information from agent j, then edge $(j, i)\in E$ and $a_{ij}=1$; otherwise, $a_{ij}=0$. The neighboring index set of agent i is denoted by $N_i=\{j: j\in V, (j, i)\in E\}$. $|N_i|$ represents the number of neighbors of agent i. Suppose that each vertex has no self-loop (i.e., $a_{ii}=0$), and that the communication network over MAS is directed (i.e., $a_{ij}\neq a_{ji}$). Furthermore, we require that each system should have at least one neighbor of information and each time instant can be measured.

To achieve consensus over MAS, each agent is modeled as

$$x_i(t+1)=f(x_i(t),u_i(t),w_i(t)),\; i\in\{1, 2, \ldots, M\}, \quad (1)$$

where $x_i(t)\in\mathbb{R}^n$, $u_i(t)\in\mathbb{R}^m$, and $w_i(t)\in W_i\subset\mathbb{R}^w$ represents the state, control inputs, additive disturbances of agent i, respectively. Here, $f:\mathbb{R}^n\times\mathbb{R}^m\times\mathbb{R}^w\to\mathbb{R}^n$ is an arbitrary nonlinear function with $f(0, 0, 0)=0$. For each agent i, define $e_i(t)=x_i(t)-\hat{x}_{-i}(t)$, where $e_i(t)$ is the state consensus error, and $\hat{x}_{-i}(t)$ represents the average state of its neighbors. The specific expression on $\hat{x}_{-i}(t)$ is given later. Let $\rho_i\triangleq\sup_{w_i(t)\in W_i}\|w_i(t)\|$ denote a known disturbance bound. Assume that the consensus error and control input are constrained as $e_i(t)\in E_i$, $u_i(t)\in U_i$. Furthermore, E_i, U_i, and W_i are compact sets containing the origin in their interiors.

Definition 1 (Robust positively invariant (RPI) set (Su et al., 2019a)) For the established system model (1), a set $E_i\subseteq\mathbb{R}^n$ is called an "RPI set" if for all $e_i(t)\in E_i$, $e_i(t+l)\in E_i$ $(l\in\mathbb{R}_{\geq 0})$ exists for all $w_i(t)\in W_i$.

Definition 2 (Regional ISpS) If there exists an RPI set $E_i\subseteq\mathbb{R}^n$ including the origin, a KL function β_i, a K function α_i, and a constant $d_i\in\mathbb{R}_{\geq 0}$ satisfying

$$\|e_i(t)\|\leq\beta_i(\|e_i(0)\|,t)+\alpha_i(\sup_{\tau\in[0,t-1]}\|w_i(\tau)\|)+d_i, \quad (2)$$

where each $e_i(0)\in E_i$ and $w_i(t)\in W_i$, then the state consensus error dynamics of system (1) is said to be ISpS in E_i with respect to w_i. $e_i(0)$ is the initial state consensus error, and w_i is the disturbance.

Lemma 1 For every agent $i\in\{1, 2, \ldots, M\}$, let $\sigma_i(\cdot)\in K$, $\alpha_1(\cdot)$, $\alpha_2(\cdot)$, $\alpha_3(\cdot)\in K_\infty$, and $\tau_1, \tau_2\in\mathbb{R}_{>0}$. Given a set E_i as defined in Definition 2 and a function $V_i(e_i)$ $\mathbb{R}^n\to\mathbb{R}_{\geq 0}$ to satisfy the following two conditions:

1. $\alpha_1(\|e_i\|)\leq V_i(e_i)\leq\alpha_2(\|e_i\|)+\tau_1$;
2. $V_i(e_i(t+1))-V_i(e_i(t))\leq-\alpha_3(\|e_i\|)+\sigma_i(\|w_i\|)+\tau_2$,

for all $e_i\in E_i$, $w_i\in W_i$, the state consensus error dynamics of system (1) reaches ISpS in E_i with respect to w_i, where function $V_i(\cdot)$ is called an ISpS–

Lyapunov function. When $\tau_1=\tau_2=0$, $V_i(\cdot)$ is an ISS–Lyapunov function. The specific proof can be found in Lazar et al. (2008).

We design a self-triggered robust DMPC consensus algorithm to obtain when and how to select control inputs for system (1) so that all agents can decrease the communication and computing resources and achieve consensus. We also assume that there is no delay in the transmission. The whole system operation procedure can be stated as follows: at the triggering instant t_k^i, each agent first deduces the self-triggered conditions according to system stability, and then determines the optimal control input sequence $\boldsymbol{u}_i^*(t_k^i)=\{u_i^*(t_k^i\,|\,t_k^i),u_i^*(t_k^i+1\,|\,t_k^i),\cdots,u_i^*(t_k^i+T-1\,|\,t_k^i)\}$ by solving an optimization problem. Before the next triggering instant t_{k+1}^i, each agent receives a predicted state sequence from every neighbor $j\in N_i$. Meanwhile, agent i stores its own predicted state sequence into a buffer area, and waits for other agents who demand it. The next calculation begins at t_{k+1}^i and repeats the above procedure.

Remark 1 Although this study discusses the discrete-time nonlinear MAS, we can apply a nearly equal process to solve the consensus problem of the continuous-time nonlinear MAS with periodic sampling.

