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ORIGINAL ARTICLE

Expansion of mortar joints in direct shear tests of masonry samples: implications on shear strength and experimental characterization of dilatancy

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Abstract The expansion of masonry specimens during direct shear tests has been reported in several research studies. This phenomenon, known as dilatancy, is caused by the formation of cracking surfaces in mortar joints. In particular, when the cracking surface is not perfectly flat, the shear displacements tend to increase the volume of the sample. Experimental investigations focused on the characterization of this phenomenon are rather limited for masonry and the effects on shear strength have received little attention, with consequent issues for a correct interpretation of the results. The present article reports the results of an ongoing research on brick masonry aimed to characterize experimentally the dilatancy and to evaluate the role of this phenomenon in the interpretation of the direct shear test. If the expansion of the specimen is significantly restrained, the standard approaches used for the characterization of the mechanical parameters (as per EN 1052-3 and ASTM C1531) tend to overestimate the initial shear strength (f_{vo}) and underestimate friction. Moreover, no indications are generally given to characterize dilatancy with experimental data. This aspect is particularly important for the micro-modelling of masonry because the constitutive models commonly used for mortar joints require this information. One of the objectives of the present article is to propose a simple model for a sound interpretation of the direct shear test of masonry samples taking into account the dilatancy. Several masonry samples composed of calcium silicate units and cement mortar joints have been subjected to triplet tests (EN 1052-3) and laboratory-simulated shove tests. First, a repeatable and objective methodology to measure and characterize the dilatancy is provided. Then, an extension of the standard methodology of the EN 1052-3 and ASTM C1531 that includes the contribution of this phenomenon is proposed. The novel formulation offers the possibility to characterize dilatancy with experimental data and the definition of mechanical parameters that are not biased by the presence of this phenomenon. The model presented in this article has proven to be consistent with the experimental data and it has been validated numerically in another recent research study.

Keywords Masonry · Shear strength · Expansion · Dilatancy · Triplet test (EN 1052-3) · In situ shove test (ASTM C1531)

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1 Introduction

The shear displacements of mortar joints during the shear failure of masonry is often accompanied by volumetric expansion. The tendency of mortar joints to dilate, when simultaneously subjected to shear stress and normal compression, increases the shear resistance because it opposes the compression force. This mechanism, known as dilatancy, has already been described for masonry by various researchers (e.g. [1–13]). In structural mechanics, this phenomenon is known as the so-called '*aggregate interlock*', which influences the shear strength of granular materials such as concrete (e.g. [14, 15]). Although the dilatancy is rather well framed in the study of rock mechanics (e.g. [16, 17]) and geotechnical engineering (e.g. [18, 19]), the study of the relationship between this mechanism and the shear strength has received comparatively little attention in masonry.

The interpretation of the triplet test EN 1052-3 [20], which is the standard laboratory test in Europe for the characterization of the initial shear strength of bed joints in masonry, and the "shove" test (ASTM C1531 [21]), which can be considered the equivalent test executed on site, are carried out with a friction model based on Coulomb's law. It is worth noting that, by comparing the strength parameters determined through laboratory shove tests with those obtained via triplet tests, may result in significant differences [22, 23]. It has been observed that this discrepancy may be caused by a wrong estimation of the normal compressive stress acting on the mortar joints under examination [1]. In particular, the actual compressive stress induced by the flat-jacks on the tested bed joints is influenced by the removal of the adjacent units. The normal stress (σ_{av}) may include a non-negligible contribution coming from far-field boundary conditions (σ_{ff}), such as those due to pre-existing overburden loads. It was found that also the phenomenon of dilatancy play a role in the definition of σ_{av}. The partial constraint imposed on the expansion by the masonry surrounding the test unit may locally increase the amount of compression in the bed joints [1]. Since the shear strength of a joint is affected by the normal stress, a correct comparison between the triplet test and the shove test implies the definition of a reliable value of σ_{av}. This type of research required both experimental and numerical investigations.

On the other hand, the numerical simulation of masonry with the micro-modeling approach (e.g. [3, 24]) requires the correct definition of the dilatancy angle (ψ) to accurately reproduce the expansion of mortar joints during shear failure. Since there are few indications for the experimental characterization of this mechanical parameter, most of the time, it is defined arbitrarily and sometimes divergent values are assumed (e.g. [25, 26]).

This article reports the results of an ongoing research aimed to quantify the contribution of dilatancy in the definition of the shear strength of masonry. The main objective of the study presented in this article is to investigate the phenomenon experimentally and propose a theoretical formulation that can be applied to other types of masonry. Several masonry samples composed of calcium silicate units and 10-mm-thick cement mortar joints have been subjected to triplet tests and laboratory shove tests (Fig. 1). This type of masonry was selected as a benchmark in this study because it highlighted a significant dilation in triplet tests (e.g. [10]). This article describes how to characterize experimentally the dilatancy, highlighting some issues of the current standards (EN 1052-3 and ASTM C1531) that occur in the characterization of the parameters when significant dilatancy is recorded during the tests. Contrary to the standard methodologies, the proposed procedure allows the experimental characterization of the initial shear strength under zero compressive load (f_{vo}) and friction coefficient (μ) that result unbiased by the presence of dilatancy. When specimens do not exhibit expansion during the tests, the parameters defined with the proposed method converge to those defined with the standard procedure.

The proposed formulation has been validated numerically in Andreotti et al. [1] by means of the micro-modelling technique and it also contributed to the proposal for an improved procedure and interpretation of ASTM C1531 for the in situ determination of brick-masonry shear strength [27].

2 Experimental observations

Dilatancy can be measured experimentally with the following equation expressing the dilatancy angle [4, 10]:

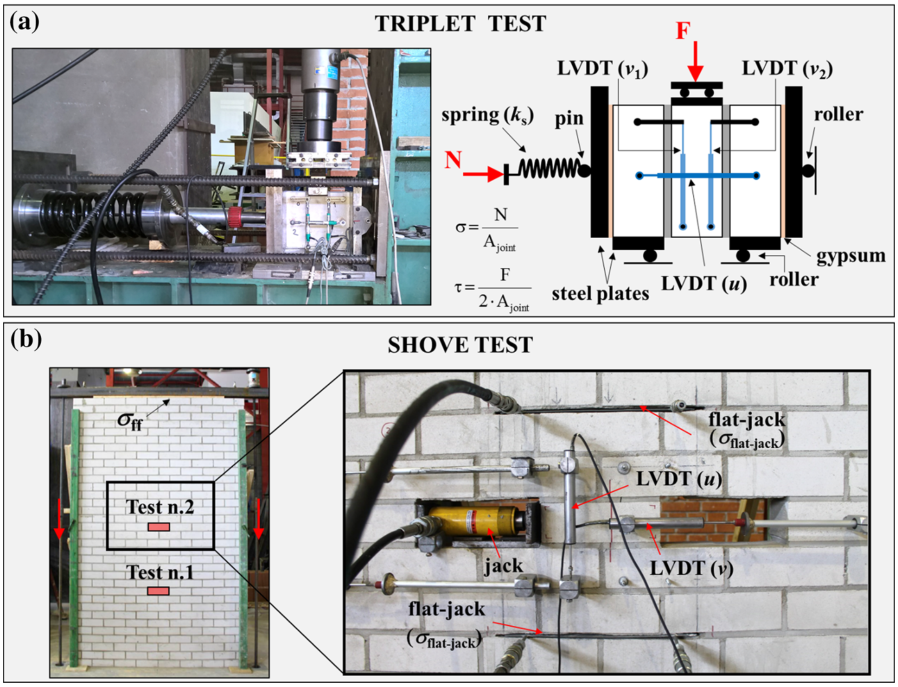

Fig. 1 Experimental setup of triplet tests (**a**) and laboratory shove tests (**b**)