3 Robust self-triggered DMPC consensus algorithm

Let t_k^i denote the k^{th} triggering instant of agent i with $k\geq0$. The cost function of each agent at the triggering instant can be defined as

$$\begin{aligned}&J_i^{H_k^i}(x_i(t_k^i),\hat{\boldsymbol{x}}_{-i}(t_k^i),\boldsymbol{u}_i(t_k^i),\boldsymbol{w}_i(t_k^i),T)\\&=\sum_{l=0}^{H_k^i-1}\gamma L_i(x_i(t_k^i+l|t_k^i),\hat{x}_{-i}(t_k^i+l|t_k^i),u_i(t_k^i+l|t_k^i),w_i(t_k^i+l|t_k^i))\\&\quad+\sum_{l=H_k^i}^{T-1}L_i(x_i(t_k^i+l|t_k^i),\hat{x}_{-i}(t_k^i+l|t_k^i),u_i(t_k^i+l|t_k^i),w_i(t_k^i+l|t_k^i))\\&\quad+F_i(x_i(t_k^i+T|t_k^i),\hat{x}_{-i}(t_k^i+T|t_k^i)),\end{aligned}\tag{3}$$

where $T\in\mathbb{N}_{\geq1}$ is the prediction horizon, H_k^i is the triggering interval calculated by $H_k^i=t_{k+1}^i-t_k^i$ $(H_k^i\in\mathbb{N}_{[1,H_{\max}]})$. $H_{\max}$ is the maximum triggering interval with $H_{\max}\in\mathbb{N}_{[1,T]}$. $x_i(t_k^i+l\,|\,t_k^i)$ is the state prediction of agent i regarding the future step t_k^i+l at time step t_k^i. $L_i(x_i(t_k^i+l\,|\,t_k^i),\hat{x}_{-i}(t_k^i+l\,|\,t_k^i),u_i(t_k^i+l\,|\,t_k^i),w_i(t_k^i+l\,|\,t_k^i))=\left\|x_i(t_k^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_k^i+l|t_k^i)\right\|+\lambda\left\|u_i(t_k^i+l\,|\,t_k^i)\right\|-\psi\left\|w_i(t_k^i+l\,|\,t_k^i)\right\|$ is the stage cost and its concrete form is a continuous function. Both λ and ψ are given weighing scalars and generally adopt $\lambda\in\mathbb{R}_{>0}$ and $\psi\in\mathbb{R}_{>0}$. F_i is the terminal cost, and its concrete form is also a continuous function $F_i(x_i(t_k^i+T\,|\,t_k^i),\hat{x}_{-i}(t_k^i+T\,|\,t_k^i))=\beta_i\left\|x_i(t_k^i+T\,|\,t_k^i)-\hat{x}_{-i}(t_k^i+T\,|\,t_k^i)\right\|$, and $\beta_i>0$ is a weighing scalar. Let $L_i(0, 0, 0)=0$ and $F_i(0, 0)=0$. $\gamma\in(0, 1)$ is a control parameter reflecting triggering or a communication effect. $\boldsymbol{u}_i(t_k^i)=\{u_i(t_k^i\,|\,t_k^i),u_i(t_k^i+1\,|\,t_k^i),\cdots,u_i(t_k^i+T-1\,|\,t_k^i)\}$ represents the future control inputs to be obtained; $\boldsymbol{w}_i(t_k^i)=\{w_i(t_k^i|\,t_k^i),\ w_i(t_k^i+1\,|\,t_k^i),\ \cdots,\ w_i(t_k^i+T-1\,|\,t_k^i)\}$ represents the additive disturbance sequence. The averaged state trajectory of the neighbors of agent i is $\hat{\boldsymbol{x}}_{-i}(t_k^i)=\{\hat{x}_{-i}(t_k^i|t_k^i),\hat{x}_{-i}(t_k^i+1|\,t_k^i),\cdots,\hat{x}_{-i}(t_k^i+2T\,|\,t_k^i)\}$ with

$$\hat{x}_{-i}(t_k^i+l\,|\,t_k^i)=\sum_{j\in N_i}\frac{\hat{x}_j(t_k^i+l\,|\,t_k^i)}{|N_i|},$$

where $\hat{x}_j(t_k^i+l\,|\,t_k^i)$ denotes the assumed state trajectory of agent j at t_k^i, which is obtained based on the received information of agent j at triggering instant $\Gamma_j(t_k^i)$. $\Gamma_j(t_k^i)$ stands for the triggering instant that is closest and occurs before t_k^i of agent j. In Gao et al. (2017) and Zhan et al. (2019a), the assumed state trajectory of agent j can be expressed as

$$\hat{x}_j(t_k^i+l\,|\,t_k^i)=\begin{cases}x_j^*(t_k^i+l\,|\,\Gamma_j(t_k^i)),\ l\in\mathbb{N}_{[0,T)},\\ \mu\hat{x}_j(t_k^i+l\,|\,\Gamma_j(t_k^i)),\ l\in\mathbb{N}_{[T,2T]},\end{cases}\tag{4}$$

where $\hat{x}_j(\Gamma_j(t_k^i)+T\,|\,\Gamma_j(t_k^i))=x_j^*(\Gamma_j(t_k^i)+T\,|\,\Gamma_j(t_k^i))$. To relieve the calculation burden, a fixed parameter μ

$(\mu \in \mathbb{R}_{>0})$ is chosen to determine which information from neighbor agents is useful for cooperation.

Remark 2 In the cost function, we use term $\left\| x_i(t_k^i + l \mid t_k^i) - \hat{x}_{-i}(t_k^i + l \mid t_k^i) \right\|$ instead of term $\sum_{j \in N_i} a_{ij} \cdot \left\| x_i(t_k^i + l \mid t_k^i) - \hat{x}_j(t_k^i) \right\|_Q^2$ (Li and Yan, 2015), where the setting of $\hat{x}_{-i}(t_k^i + l \mid t_k^i)$ can reduce part of the calculation. Meanwhile, we use the Euclidean norm instead of the usual quadratic function. Note that for every agent i, we assume some predicted trajectories for its neighbors in Eq. (4) before the next state update, because the current actual predicted trajectories $x_j(t_k^i + l \mid t_k^i)$ $(j \in N_i)$ are unknown as the self-triggered communication mechanism.

3.1 Min–max optimization

According to the defined cost function (3), each agent solves the following optimization problem SP$_i$:

$$V_i^{H_k^i}(e_i(t_k^i), T) \triangleq \min_{u_i(t_k^i)} \max_{w_i(t_k^i)} J_i^{H_k^i}\left\{ x_i(t_k^i), \hat{\boldsymbol{x}}_{-i}(t_k^i), \boldsymbol{u}_i(t_k^i), \boldsymbol{w}_i(t_k^i), T \right\} \tag{5a}$$

subject to

$$u_i(t_k^i + l \mid t_k^i) \in U_i, \tag{5b}$$

$$w_i(t_k^i + l \mid t_k^i) \in W_i, \tag{5c}$$

$$x_i(t_k^i + l + 1 \mid t_k^i) = f(x_i(t_k^i + l \mid t_k^i), u_i(t_k^i + l \mid t_k^i), w_i(t_k^i + l \mid t_k^i)), \tag{5d}$$

$$\left\| x_i(t_k^i + l \mid t_k^i) - \hat{x}_i(t_k^i + l \mid t_k^i) \right\| \le \frac{a}{T - H_k^i} \min_{j \in N_i} \left\| x_j(\Gamma_j(t_k^i)), - \hat{x}_{-j}(\Gamma_j(t_k^i) \mid \Gamma_j(t_k^i)) \right\|, \tag{5e}$$

$$x_i(t_k^i + T \mid t_k^i) - \hat{x}_{-i}(t_k^i + T \mid t_k^i) \in E_i^f, \tag{5f}$$

where $x_i(t_k^i \mid t_k^i) = x_i(t_k^i)$, $e_i(t_k^i) = x_i(t_k^i) - \hat{x}_{-i}(t_k^i)$, and $a \in \mathbb{R}_{(0,1)}$ is a constant. E_i^f is the state consensus error terminal region including the origin. In Zhan et al. (2019a), the compatibility constraint (Eq. (5e)) ensures a certain degree of consensus, which implies that the predicted trajectory cannot be far away from the assumed one.

We design the state consensus error terminal region E_i^f to satisfy

$$E_i^f \triangleq \left\{ x \in X_i \left| \left\| x - \hat{x}_i(t_k^i + l \mid t_k^i) \right\| \le a/T \min_{j \in N_i} \left\| x_j(\Gamma_j(t_k^i)) - \hat{x}_{-j}(\Gamma_j(t_k^i) \mid \Gamma_j(t_k^i)) \right\|, \ l > T - (H_k^i)^* \right. \right\}. \tag{6}$$

Assumption 1 At any triggering instant in system (1), an auxiliary local feedback control law $\bar{u}_i = k_i(x_i, \hat{x}_{-i})$ exists, where $k_i : \mathbb{R}^n \to \mathbb{R}^m$ and $k_i(0, 0)=0$, such that the state consensus error terminal region E_i^f is an RPI set. Meanwhile, $\bar{u}_i \in U_i$ holds for all $(x_i - \hat{x}_{-i}) \in E_i^f$; i.e., $e_i \in E_i^f$.

Remark 3 For the auxiliary local feedback control law, we design a set of fixed gains k_i offline (Lazar et al., 2008) to satisfy E_i^f as an RPI set. In addition, both control behavior and communication behavior are involved in Eq. (3). To prove that the considered MAS can reach a consensus, for each agent, we denote the cost function as $J_i(x_i(t_k^i), \hat{\boldsymbol{x}}_{-i}(t_k^i), \boldsymbol{u}_i(t_k^i), \boldsymbol{w}_i(t_k^i), T)$ when γ=1 and $H_k^i = 1$. Then its optimal cost is $V_i(e_i(t_k^i), T)$. This is the so-called "time-driven DMPC" without considering communication cost.

3.2 Self-triggering in optimization

Between any two successive triggering instants, the control input is in the form of

$$u_i^{\mathrm{ST}}(x_i(t_k^i + l \mid t_k^i)) = \boldsymbol{u}_i^*(t_k^i), \ l \in \mathbb{N}_{[0, t_{k+1}^i - t_k^i - 1]}, \tag{7}$$

where $\boldsymbol{u}_i^*(t_k^i)$ is a set of the optimal control sequence obtained at t_k^i by solving the optimization problem SP$_i$. The triggering instant is defined as follows:

$$\begin{aligned} & t_{k+1}^i = t_k^i + (H_k^i)^*, \\ & (H_k^i)^* \triangleq \max\left\{ H_k^i \left| H_k^i \in \mathbb{N}_{[1, H_{\max}]}, V_i^{H_k^i}(e_i(t_k^i), T) \right. \le V_i^1(e_i(t_k^i), T) \right\}. \end{aligned} \tag{8}$$

Remark 4 Note that for each agent, the control input from $t_k^i + 1$ to $t_{k+1}^i - 1$ is derived from an open-loop min–max optimization problem, which depends on the previous sampling instant t_k^i. Because of the self-triggered mechanism, communication resources can be saved as the communication period increases.

We obtain the optimal triggering interval $(H_k^i)^*$ by checking whether the optimal cost is dropping and by choosing a satisfied maximum triggering interval.

The self-triggered robust DMPC consensus algorithm for system (1) is summarized in Algorithm 1.

Algorithm 1 Self-triggered robust DMPC consensus

Off-line:

Require: $\lambda, \psi, \mu, \gamma, \alpha, \beta_i, k_i, T, H_{\max}$

On-line:

for each agent $i \in \{1, 2, \ldots, M\}$ **do**
- set $t_0^i = 0$ as the first triggering instant with k=0;
- transmit its state sequence $\hat{x}_i(\tau \mid 0)$ to its neighbors and receive $\hat{x}_j(\tau \mid 0)$ from every number $j \in N_i$, $\tau \in [0, T)$;
- solve problem SP_i to obtain $(H_k^i)^*$ and $u_i^*(0)$;

end for // Initialization

while each $e_i(t_k^i + l \mid t_k^i) \notin E_i^f$ **do**
- **while** t_k^i $(k \geq 1)$ is not triggered **do**
  - apply $u_i^{\text{ST}}(t_{k-1}^i)$ at $t \in [t_{k-1}^i, t_{k-1}^i + T)$;
  - obtain $\hat{x}_i(\tau \mid t_k^i)$ $(\tau \in [t_k^i, t_{k+1}^i])$ based on Eqs. (3) and (4);
  - store $u_i^*(t_k^i \mid t_{k-1}^i)$ and $x_i^*(t_k^i \mid t_{k-1}^i)$;
- **end while**
- measure the current state $x(t_k^i)$;
- solve SP_i in problem (5) and DPMC mechanism (8) to obtain $u_i^*(\tau \mid t_k^i)$ and $H_i^*(t_k^i)$ $\left(\tau \in [t_k^i, t_k^i + T)\right)$;
- set k=k+1;

end while

apply the auxiliary feedback control law $\bar{u}_i = k_i(x_i, \hat{x}_{-i})$ to the corresponding subsystems;