$$\psi = \arctan\left(-\frac{du_{pl}}{dv_{pl}} \cdot n\right) \quad (1)$$

where dv_{pl} and du_{pl} are respectively, the plastic displacement in the shear direction and in the direction perpendicular to shear displacement, expressed in incremental terms. Equation (1) is not valid when the bed joint is in the elastic phase because dx_{pl} is zero. The plastic phase is identified by the onset of sliding (i.e. $v_{pl} \neq 0$). n is a coefficient that has been introduced in Eq. (1) to take into account the number of bed joints that contribute to the mechanism of dilatancy. In the triplet test, when the onset of sliding has been detected in the two mortar joints, n is equal to 0.5 because it is assumed that both joints contribute equally to dilatancy (see Fig. 2). When the onset of sliding in the two bed joints occurs in different time instants, this type of failure is called '*sequential*'. Once this condition arises, friction and dilatancy are mobilized only in one joint and $n = 1$ until the failure occurs also in the other bed joint. At this stage, when both joints are sliding, n is set equal to 0.5 (see Fig. 2). It is worth noting that in the couplet test, where the specimen has only one bed joint, n is always equal to one.

Lourenço [3, 4] and Van der Pluijm [10] found that dilatancy angle (ψ) tends to decrease with the increase of shear displacement and compression. According to these observations, the experimental tests discussed in this article have been organized in several time windows defined by subsequent executions of the test on the same specimen. The objective is to measure experimentally the dilatancy angle and shear strength at different levels of shear displacement and compression. The experimental campaign consists of several triplet tests executed according to EN 1052-3 on 9 brick masonry specimens (see Fig. 1 and Appendix A: Figures A5 and A6 in the online supplementary material) and of 6 laboratory shove tests executed according to ASTM C1531 at different positions of one masonry wall (see Fig. 1 and Figure A7 in the online supplementary material, Appendix A).

All specimens are characterized by calcium silicate (CS) bricks, with dimensions of 212 × 102 × 72 mm, and cement mortar joints with thickness equal to 10 mm. The density of masonry and bricks are, respectively 1835 kg/m^3 and 1900 kg/m^3. The experimental program includes also characterization tests on units, mortar and masonry assemblies. Tests have been carried out at the DICAr Laboratory of University of Pavia in 2015 (for more information see technical report [28, 29]). The strength of mortar has been defined according to EN 1015-11: the

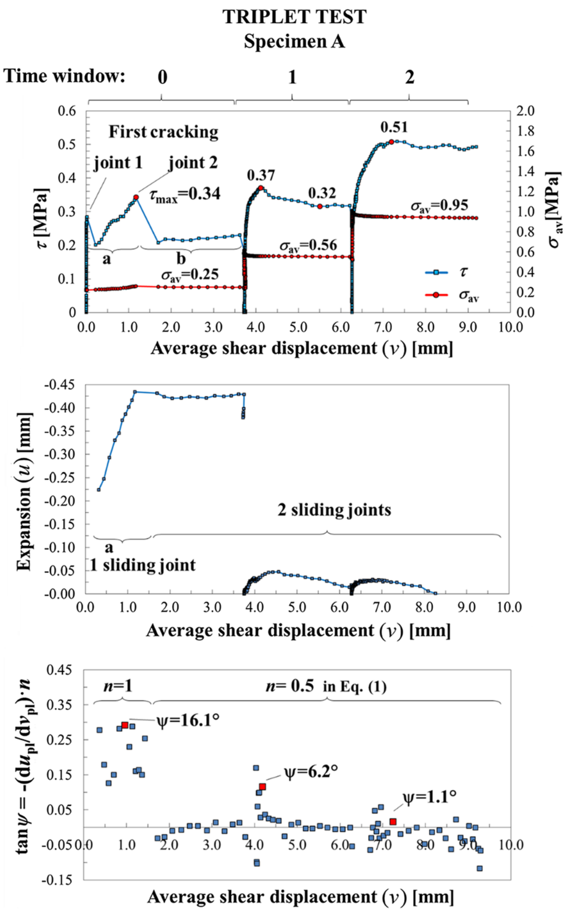

Fig. 2 Data of tests executed on specimen A. Shear stress (τ), volumetric expansion (u) and dilatancy angle (ψ) versus shear displacement curves (v)

compressive strength is 7.24 MPa and the flexural strength is 2.870 MPa. The compressive strength of bricks is 18.67 MPa. The compressive strength of masonry (6.20 MPa) and Young's modulus of masonry in compression (4182 MPa) have been defined according to EN 1052-1. The flexural bond strength of masonry (0.24 MPa) has been defined according to EN 1052-5.

2.1 Triplet test

2.1.1 Description of the results

Since EN 1052-3 refers to the peak values of shear stress (τ_{max}), the triplet tests are commonly executed under controlled force. The shear force is applied with a static hydraulic jack according to the EN 1052-3 prescriptions. The oil pressure is slowly increased. If the sampling frequency is adequate (e.g. 60 Hz), the peak and the immediate post-peak phase can be recorded with reasonable accuracy. However, if the sampling frequency is low (e.g. 25 Hz), the acquisition of data concerning the first cracking of the sample could be less accurate because the sliding of the brick is relatively fast. The triplet tests were more susceptible to this phenomenon than the shove tests because the sampling frequency was lower (e.g. 25 Hz).