Remark 5 In Algorithm 1, the initial state trajectory $\hat{\boldsymbol{x}}_{-i}(t_0^i)$ of the neighbors of agent i is assumed by applying zero control without constraint (5e) or (5f). Constraints (5e) and (5f) are adopted to solve problem SP_i only when $k \geq 1$. Meanwhile, the control actions switch from solving problem SP_i to the auxiliary feedback control law as long as all subsystems' state consensus error trajectories enter the error terminal region, which further saves computation resources. The above control idea is called a "dual-mode strategy" (Dunbar, 2005). In addition, we must point out that although the self-triggered robust consensus algorithm effectively eases the communication load in this study, it also adds optimization computation due to the triggering mechanism. The quantization technique is an effective tool for saving control costs and communication resources, and many interesting results have been reported, such as in Feng et al. (2018), Xu et al. (2018), Yang et al. (2018), and Wan et al. (2019). Therefore, it is important to use quantized self-triggered control to optimize control and communication costs.

4 Feasibility and stability analysis

The optimization independence of MPC in adjacent time shows that the optimization feasibility of SP_i at the current moment does not guarantee the feasibility of the next moment. Thus, we must provide conditions to ensure that Algorithm 1 has iterative feasibility. Moreover, the property of iterative feasibility ensures that the optimization problem (5) is solvable.

Assumption 2 E_i^f satisfies $E_i^f \subseteq E_i$ and $0 \in E_i^f$.

Assumption 3 There exist α_l, α_f, $\alpha_{\text{F}} \in K_\infty$, α_w, and $\sigma \in K$, such that

(1) $L_i(x_i, \hat{x}_{-i}, u_i, w_i) \geq \alpha_l\left(\|x_i - \hat{x}_{-i}\|\right) - \alpha_w\left(\|w_i\|\right) \geq 0$, $\forall (x_i - \hat{x}_{-i}) \in E_i, \forall u_i \in U_i, \forall w_i \in W_i$;

(2) $\alpha_f\left(\|x_i - \hat{x}_{-i}\|\right) \leq F_i(x_i, \hat{x}_{-i}) \leq \alpha_{\text{F}}\left(\|x_i - \hat{x}_{-i}\|\right)$, $\forall (x_i - \hat{x}_{-i}) \in E_i^f$;

(3) $F_i(x_i(t_k^i+T+1 \mid t_k^i), \hat{x}_{-i}(t_k^i+T+1 \mid t_{k+1}^i)) - F_i(x_i(t_k^i+T \mid t_k^i), \hat{x}_{-i}(t_k^i+T \mid t_{k+1}^i)) \leq -L_i(x_i(t_k^i+T \mid t_k^i), \hat{x}_{-i}(t_k^i+T \mid t_{k+1}^i), u_i(t_k^i+T \mid t_k^i), w_i(t_k^i+T \mid t_k^i)) + \sigma\left(\|w_i(t_k^i+T)\|\right)$, $\forall (x_i - \hat{x}_{-i}) \in E_i^f, \forall w_i \in W_i$.

It can be observed that $F_i(\cdot,\cdot)$ is an ISS–Lyapunov function in E_i^f.

Lemma 2 If Assumption 3 is satisfied for any state consensus error $(x_i(t_0) - \hat{x}_{-i}(t_0)) \in E_i^f$ and admissible additive disturbance $w_i \in W_i$, then

$$F_i(x_i(t_m), \hat{x}_{-i}(t_m)) - F_i(x_i(t_0), \hat{x}_{-i}(t_0)) \leq -\sum_{p=0}^{m-1}\left(L_i(x_i(t_p), \hat{x}_{-i}(t_p), \bar{u}_i(t_p), w_i(t_p)) - \sigma\left(\|w_i(t_p)\|\right)\right), \tag{9}$$

where $x_i(t_m)$ is calculated by applying the auxiliary feedback control law $\bar{u}_i$, $m\in\mathbb{N}_{[1,T]}$.

Proof According to Assumption 3(3), we have

$$\begin{aligned}&F_i(x_i(l+1\mid t_k),\hat{x}_{-i}(l+1\mid t_{k+1}))-F_i(x_i(l\mid t_k),\hat{x}_{-i}(l\mid t_{k+1}))\\&\quad\le-(L_i(x_i(l\mid t_k),\hat{x}_{-i}(l\mid t_{k+1}),\bar{u}_i(l\mid t_k),w_i(l\mid t_k))\\&\quad-\sigma\left(\|w_i(l\mid t_k)\|\right),\end{aligned}\tag{10}$$

for all $(x_i-\hat{x}_{-i})\in E_i^f$. Because E_i^f is an RPI set, by summing inequality (10) from l=0 to l=m−1, inequality (9) can be proved for agent i.

Theorem 1 (Feasibility and stability) For each agent i under Assumption 3, if SP$_i$ is feasible at the initial triggering instant t_0^i, then Algorithm 1 is iteratively feasible. Furthermore, the state consensus error dynamics of system (1) reaches ISpS with respect to additive disturbances and it follows that as $t\to\infty$, $\|x_i-x_j\|=0$ for all $i, j\in\{1, 2, \ldots, M\}$, MAS with self-triggered robust DMPC mechanisms (7) and (8) can reach consensus.

The proof of Theorem 1 involves two parts: feasibility and consensus analysis.

4.1 Feasibility analysis

Definition 3 (Initial feasible set) For every agent i, the set $E_i^{\mathrm{MPC}}(T)\in E_i$ is called the "initial feasible set" of Algorithm 1, which indicates that the set of state consensus errors can be controlled robustly into E_i^f in T steps for all $e_i(t_0)\in E_i^{\mathrm{MPC}}(T)$ and all $w_i\in W_i$.

Suppose that we obtain a feasible solution $\boldsymbol{u}_i^*(t_k^i)$ of problem SP$_i$ and $(H_k^i)^*$ at t_k^i. Then we construct a feasible solution at triggering instant t_{k+1}^i. This can be expressed as

$$\begin{aligned}\tilde{\boldsymbol{u}}_i(t_{k+1}^i)=\big\{&u_i^*(t_{k+1}^i\mid t_k^i),u_i^*(t_{k+1}^i+1\mid t_k^i),\cdots,u_i^*(t_{k+1}^i+T-(H_k^i)^*\\&-1\mid t_k^i),\bar{u}_i(t_{k+1}^i+T-(H_k^i)^*\mid t_{k+1}^i),\bar{u}_i(t_{k+1}^i+T\\&-(H_k^i)^*+1\mid t_{k+1}^i),\cdots,\bar{u}_i(t_{k+1}^i+T-1\mid t_{k+1}^i)\big\},\end{aligned}\tag{11}$$

with $\bar{u}_i(t_{k+1}^i+q\mid t_{k+1}^i)=k_i(x_i(t_{k+1}^i+q\mid t_{k+1}^i),\hat{x}_{-i}(t_{k+1}^i+q\mid t_{k+1}^i))$ and $q\in\mathbb{N}_{[T-(H_k^i)^*,T-1]}$.

When $t_{k+1}^i\le t< t_{k+1}^i+T-(H_k^i)^*$, constraints (5b)–(5f) can be easily satisfied since $\tilde{u}_i(t\mid t_{k+1}^i)=u_i^*(t\mid t_k^i)$ during this period. When $t_{k+1}^i+T-(H_k^i)^*\le t< t_{k+1}^i+T$, using Assumption 1, constraints (5b)–(5d) and (5f) can be satisfied. In addition, according to the definition of the terminal region of the state consensus error in inequality (6), we can obtain compatibility constraint (5e) when $t_{k+1}^i+T-(H_k^i)^*\le t<t_{k+1}^i+T$. Thus, all constraints in problem SP$_i$ are satisfied, and $\tilde{\boldsymbol{u}}_i(t_{k+1}^i)$ is indeed a feasible solution at t_{k+1}^i. In summary, as long as problem SP$_i$ admits a feasible solution at the initial instant t_0^i, from the induction principle we can obtain feasible solutions for all k≥0.

4.2 Consensus analysis

Due to the iterative feasibility, we know that the optimization calculation between two successive triggering instants in Algorithm 1 is relevant, and that the value of the cost function defined in inequality (2) is relevant and comparable.

Lemma 3 For the optimal cost defined in problem (5), $V_i^1(e_i(t_k^i),T)\le V_i(e_i(t_k^i),T)$ exists.

Proof Suppose that $\boldsymbol{u}_i^*(t_k^i)=\{u_i^*(t_k^i\mid t_k^i),u_i^*(t_k^i+1\mid t_k^i),\cdots,u_i^*(t_k^i+T-1\mid t_k^i)\}$ and $\boldsymbol{w}_i^*(t_k^i)=\{w_i^*(t_k^i\mid t_k^i),w_i^*(t_k^i+1\mid t_k^i),\cdots,w_i^*(t_k^i+T-1\mid t_k^i)\}$ are the solutions obtained using $V_i(e_i(t_k^i),T)$. Then according to optimality, we obtain

$$\begin{aligned}V_i^1(e_i(t_k^i),T)&\le\max_{\boldsymbol{w}_i(t_k^i)}J_i^1(x_i(t_k^i),\hat{\boldsymbol{x}}_{-i}(t_k^i),\boldsymbol{u}_i^*(t_k^i),\boldsymbol{w}_i(t_k^i),T)\\&=\max_{\boldsymbol{w}_i(t_k^i)}J_i(x_i(t_k^i),\hat{\boldsymbol{x}}_{-i}(t_k^i),\boldsymbol{u}_i^*(t_k^i),\boldsymbol{w}_i(t_k^i),T)\\&\quad+(\gamma-1)L_i(x_i(t_k^i),\hat{x}_{-i}(t_k^i),u_i^*(t_k^i),w_i(t_k^i))\\&=V_i(e_i(t_k^i),T)+(\gamma-1)L_i(x_i(t_k^i),\hat{x}_{-i}(t_k^i),\\&\quad u_i^*(t_k^i),w_i(t_k^i)).\end{aligned}$$

According to Assumption 3(1), $L_i(x_i(t_k^i),\hat{x}_{-i}(t_k^i),u_i^*(t_k^i),w_i(t_k^i))\ge0$, and $\gamma\in(0,1)$. Lemma 3 holds.

Using the definition of SP$_i$ in problem (5), for all $e_i(t_k^i)\in E_i^{\mathrm{MPC}}(T)$, we obtain

$$V_i^{(H_k^i)^*}(e_i(t_k^i),T)=J_i^{(H_k^i)^*}(x_i(t_k^i),\hat{\boldsymbol{x}}_{-i}(t_k^i),\boldsymbol{u}_i^*(t_k^i),\boldsymbol{w}_i^*(t_k^i),T)$$
$$\geq \min_{I_i(t_k^i)} J_i^{(H_k^i)^*}(x_i(t_k^i),\hat{\boldsymbol{x}}_{-i}(t_k^i),\boldsymbol{u}_i(t_k^i),0,T)$$
$$\geq \gamma\alpha_l\left(\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i)\right\|\right). \tag{12}$$