The average shear stress in bed joints (τ) is computed by dividing the shear force with the area of bed joints (see Fig. 1). The average compressive stress in bed joints is similarly evaluated as $\sigma_{av} = N/A_{joint}$, where N is the compression force applied by the horizontal jack. The relative displacements have been recorded with 6 linear variable differential transformers (LVDTs), three on each side of the specimen. The average shear displacement (v) is the mean value of the displacements measured by the four LVDTs parallel to the shear force (see Fig. 1). The average expansion (u) is the mean value of the displacements measured by the transducers perpendicular to the shear force, measured on the two opposite sides of the specimen (see Fig. 1). The setup of the triplet test has been prepared according to the EN 1052-3. The boundary conditions are reported in detail in Fig. 1a. A tiny gypsum layer has been interposed at the interfaces between the steel plates and the bricks. The steel plates rest on rollers that allow rotations. A soft spring with stiffness k_s = 1070 N/mm is interposed between the specimen and the horizontal jack that applies the compression in bed joints (σ_{av}) to allow the expansion (dilation) of the joints with minimal variation of normal stress, which should be kept constant during the execution of the test.

Each triplet test has been executed in three time windows that are defined by successive executions of the test on the same specimen but with modified initial conditions. In each time window, the shear force is characterized by loading with a static hydraulic jack and complete unloading. The value of compression N is imposed to the specimen before the application of the shear force. The time window 0 represents the first execution of the test on the undamaged sample. This time window gives information about the fist cracking of mortar joints and is used for the definition of the maximum shear strength according to EN 1052-3 (τ_{max}) and maximum dilatancy angle. In the time windows 1 and 2, the test is repeated on the same sample with mortar joints that have already been damaged in the previous time window. As previously introduced, the triplet tests are characterized by a sequential failure of the two bed joints (time window 0-a and b). This behaviour is favoured by the aleatoric variability of the mechanical properties of the mortar. The first cracking is generally characterized by two peaks (see Fig. 2), which represent the complete crack formation and sliding in the two joints occurred in different time instants. As shown in Fig. 2, the Time window 0 is divided in two sub-windows: a) when the complete cracking has already formed only in one joint and b) when both joints are sliding. In time window 0, the specimen is initially intact and the test is generally stopped at the onset of sliding in both bed joints (time window 0-b). The expansion of the specimen (i.e. u in Fig. 1) is also recorded during the test. The dilatancy angle has been computed with Eq. (1). When the first cracking is sequential the triplet has only one sliding joint, $n = 1$, and dv_{pl} is computed excluding the elastic displacement of the other non-sliding joint. When both joints are sliding, $n = 0.5$ and dv_{pl} is the mean value of the displacements measured by the four LVDTs parallel to the shear force, minus the elastic component. The objective of the subsequent runs (i.e. time windows 1 and 2) is to study the variation of the shear strength in relation to the dilatancy angle by varying the initial conditions.

According to the European standard EN 1052-3, the first cracking, which in this study is identified by the time window 0, should be investigated using at least three samples for each of the three reference compression levels (e.g. about 0.2, 0.6 and 1 MPa). In the proposed procedure the test on a single specimen should continue with the same logic of EN 1052-3, considering two further time windows (1 and 2) with at least one variation of compression level. Figure 2 shows the post-processing of the data acquired during the tests executed on one of the nine samples.

Data of specimens A and B have been discussed in detail because they are representative of the mechanical behaviour of all samples investigated with the triplet test (see also the online supplementary material in Appendix A: Figure A5 related to specimen A and Figures A1 and A6 on specimen B).

The initial conditions of the time windows of specimen A are characterized by a progressive increase in compression and shear displacement. The peak values of shear stress (τ_{max} in time window 0) are defined according to EN 1052-3. When the crack is fully formed in one joint, the shear stress drops significantly with the rapid increment of shear sliding. This means that decohesion is completed. The run is stopped when both joints are sliding. As reported in Fig. 2, the specimen A showed significant expansion during time window 0. If compared to specimen B, the dilatancy recorded in specimen A is significantly higher. The failure mechanism of the two samples is also different (see Figures A5 and A6 in the online supplementary material, Appendix A). The sample A shows an evident crack through one mortar joint while the specimen B is dominated by decohesion at the unit-mortar interface. This behaviour is consistent with the observations of Van der Pluijm [10].

In the time window 1, the compression has been increased before the application of the shear force. It is important to underline that the cracks in both bed joints were fully formed during the previous time window because both joints exhibited shear sliding. In spite of this, the shear-displacement curve (τ–v) shows a peak and softening behaviour even if the compression is constant (Fig. 2). It can be noted that the attainment of the peak shear strength is associated to a non-negligible value of dilatancy angle ($\psi = 6.2°$). Moreover, the softening of the shear stress goes hand in hand with reduction of the dilatancy angle. Finally, in time windows 2, the compression and the initial shear displacement are higher than in the previous time windows. The dilatancy angle at peak shear approaches zero and the τ-v curve does not show appreciable softening.

The experimental tests carried out on specimen B are characterized by a different configuration of the compression time history (see Fig. 5 and online supplementary material in Appendix A: Figures A1 and A6). The value of σ_{av} has been set in time window 0, it has been reduced in time window 1 and then reset to the initial value in time window 2. According to the observations of Lourenço [4] and Van der Pluijm [10], the variation of the value of dilatancy angle is strictly connected to the level of compression. At the beginning of the time window 1, specimen B is characterized by shear displacements higher than in time window 0. However, in time window 1, the level of compression is lower and the recorded dilatancy is higher. The variation of the dilatancy angle with the shear displacement (v) and the compression (σ_{av})

recorded in the experimental campaign of triplet tests are reported in the supplementary materials of this article (see Figure A2 in the online supplementary material, Appendix A). Despite the correlation is not strong, the general trend is clear. The high scattering of the dilatancy angle is partially due to the geometrical complexity of the cracking surface in mortar joints and the type of failure mechanism activated in the sample. As will be discussed more in detail later in this article, the dilatancy angle is related to the primary asperities of the cracking surface (see Fig. 4) and it is generally higher when the crack passes through the mortar rather than decohesion takes place at the unit-mortar interface.

As already described for specimen A, at the beginning of time windows 1 the bed joints of specimen B are already cracked, nevertheless, the τ-v curve shows a slight softening as the dilatancy angle decreases. In time windows 2, the dilatancy angle is very low and the τ-v curve does not show appreciable softening. For the specimen B, the comparison between the time window 2 and the time window 0, at approximately $v = 4$ mm, is important (see Figure A1 in the online supplementary material, Appendix A). It shows that the "steady state" value of shear strength (or constant volume shear stress τ_{cv}) has already been reached at the end of the time window 0, when the value of dilatancy angle is close to zero. The repetition of the test with the same level of compression (i.e. time window 2) converges to the same value of shear strength and dilatancy angle.

Finally, Fig. 3 shows the experimental values of the initial shear strength of masonry under zero compressive load (f_{vo}) and friction coefficient (μ) defined with the data collected in time window 0, according to EN 1052-3 as recommended by Eurocode 6. The European standard EN 1052-3 gives the possibility to interpret data from triplet tests by using at least nine different samples and then fitting the values of peak shear stress (τ_{max}) in the τ–σ plane with Coulomb's law. It is worth emphasizing that all the data reported in Fig. 3 refer to the peak shear stress defined in time window 0. This figure reports also the empirical relation found between the dilatancy measured at the peak value of shear strength and average compression.