To obtain the upper bound of the class K_∞ function of the value function, define a field of origin $O_r=\left\{(x_i-\hat{x}_{-i})\middle|(x_i-\hat{x}_{-i})\in E_i,\|x_i-\hat{x}_{-i}\|< r\right\}$ that satisfies $O_r\subseteq E_i^f$. O_r exists as E_i^f includes the origin. Then, consider the following two situations:

Case1: for all $e_i(t_k^i)\in E_i^f$. We have

$$\begin{aligned}&J_i^1(x_i(t_k^i),\hat{\boldsymbol{x}}_{-i}(t_k^i),\tilde{\boldsymbol{u}}_i(t_k^i),\boldsymbol{w}_i(t_k^i),T+1)\\&=J_i^1(x_i(t_k^i),\hat{\boldsymbol{x}}_{-i}(t_k^i),\boldsymbol{u}_i^*(t_k^i),\boldsymbol{w}_i(t_k^i),T)\\&\quad+F_i(x_i(t_k^i+T+1\,|\,t_k^i),\hat{x}_{-i}(t_k^i+T+1\,|\,t_k^i))\\&\quad-F_i(x_i(t_k^i+T\,|\,t_k^i),\hat{x}_{-i}(t_k^i+T\,|\,t_k^i))+L_i(x_i(t_k^i+T\,|\,t_k^i),\\&\quad\hat{x}_{-i}(t_k^i+T\,|\,t_k^i),\overline{u}_i(t_k^i+T\,|\,t_k^i),w_i(t_k^i+T\,|\,t_k^i)),\end{aligned} \tag{13}$$

where $\tilde{\boldsymbol{u}}_i(t_k^i)=[\boldsymbol{u}_i^*(t_k^i),\overline{u}_i(t_k^i+T\,|\,t_k^i)]$ and $\overline{u}_i(t_k^i+T\,|\,t_k^i)=k_i(x_i(t_k^i+T\,|\,t_k^i),\hat{x}_{-i}(t_k^i+T\,|\,t_k^i))$. Under Assumption 3(3) and the sub-optimality of the input control signals $\tilde{\boldsymbol{u}}_i(t_k^i)$, we have

$$\begin{aligned}V_i^1(e_i(t_k^i),T+1)&\leq\max_{w_i(t_k^i)}J_i^1(x_i(t_k^i),\hat{\boldsymbol{x}}_{-i}(t_k^i),\tilde{\boldsymbol{u}}_i(t_k^i),\boldsymbol{w}_i(t_k^i),\\&\qquad T+1)\leq V_i^1(e_i(t_k^i),T)+\sigma(\rho_i).\end{aligned} \tag{14}$$

Similarly, according to the triggering mechanism (8) and the above idea, we have

$$\begin{aligned}&V_i^{H_k^i}(e_i(t_k^i),T)\leq V_i^1(e_i(t_k^i),T)\\&\leq V_i^1(e_i(t_k^i),1)+(T-1)\sigma(\rho_i)\\&\leq F_i(x_i(t_k^i),\hat{x}_{-i}(t_k^i))+T\sigma(\rho_i)\\&\quad+(\gamma-1)L_i(x_i(t_k^i),\hat{x}_{-i}(t_k^i),\overline{u}_i(t_k^i),w_i(t_k^i))\\&\leq\alpha_F(\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i)\|)+T\sigma(\rho_i).\end{aligned} \tag{15}$$

Case 2: for all $e_i(t_k^i)\in E_i^{\mathrm{MPC}}(T)\notin E_i^f$, $e_i(t_k^i)\notin O_r$, and $\|x_i-\hat{x}_{-i}\|\geq r$. Since the iterative feasibility of the optimization problem has been proved previously, a group of feasible control solutions that can satisfy all constraints of the optimization problem exists. Meanwhile, the optimal cost is bounded. Therefore, for the finite prediction horizon T, a large positive number $D<+\infty$ is admitted such that $V_i^{H_k^i}(e_i(t_k^i),T)<D$ for all time instants. Define $\theta=\max(1,D/\alpha_F(r))$ and a class of K_∞: $\overline{\alpha}(s)=\theta\alpha_F(s)$. Apparently, $\overline{\alpha}(s)\geq\alpha_F(s)$ for all $s\in\mathbb{R}_{>0}$. The result is represented as

$$\begin{aligned}V_i^{H_k^i}(e_i(t_k^i),T)&\leq D\frac{\alpha_F\left(\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i)\right\|\right)}{\alpha_F\left(\|r\|\right)}+T\sigma(\rho_i)\\&\leq\overline{\alpha}\left(\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i)\right\|\right)+T\sigma(\rho_i).\end{aligned} \tag{16}$$

Combining inequalities (15) and (16), we can conclude that

$$V_i^{H_k^i}(e_i(t_k^i),T)\leq\overline{\alpha}\left(\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i)\right\|\right)+T\sigma(\rho_i). \tag{17}$$

In accordance with the triggering mechanism and Lemma 3, the result is inequality (18) (on the next page).

For $l\in\mathbb{N}_{[0,T-(H_k^i)^*-1]}$ and using the triangle inequality and constraint (5e), it holds that

$$\begin{aligned}&\left\|x_i^*(t_{k+1}^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_{k+1}^i)\right\|-\left\|x_i^*(t_{k+1}^i+l\,|\,t_k^i)\right.\\&\left.-\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_k^i)\right\|\leq\left\|\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_{k+1}^i)-\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_k^i)\right\|\\&=\left\|\sum_{j\in N_i}\frac{\hat{x}_j(t_{k+1}^i+l\,|\,t_{k+1}^i)-\hat{x}_j(t_{k+1}^i+l\,|\,t_k^i)}{|N_i|}\right\|\\&\leq\frac{a}{T-H_k^i}\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i\,|\,t_k^i)\right\|.\end{aligned} \tag{19}$$

Substituting inequality (19) into inequality (18) and considering Lemma 2, we obtain inequality (20) (on the next page).