The dilatancy has been computed in terms of tan(ψ) with Eq. (1). As already discussed, the correlation is weaker than the τ–σ relation because the geometrical complexity of the cracking surface in mortar joints and

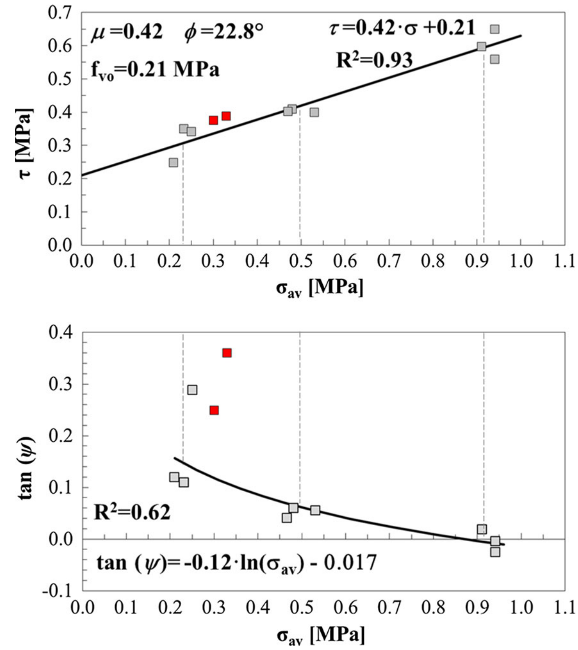

Fig. 3 Characterization of the mechanical parameters defined according to the EN 1052-3 (i.e. Time window 0), with illustration of the variation of dilatancy tan(ψ) at peak values of shear strength (τ_{max}) with average compression. The results of the shove tests, which are not used for the definition of the empirical relations, are shown in red. (Color figure online)

the different types of failure mechanisms activated in the samples generate high dispersion of the dilatancy angle. However, the trend of this parameter is well defined. It has a high value at low compression approaching zero at high compressive stresses. Since the dilatancy angle increases the shear resistance (e.g. [1, 13]), at low compressive stresses the shear strength is significantly influenced by the dilatancy angle. On the other hand, the shear strength at high levels of precompression is unaffected by the dilatancy phenomenon. As a term of comparison, Fig. 3 also reports the results of the laboratory shove tests in time window 0 using the numerical values of σ_{av} reported in Andreotti et al. [1]. These data points were not considered for the definition of the empirical relations.

When the expansion of specimens is restrained, there is a coupling effect between compression, dilatancy and shear strength because the increase of dilatancy tends to increase the compression, resulting in the increase of shear strength. Figure 3 shows that the average values of dilatancy recorded in the shove tests tend to be similar to the upper bound of dilatancy recorded in the triplet tests. As confirmed by the numerical simulations (see [1]), the compression in the bed joints at the beginning of the tests are similar but, with the application of shear stress, the compression increases more in the shove test due to higher values of dilatancy angle. The higher values of peak shear strength recorded in the shove test are consistent with the results of the triplet tests if interpreted considering the actual vertical compression in the in situ test (see [1, 27]).

2.2 Shove tests (executed in laboratory)

The experimental campaign has been extended with several force-controlled shove tests (ASTM C1531 2016) repeated at two different heights of a masonry wall (1766 × 102 × 2761 mm). The laboratory specimen has been built under laboratory conditions in the same period and with the same materials used for the preparation of the samples subjected to triplet tests (Fig. 1). The choice to report the results of this test in the present article has been undertaken mainly for two reasons. On the one hand, to study the phenomenon of dilatancy under more realistic boundary conditions (e.g. running bond pattern). On the other hand, evaluate consistency in the interpretation of two theoretically identical tests, executed on the same material (see [1, 27]). Furthermore, it is important to underline that the setup of the shove test generates slower failures in the mortar joints giving complementary information about dilatancy. As will be discussed, this type of test is significant to the scopes of the present article because it provides high quality recordings about the evolution of dilatancy in the immediate pre- and post-peak phases of the shear stress.

A constant level of compression was imposed to the wall before the execution of the tests by means of a beam, located on the top of the unreinforced masonry pier (i.e. "far field" stress σ_{ff} = 0.22 MPa), which is connected to the ground with two pretensioned tie rods. A variable amount of compression is locally enforced with two flat-jacks just above and below the brick that is subjected to the shear force. The amount of pressure imposed by the flat jacks represents the "near field" compression ($\sigma_{flatjacks}$). Unlike the triplet test, the average pressure in the bed joints under investigation (σ_{av}) is not measurable experimentally because of the distance of the flat jacks and the perturbation of the stress field generated by the holes in masonry. Moreover, the mechanical behaviour of the portion of masonry under investigation is also influenced by the presence of the head joints at midlength of the tested brick. When the head joints are only partially mortared, the head joint may represent a discontinuity that may alter significantly the load transfer mechanism and the state of stress in the two consecutive halves of the bed joint, not allowing to establish a similarity between the shove test and the triplet test. In such case, even the execution of the shove test is questionable. However, it should be highlighted that the head joints in the masonry of the laboratory shove tests were fully mortared. Moreover, the similarity between the results of the two types of tests (see Figs. 3, 6) suggests that there are no significant differences between the two configurations. The shove tests were performed with the same loading phases of the triplet tests (see also Figures A3 and A7 in the online supplementary material, Appendix A): each time window is characterized by an increasing value of near field pressure ($\sigma_{flatjacks}$) whereas the far field stress is kept constant (σ_{ff}).

The experimental data show the occurrence of volumetric expansion in the region of masonry subjected to shear force. The dilatancy angle is maximum in time window 0 and it starts to grow before the peak value of shear stress is reached (e.g. see the online supplementary material: test n.2, Figure A3 in Appendix A). From a phenomenological viewpoint, unlike the triplet test where the shear-displacement curves show two peaks, in the shove test the failure (i.e. complete crack formation) of the two bed joints is closer in time and the shear-displacement curves are characterized by one peak. However, also in this test, the onset of sliding in the two bed joints does not take place simultaneously giving rise to the mobilization of the friction and dilatancy before and beyond the peak phase. This occurrence is particularly evident if we look at the time window 0 of the shove test n.2 (see Figure A3 in the online supplementary material, Appendix A). The onset of sliding in the bed joints and the expansion of the specimen take place before the peak shear stress is reached (i.e. v = 0.1 mm). This important experimental observation reveals that the mobilization of the friction and dilatancy starts before the peak shear stress is reached. The dilatancy angle reduces approximately to zero in the time window 1 and becomes negative in the time window 2.

The comparison between the shove test n.1 and n.2 in the time window 0 (see Figure A3 in the online supplementary material, Appendix A) is interesting because the level of compression is different but the peak value of shear stress is approximately equal. The shove test n.2 has been carried out with a lower level of compression and the mobilization of dilatancy before the peak phase is more evident. Furthermore, the maximum dilatancy angle at the peak shear stress is higher in the test n.2. The numerical simulations carried out with the proposed formulation and presented in Andreotti et al. [1] show that the higher dilatancy angle recorded in the test n.2 generates a higher value of shear strength.

In order to provide a reliable comparison between the results of this test and the triplet test, the mean pressure in the bed joints (σ_{av}) of the shove test should be defined numerically. The detailed micro-modelling approach [3] is the most suitable numerical strategy for this purpose. The theoretical model developed in this article starting from the experimental observations has been implemented in ABAQUS to perform detailed micro-modelling analyses capable to simulate the expansion of masonry samples due to dilatancy. The constitutive model used for mortar joints requires three parameters: the initial shear strength under zero compressive load (f_{vo}), friction coefficient at constant volume (μ_{cv}) and dilatancy angle (ψ). The characterization procedure of these parameters is described later in this article. Moreover, the numerical values of σ_{av} have been used to compare the results of shove test and triplet test in the τ-σ plane (see Figs. 3, 6). The maximum values of shear strength recorded in time window 0 are consistent to the results of the triplet tests.

3 Shear strength model of mortar joints accounting for dilatancy

The response of masonry structural elements subjected to shear action is usually characterized by a peak shear stress followed by the softening, until reaching a steady state. According to Stupkiewicz and Mróz [30], this type of behaviour is observed for both cohesive (e.g. [2, 8, 31]) and non-cohesive joints (e.g. [32–34]).

One of the objectives of the present article is to propose a simple friction model for the interpretation of the direct shear test of masonry samples which is based on experimental evidence and, at the same time, it considers the phenomenon of dilatancy. The aim is to extend the standard model based on Coulomb's law of the EN 1052-3 and ASTM C1531 in order to include the dilatancy angle. According to the friction model used for masonry (e.g. EN 1052-3, EC6), the two main parameters are the initial shear strength in absence of compression (f_{vo}) and friction (μ). To better understand the physical meaning of friction in absence and in presence of the dilatancy mechanism, let us first consider $f_{vo} = 0$ (e.g. complete crack formation in mortar joints) and the presence of only one mortar joint. According to Patton [17] and Stupkiewicz and Mróz [30], the cracking surface in one mortar joint may be seen as a composition of asperities with different size (Fig. 4). Primary asperities are the largest ones and define the dilatancy angle (ψ) while secondary asperities, which act at a smaller scale, characterize the friction angle at constant volume (ϕ_{cv}). When the cracking surface is relatively flat (e.g. debonding at the unit-mortar interface), the shear displacement is not accompanied by significant expansion ($\psi = 0$). As shown in Fig. 4, friction is governed by the friction angle at constant volume (ϕ_{cv}) and level of compression. On the other hand, when the cracking surface is not flat (e.g. crack passing through the mortar joint), the shear sliding generates expansion. At the microscopic level, the dilatancy angle (ψ) provides a quantitative indication on the complex geometry of the primary asperities in the cracking surface (Fig. 4).

Let us now consider the initial shear strength in absence of compression (f_{vo}). This parameter characterizes the complex bonding within the mortar and between the unit-mortar interfaces (e.g. [1, 10, 12]). According to the friction model, the contribution of f_{vo} to the overall shear strength is valid up to the complete crack formation, in other words when f_{vo} becomes equal to zero. However, the softening of this parameter begins at the onset of cracking and it continues with the propagation of the cracking surface (e.g. [1]). Unlike the friction coefficient, f_{vo} by definition (i.e. EN 1052-3 and EC6) is independent of the compression level (σ_{av}). If now we neglect the contribution of friction (e.g. $\sigma = 0$) to better highlight the features of f_{vo}, the shear strength shows a peak immediately

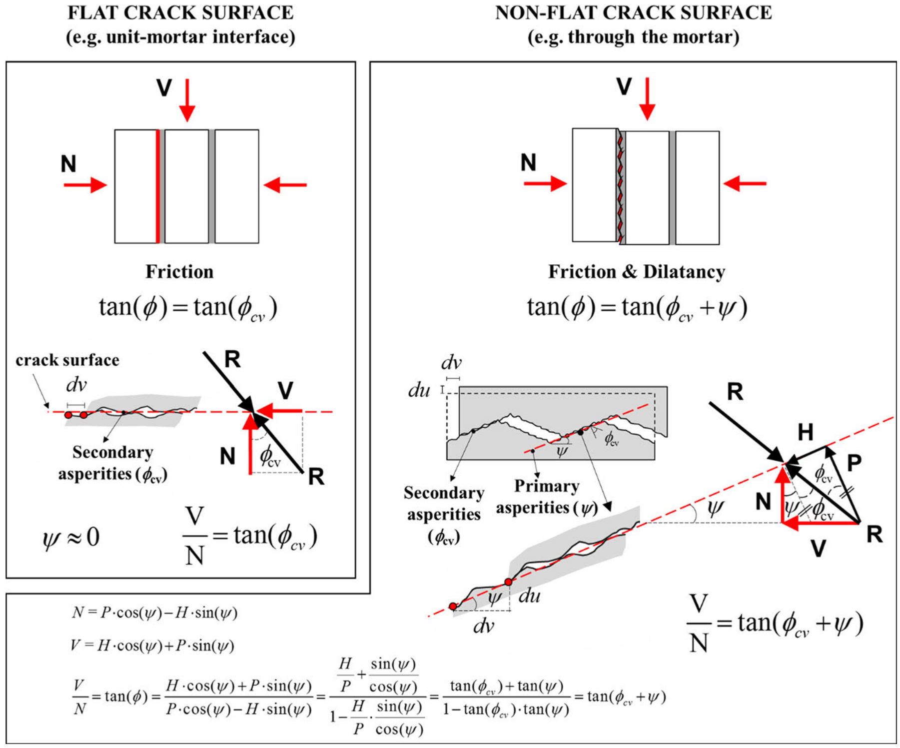

Fig. 4 Conceptual model used to describe the shear failure mechanism of one mortar joint accounting for the expansion during shearing. dv_{pl} and du_{pl} are plastic displacements in incremental terms

before the onset of cracking. The formation and propagation of the cracking surface reduce the shear strength (i.e. softening) until the complete formation of the crack ($f_{vo} = 0$).