The last two terms in inequality (20) can be treated as a constant τ_2. According to the sufficient conditions of ISpS in Lemma 1, we show that the state consensus error dynamics of system (1) is ISpS at triggering instants in $E_i^{\mathrm{MPC}}(T)$ with respect to w_i, and it follows that $\lim_{t\to\infty}\left(x_i(t)-\hat{x}_{-i}(t)\right)=0$, which implies that the considered MAS can reach consensus.

$$
\begin{aligned}
&V_i^{(H_{k+1}^i)^*}(x_i(t_{k+1}^i),T)-V_i^{(H_k^i)^*}(x_i(t_k^i),T)\\
&\quad\le V_i^1(x_i(t_{k+1}^i),T)-V_i^{(H_k^i)^*}(x_i(t_k^i),T)\\
&\quad\le \sum_{l=0}^{T-(H_k^i)^*-1}\left(\left\|x_i^*(t_{k+1}^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_{k+1}^i)\right\|+\left\|x_i^*(t_{k+1}^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_k^i)\right\|\right)\\
&\qquad+\sum_{l=T-(H_k^i)^*}^{T-1}\left(\left\|x_i(t_{k+1}^i+l\,|\,t_{k+1}^i)-\hat{x}_{-i}(t_{k+1}^i+l\,|\,t_{k+1}^i)\right\|+\lambda\left\|\overline{u}_i(t_{k+1}^i+l\,|\,t_{k+1}^i)\right\|-\psi\left\|w_i(t_k^i+l\,|\,t_k^i)\right\|\right)\\
&\qquad-\sum_{l=0}^{(H_k^i)^*-1}\gamma\left(\left\|x_i^*(t_k^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_k^i+l\,|\,t_k^i)\right\|+\lambda\left\|u_i^*(t_k^i+l\,|\,t_k^i)\right\|-\psi\left\|w_i(t_k^i+l\,|\,t_k^i)\right\|\right)\\
&\qquad+F_i(x_i(t_{k+1}^i+T\,|\,t_{k+1}^i),\hat{x}_{-i}(t_{k+1}^i+T\,|\,t_{k+1}^i))-F_i(x_i(t_k^i+T\,|\,t_k^i),\hat{x}_{-i}(t_k^i+T\,|\,t_k^i)).
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&V_i^{(H_{k+1}^i)^*}(x_i(t_{k+1}^i),T)-V_i^{(H_k^i)^*}(x_i(t_k^i),T)\\
&\quad\le \sum_{l=0}^{T-(H_k^i)^*-1}\frac{a}{T-H_k^i}\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i\,|\,t_k^i)\right\| +(H_k^i)^*\sigma(\rho_i)\\
&\qquad-\sum_{l=0}^{(H_k^i)^*-1}\gamma\left(\left\|x_i^*(t_k^i+l\,|\,t_k^i)-\hat{x}_{-i}(t_k^i+l\,|\,t_k^i)\right\|+\lambda\left\|u_i^*(t_k^i+l\,|\,t_k^i)\right\|-\psi\left\|w_i(t_k^i+l\,|\,t_k^i)\right\|\right)\\
&\quad\le-\gamma\alpha_l\left(\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i\,|\,t_k^i)\right\|\right)+\gamma\alpha_w\left(\left\|w_i(t_k^i+l\,|\,t_k^i)\right\|\right)+a\left\|x_i(t_k^i)-\hat{x}_{-i}(t_k^i\,|\,t_k^i)\right\|+(H_k^i)^*\sigma(\rho_i).
\end{aligned}
\tag{20}
$$

5 Simulations

Considering a four-agent cart-damper-spring system, the dynamics of each agent is

$$
\begin{cases}
x_{i,1}(t_{k+1}^i)=x_{i,1}(t_k^i)+x_{i,2}(t_k^i)T_i,\\
x_{i,2}(t_{k+1}^i)=-\dfrac{k_iT_i}{M_i}\mathrm{e}^{-x_{i,1}(t_k^i)}x_{i,1}(t_k^i)+\dfrac{M_i-h_iT_i}{M_i}x_{i,2}(t_k^i)\\
\qquad+\dfrac{T_i}{M_i}u_i(t_k^i)+\dfrac{T_i}{M_i}w_i(t_k^i),
\end{cases}
\tag{21}
$$

where $x_{i,1}$ and $x_{i,2}$ express the displacement of the cart and its velocity, respectively, and k_i=0.25 N/m is the linear spring factor, h_i=1.20 N·s/m is the damper factor, M_i=1 kg is the mass of the cart, and T_i=0.4 s is the sampling period. For simplicity, each agent has the same system parameters. The input control force is u_i, which is limited to $-2\le u_i\le 2$. The additive disturbance constraint is set as $-0.2\le w_i\le 0.4$. In addition, each agent can communicate with its neighbor agents.

For the four agents, their neighboring sets are N_1={2}, N_2={1, 3}, N_3={2, 4}, and N_4={3}. Some parameters obtained offline are selected as T=5, λ=0.01, $H_{\max}$=4, ψ=2, α=0.2, $E_i^f=2$, and β_i=3 for all i's. The auxiliary local feedback control gain and the initial state of four agents are designed as k=[−0.6, −0.4; −0.6, −0.4; −0.5, −0.3; −0.5, −0.4] and x=[3.4, −1.5; 0.6, 0.5; −1.2, 2; 2.5, −1.2], respectively. Using the MATLAB fminimax module, the proposed self-triggered DMPC consensus algorithm is executed. To show the performance level under Algorithm 1, we consider two configurations of γ=0.85 and γ=0.5 on the constrained min–max optimization problems. Parameter k represents the number of samples. The performance results are shown in Figs. 1 and 2. For each agent, Figs. 1a and 1b display the state trajectories of each agent, while Fig. 1c displays the control inputs. Figs. 2a and 2b display the corresponding triggering instants of each agent, showing that all triggering intervals converge to $H_{\max}$=4. It can be observed that a smaller γ has a lower triggering frequency, which suggests that the burden of communication can be reduced. For further comparisons, we use time-driven DMPC with the same parameters when γ=0.85 to obtain results. The comparison of number of triggering times regarding different γ's and time-driven parameter is presented in Table 1, which shows that the self-triggered approach significantly reduces the communication cost. Figs. 3a and 3b plot the evolution of system states using self-triggered and time-driven control, respectively. Note that the performance of self-triggered control is comparable with that of time-driven control, where the considered MAS reaches consensus in two strategies. Fig. 3c provides the control inputs of two strategies.