The direct shear tests executed on masonry samples and interpreted according to the standard friction model (e.g. [2, 3, 8, 10]) show that the shear strength of mortar joints increases with the level of compression. According to the friction model based on Coulomb's law, this experimental evidence can be explained with the mobilization of the friction in the definition of the shear strength (i.e. peak phase). According to the friction model, this experimental evidence may be physically explained with the local mobilization of friction during the crack propagation. In other terms, when $\sigma \neq 0$, the softening of the shear strength does not go hand in hand with the softening of f_{vo}, which starts at the onset of cracking, because the reduction of f_{vo} is compensated by the presence of friction, depending on the compression level. However, as dilatancy is strictly connected to friction and the features of the cracking surfaces, the shear strength is also influenced by this mechanism because the work generated by the expansion is of opposite sign to the work done by the compression load. Before sliding, the test unit must overcome the resistance mechanism offered by the dilatancy. As already pointed out in other research works on masonry and concrete (e.g. [1, 15, 24, 35]), the experimental results discussed in the present article show that also dilatancy play a non-negligible role in the definition of the peak the shear stress (i.e. peak phase).

It is worth remembering that the response of masonry samples in the triplet test and shove test is generally more complex than that previously described because this setup is characterized by two bed joints whose failure commonly takes place in sequence. In particular, the complete mobilization of friction and dilatancy may occur in one bed joint before the complete formation of the crack in the other joint. From a phenomenological viewpoint, the experimental data discussed in the present article clearly show that the mechanisms of friction and dilatancy mobilize before the peak phase of shear stress (see Fig. 2 for triplet test and shove test n. 2 in the online supplementary material: Figure A3, Appendix A). The mechanism of dilatancy at and beyond the peak shear stress has been reported for the couplet test and concrete also by other researchers (e.g. [10, 15, 24]). A conceptualization of the proposed model is also reported in the online supplementary material (see Figure A4 in Appendix A). The analytical formulation proposed in this article is the result of the adaptation for masonry of a model that is used in the field of geotechnical engineering and rock mechanics (e.g. [17, 19, 36]).

The friction angle (ϕ) of mortar joints, which defines the friction coefficient $\mu = \tan(\phi)$, is defined by two components (see Fig. 4):

$$\phi = \phi_{cv} + \psi \tag{2}$$

where ϕ_{cv} is the friction angle at constant volume. It depends exclusively on the material without generating volume changes of the sample. ψ is the dilatancy angle that can be computed with Eq. (1). It governs the expansion of the specimens because it is a measure of the primary asperities in the cracking surface of mortar joints (see Fig. 4). Equation (2) can be rewritten in terms of coefficient of friction as follows:

$$\mu = \tan(\phi_{cv} + \psi) = \frac{\mu_{cv} + \mu_{\psi}}{1 - \mu_{cv} \cdot \mu_{\psi}} \tag{3}$$

where $\mu_{cv} = \tan(\phi_{cv})$ is the friction coefficient at constant volume and $\mu_{\psi} = \tan(\phi_{\psi})$ is the variable amount of friction coefficient due to the dilatancy. It is important to underline that dilatancy angle (ψ) varies depending on the level of compression and shear displacement. On the other hand, ϕ_{cv} is relatively constant because it depends exclusively on the roughness of the sliding surface.

Coulomb's law is used for the characterization of the shear strength of mortar joints (EN 1052-3) and for the shear strength of unreinforced masonry (Eurocode 6). The introduction of Eqs. (2) and (3) in Coulomb's law couples the shear strength and the dilatancy angle in the same equation:

$$f_v = f_{vo} + \sigma \cdot \tan(\phi_{cv} + \psi) \tag{4}$$

$$f_v = f_{vo} + \sigma \cdot \left(\frac{\mu_{cv} + \mu_{\psi}}{1 - \mu_{cv} \cdot \mu_{\psi}} \right) \tag{5}$$

When the dilatancy angle is zero, Coulomb's law assumes the standard form. From a theoretical point of view, it is important to underline that, when the boundary conditions partially constraints the expansion (e.g. shove test), the tendency of masonry to expand generates a local increase of normal compression (σ) (see also [1, 3, 12]). The effect of the dilatancy mechanism on shear stress is therefore twofold. On the one hand, the dilatancy angle directly increases the shear strength. On the other hand, depending on the boundary conditions, the local increase of the normal compression generates a secondary mechanism that further increases the shear strength.

A direct comparison between experimental data and analytical values of shear strength computed with Eqs. (4) or (5) is reported in Table 1. The analytical model gives predictions consistent with the experimental data. It is important to underline that the theoretical model described in this article has also been validated numerically in the research work described in Andreotti et al. [1] and Graziotti et al. [27].

Figure 4 reports the results of the triplet tests executed on three different specimens. The time windows with the same level of compression are reported as solid lines. It is important to note that, consistently with the proposed formulation, when the dilatancy angle is close to zero (i.e. constant volume phase) the shear strength of different samples converges to the same residual value (τ_{cv}). At this stage, the residual shear strength is governed essentially by the friction angle at constant volume. As shown in Fig. 4, this parameter can be computed with Eq. (4) by using the values of compression and shear strength at constant volume (τ_{cv}). Focusing on specimen B and C, it can be noted that the constant volume condition has been reached in time window 0, just after the decohesion of both joints (i.e. beyond the second peak).

It is important to underline that, if the samples show expansion during the test, the initial shear strength (f_{vo}) and friction defined according to the EN 1052-3 may be biased by the presence of dilatancy. For this reason, a modification of the standard procedure for the characterization of friction and initial shear strength of mortar joints is proposed. The novel approach, which allows the characterization of the dilatancy angle, may be used as a reference for the calibration of micromechanical models that require the definition of this parameter.

4 Interpretation of direct shear tests in case of expansion of the specimens: issues related to EN 1052-3

EN 1052-3 gives the possibility to define the shear strength of mortar joints by referring to the peak values

Table 1 Comparison between experimental shear strength and analytical values defined with the proposed model

Specimen (triplet test)	Time window	Experimental						Analytical
		x (mm)	$f_{v0,av}$* (MPa)	σ_{av} (MPa)	ψ (°)	ϕ_{cv}** (°)	τ (MPa)	$\tau = f_v = f_{v0,av} + \sigma \cdot \tan(\phi_{cv} + \psi)$ (MPa)
A (Fig. 2, 5)	0 (I cracking)	1.2	0.12	0.25	16.1	28.2	0.34	0.36
	1 (residual)	4.1	0.0	0.56	6.2	28.2	0.37	0.38
	2 (residual)	7.4	0.0	0.95	1.1	28.2	0.51	0.53
B (Fig. 5 and A1 in the online supplementary material, Appendix A)	0 (I cracking)	2.1	0.12	0.52	3.0	28.2	0.40	0.43
	1 (residual)	4.7	0.0	0.22	8.5	28.2	0.14	0.16
	2 (residual)	6.3	0.0	0.56	1.1	28.2	0.32	0.31
C (Fig. 5)	0 (I cracking)	4.6	0.12	0.48	3.5	32.0	0.41	0.46
	0 (residual)	7.7	0.0	0.48	0.9	32.0	0.30	0.31

*$f_{vo,av}$: average initial shear strength under zero compressive load as defined in Fig. 6

**ϕ_{cv}: friction angle at constant volume computed with the value of τ_{cv} defined in Fig. 5

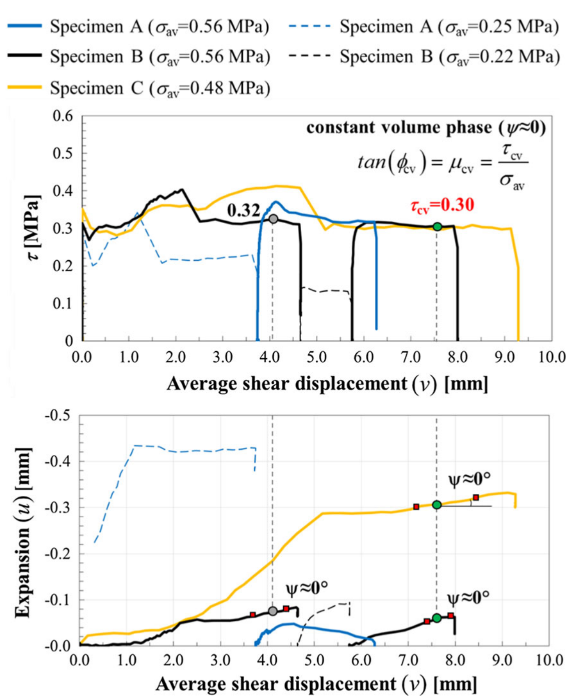

Fig. 5 Identification of the constant volume phase ($\psi \approx 0$) for the definition of residual shear strength (τ_{cv}) and friction angle at constant volume (ϕ_{cv})

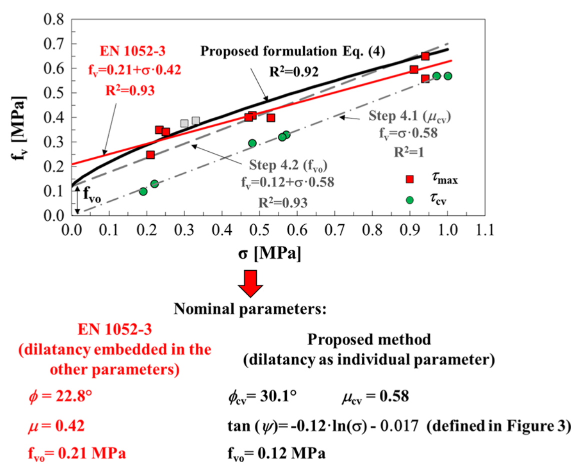

Fig. 6 Model parameters defined according to the proposed method and comparison with theoretical formulation and the European Standard EN 1052-3. The colours of the data points follow the framework of Figure A4 in online supplementary material, Appendix A. The data points related to the shove tests are shown in grey

of shear stress (τ_{max}) at different levels of compression (σ_{av}). Implications of the dilatancy angle on the definition of the shear strength are neglected. However, Eq. (4) and Fig. 3 show that the influence of the dilatancy angle on τ_{max} depends on the level of compression. The dilatancy angle tends to increase the shear strength more at low σ rather than at mid-high values of compression. Furthermore, when σ is low the variability of dilatancy angle tends to be higher.

When volumetric expansion of the specimens has been recorded during the direct shear tests, the definition of the shear strength by using the peak values (τ_{max}) has important consequences. Since the τ_{max} values at low normal stress are more influenced by the dilatancy angle than those at high values of σ, the parameters of Coulomb's law defined by means of linear regression may be biased. In particular, the friction tends to be underestimated and the initial shear strength overestimated because they have embedded the effect of dilatancy (see Figs. 4, 6). However, from a theoretical viewpoint, the overestimation of the initial shear strength under zero compressive stress is not consistent with Eq. (4) because, when $\sigma = 0$, the dilatancy angle has no effects on the shear strength. At the same time, the friction coefficient (i.e. $\mu = 0.42$ in Fig. 6) loses its physical meaning because it becomes lower than the residual friction (i.e. friction coefficient at constant volume $\mu_{cv} = 0.58$ in Fig. 6). In addition, this trend may be further emphasized by the reduction of friction at high values of compression due to the damaging of the sliding surfaces.

The two parameters defined with the standard Coulomb's law of the EN 1052-3 seem not suitable for the detailed micro-modeling of masonry because the constitutive models of mortar joints require the definition of three independent parameters: f_{vo}, ϕ_{cv} and ψ. The proposed procedure allows the definition of the parameters independently, without generating bias due to dilatancy. Moreover, it offers the possibility to characterize the dilatancy angle with experimental data (see Fig. 3).

4.1 Proposed methodology for the experimental characterization of the initial shear strength, friction and dilatancy

Instead of using τ_{max}, the friction angle at constant volume (ϕ_{cv}) can be defined by fitting the steady state values of strength (τ_{cv}). Since only the friction mechanism is active at the constant volume phase, the value of friction can be defined with improved accuracy. Once the friction at constant volume has been defined, the intercept (f_{vo}) of the linear relation expressed by Eq. (4) can be defined by means of linear regression. It is worth noting that in the constant volume condition the dilatancy is not active ($\psi = 0$) therefore Eq. (4) coincides with the standard Coulomb's law. Generally, when the dilatancy angle is approximately zero, the peak strength is equal to the strength at constant volume and the proposed procedure converges to the method of EN 1052-3. The proposed methodology is described in the following steps:

(1) Execution of experimental tests.
Experimental tests should be carried out according to the EN 1052-3 or ASTM C1531 (e.g. setup, loading conditions). Vertical (u) and horizontal (v) relative displacements must be recorded by selecting an adequate sampling frequency (despite the test is static, a sampling rate of at least 50 Hz is recommended to record sudden changes coming from the brittle nature of phenomena involved). The experimental tests on each sample must be organized in several time windows. Each time window should be characterized by a constant value of σ. According to the EN 1052-3, at least three samples should be subjected to three different levels of compression, which, for brick elements with compressive strength higher than 10 MPa are about: 0.2, 0.6 and 1 MPa. In order to optimize the fitting of the data, the results of the different time windows should be equally distributed in the range of σ. Each time window should be characterized by a complete unloading and reloading. The loading and reloading phases should continue until the abrupt increase of shear displacement is detected in both bed joints. As a qualitative indication, the unloading should start once the steady state value of shear stress has been reached.

(2) Post-processing of the acquired data and collection of information.
The recorded data should be organized as shown in Fig. 2. One value of τ_{max} (i.e. Time window 0) and τ_{cv} (i.e. residual value), with the corresponding value of σ, should be collected from each sample (see also Figure A4 in the online supplementary material, Appendix A).