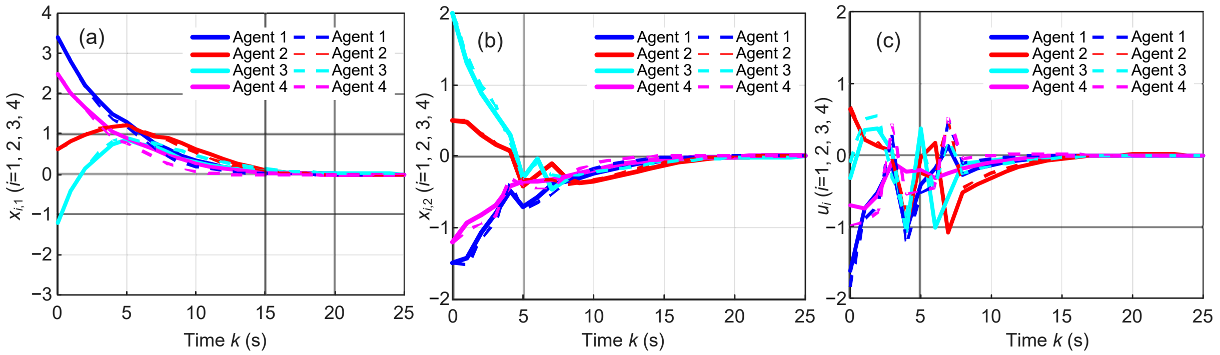

Fig. 1 Trajectories of system states $x_{i,1}$ (a), $x_{i,2}$ (b), and u_i (c) with γ=0.85 (solid lines) and γ=0.5 (dashed lines)

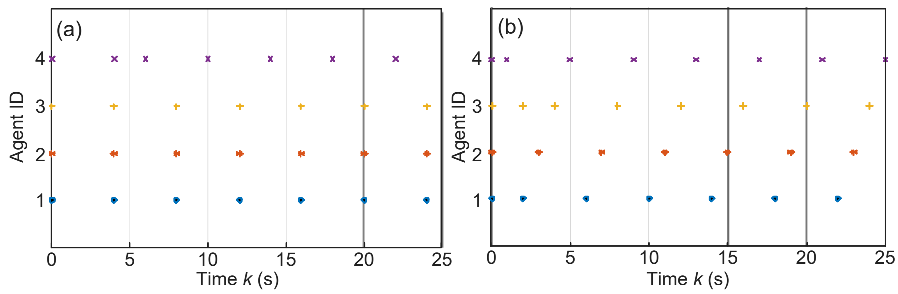

Fig. 2 Triggering instants with γ=0.5 (a) and γ=0.85 (b)

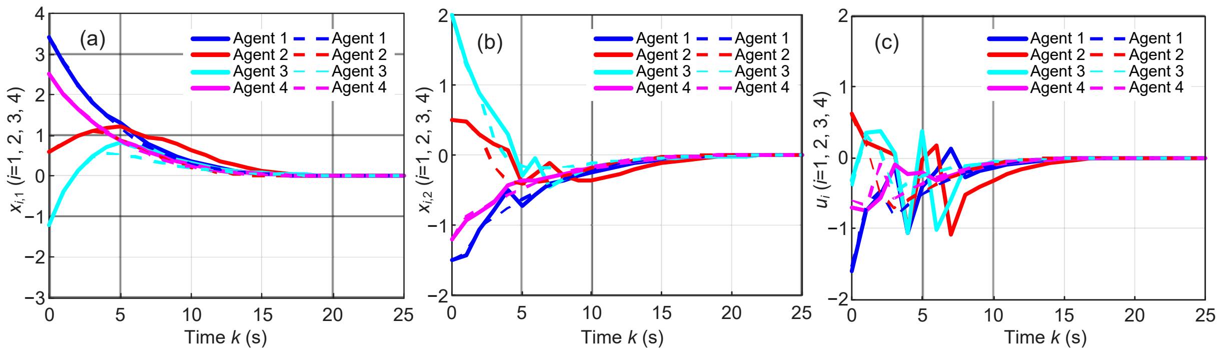

Fig. 3 State trajectories $x_{i,1}$ (a), $x_{i,2}$ (b), and u_i (c) using self-triggered (solid lines) and time-driven (dashed lines) control

Table 1 Comparison of total number of triggering times

Control parameter		Average number of triggering times	Total number of triggering times
Time-driven		1.000 (for agents 1–4)	100
Self-triggered	γ=0.85	3.571 (for agents 1 and 2) 3.125 (for agents 3 and 4)	30
	γ=0.5	3.571 (for agents 1–4)	28

6 Conclusions

In this study, a robust DMPC method has been used to study the consensus problem of discrete nonlinear MAS with additive disturbances. We have proposed a self-triggered control scheduler based on a min–max optimization problem to determine the control inputs and maximize the triggering interval. The control sequence updates and transmits at only triggering instants, which can significantly reduce communication costs. The conditions that guarantee the feasibility of the algorithm and the consensus over the perturbed nonlinear MAS are sufficient and practicable, and we have used the invariant set theory to realize ISpS with respect to the state consensus error to ensure that the closed-loop MAS can achieve consensus. Furthermore, simulation examples have shown the effectiveness of the algorithm.

Contributors

Qingling WANG, Yanxu SU, and Changyin SUN guided the research. Jiaqi LI performed the experiments and drafted the manuscript. Qingling WANG and Yanxu SU helped organize the manuscript. Jiaqi LI revised and finalized the paper.

Compliance with ethics guidelines

Jiaqi LI, Qingling WANG, Yanxu SU, and Changyin SUN declare that they have no conflict of interest.

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