(3) Computation of the dilatancy angle (ψ) at the peak value of shear strength (τ_{max}).
The dilatancy angle that contributes to the definition of the peak shear strength is that recorded in Time window 0, when the test starts with the intact specimen. The dilatancy angle can be computed with Eq. (1). When only one joint slides $n = 1$ and the increment of plastic shear displacement (dv_{pl}) is computed excluding the displacement of the non-sliding joint. When

two joints slide, $n = 0.5$ and dv_{pl} is the mean value of the displacements measured by all the LVDTs parallel to the shear force, minus the elastic component. The empirical relation between the dilatancy angle at the peak value of shear strength (τ_{max}) and σ_{av} can be defined as shown in Fig. 3.

(4) Fit data points with Coulomb's law for the definition of mechanical parameters.

(4.1) Define friction angle at constant volume (ϕ_{cv}) with Eq. (4) by fitting data points σ–τ_{cv} (see Fig. 6). At this stage, $\psi = 0$ in Eq. (4) because in the constant volume phase the dilatancy mechanism is not active. μ_{cv} identifies the friction coefficient of mortar joints more accurately because it depends exclusively on the material and not by the phenomenology of the cracking surface (i.e. dilatancy).

(4.2) Once ϕ_{cv} has been defined, the initial shear strength under zero compression (f_{vo}) can be found with the same equation ($\psi = 0$) by fitting pairs of data points σ–τ_{max} with the linear regression (see Fig. 6).

(5) Definition of the final relation of shear strength and comparison with experimental data.
Once f_{vo} and ϕ_{cv} have been characterized and the empirical relation between dilatancy angle (ψ) and compression (σ) has been defined (see Fig. 3), Eqs. (4) or (5) can be used to define the shear strength of mortar joints.

5 Interpretation of the triplet test with the proposed method and comparison with EN 1052-3

The proposed methodology has been used to interpret the data of the triplet tests discussed in this article. The methodology has not been applied to the shove test because of the limited number of tests. However, the results of the shove tests have been reported in the same plot of the triplet tests for comparison (Figs. 3, 6). Figure 6 shows the comparison between the sets of parameters defined according to the European Standard EN 1052-3 and the proposed procedure.

The novel methodology brings to the definition of the friction with less uncertainty. The nominal values of friction defined according to EN 1052-3 is lower (38%) than the constant volume friction defined with the proposed method. As expected, the initial shear strength of the EN 1052-3 is higher (43%) because the dilatancy is not considered as an independent parameter but it is embedded in the other parameters.

The difference of f_{vo} (i.e. 0.09 MPa) represents the amount of initial shear strength caused by the higher influence of the dilatancy at low compression. In addition, this figure illustrates the curve defined with the proposed formulation, which is essentially expressed by Eq. (4). The dilatancy angle at peak shear strength has been computed with the empirical relation defined in Fig. 3.

It can be noted that under zero compressive stress the dilatancy has no effect on the shear strength. On the other hand, at mid-low values of σ, the dilatancy angle increases the shear strength. Finally, at high values of compressive stress, the dilatancy angle is close to zero and the shear strength is not influenced by the expansion.

6 Final discussion

Standard methodologies for the characterization of shear strength of bed joints in masonry (EN 1052-3 and ASTM C1531) adopt the friction model based on Coulomb's law. Although the expansion of masonry specimens in direct shear tests has been reported by several researchers, the standard procedure for the interpretation of the experimental data does not consider explicitly the phenomenon of dilatancy. This approach has two important consequences: (1) disregard the phenomena induced by dilatancy may bias the interpretation of the results, (2) the constitutive models used in the micro-modelling of masonry generally require the definition of dilatancy but, given the lack of characterization procedures, this parameter is generally arbitrarily defined and sometimes divergent values are assumed (e.g. [25, 26]).

The present study reports the results of an ongoing research on brick masonry aimed to clarify the role of the expansion of mortar joints in the interpretation of direct shear tests. The main objective is to investigate the dilatancy experimentally and propose a simple theoretical model for a sound interpretation of the direct shear test of masonry samples that takes this phenomenon into account. Several masonry samples composed of calcium silicate units and 10-mm-thick cement mortar joints have been subjected to triplet tests and laboratory shove tests. This type of masonry was selected as a benchmark because it highlighted a significant dilatancy in past tests.

This article first proposes a repeatable and objective methodology to measure and characterize dilatancy through the direct shear test. Then, a relation that extends the standard friction model used for the definition of shear strength of mortar joints (i.e. EN 1052-3 and ASTM C1531) has been proposed. The methodology presented in this article has been verified and validated numerically in Andreotti et al. [1] and contributed to the proposal for an improved procedure and interpretation of ASTM C1531 for the in situ determination of brick-masonry shear strength [27]. A conceptual model to better understand the mechanical behaviour of mortar joints and the features of the proposed formulation is also provided. The reliability of the method has been evaluated by comparing the analytical results with experimental data. Finally, some issues related to the fact that the standard methodologies of EN 1052-3 and ASTM C1531 do not consider the dilatancy have been highlighted.

7 Conclusions

The experimental data discussed in the article showed that the masonry samples tested in this study are characterized by a significant amount of dilatancy when subjected to direct shear tests. In presence of significant expansion, the interpretation of the results with standard approaches (i.e. EN 1052-3 and ASTM C1531) that disregard the phenomenon of dilatancy led to the definition of mechanical parameters without a physical basis since the initial friction coefficient was found to be less than the residual friction (i.e. dry friction). Due to the presence of dilatancy, the characterization of mechanical parameters with the standard approach has generated a significant underestimation (43%) of the initial shear strength (f_{vo}) and overestimation of the initial friction coefficient (38%).

The extension to the standard procedure proposed in this article allows to take dilatancy into account, bringing to the definition of unbiased mechanical parameters. The main parameter used to quantify the phenomenon of dilatancy is the dilatancy angle. When dilatancy angle is zero, the proposed procedure converges to the standard model based on Coulomb's law. The results of the proposed analytical formulation have been found consistent with experimental data.

The novel approach offers also the possibility to characterize experimentally the dilatancy angle (ψ), allowing to directly use this data as input in the constitutive models commonly adopted for mortar joints in the micro-modelling of masonry (e.g. [1]).

According to other studies (e.g. [3, 10]), dilatancy angle has been found to decrease with the increase of compression and shear displacement. The magnitude of this phenomenon seems to depend on the type of failure mechanism activated in the sample because dilatancy angle tends to be higher when the cracking surface develops through the mortar rather than at the unit-mortar interfaces.

The experimental data presented in this article show that dilatancy angle influences the shear strength. In agreement with other research studies carried out on mortar joints and concrete (e.g. [10, 15, 24, 35]) the study clearly shows that the mechanisms of friction and dilatancy start to be mobilized before reaching the peak shear stress. This behaviour translates into the fact that both friction and dilatancy significantly contribute to the definition of the shear strength, which increases with dilatancy angle. A physical explanation is that the work generated by the expansion is of opposite sign to the work done by the compression load.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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