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Generation of Photons from Vacuum in a Cavity with Time-Dependent Eigenfrequency and Damping Coefficient

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Abstract. This is a brief review of recent achievements in the theory of non-stationary Casimir effect in non-ideal cavities and of existing proposals to observe this effect in a laboratory. In this connection, a model of quantum damped oscillator with arbitrary time-dependent frequency and damping coefficient is developed and the influence of different parameters on the photon generation rate is analyzed.

Keywords: non-stationary Casimir effect, quantum damped oscillator, time-dependent frequency, time-dependent damping
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1. Introduction

We consider the problem of photon generation from vacuum in a selected mode of electromagnetic field of a closed high-Q cavity due to periodical variations of the conductivity of a thin semiconductor layer deposited on the plane surface of a cavity wall. Fast and significant changes of electric properties can be achieved in semiconductors illuminated by laser pulses. The suggestion to use this scheme to simulate the so-called Unruh effect was put forward by Yablonovitch [1]. Man'ko [2] proposed to use semiconductors with time-dependent properties to produce the analogue of the *non-stationary Casimir effect* (see also [3, 4]). A more developed scheme, based on the creation of an electron–hole 'plasma mirror' inside a semiconductor slab, illuminated by a femtosecond laser pulse, was proposed in [5]. The idea of experiment on observation of the non-stationary Casimir effect, which is under

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preparation in the university of Padua [6], is to use an effective electron–hole 'plasma mirror' created periodically on the surface of a semiconductor slab by illuminating it with a sequence of short laser pulses. If the interval between pulses exceeds the recombination time of carriers in the semiconductor, a highly conducting layer will periodically appear and disappear on the surface of the semiconductor film, thus simulating periodical displacements of the boundary. Quite recently, some other schemes were also proposed [7, 8], but we consider here only the setup of [6].

Quantum effects caused by the time dependence of properties of thin slabs were studied by several authors [9–13]. However, only very simple models of the media were considered in that papers: ideal dielectrics or ideal conductors, suddenly removed from the cavity [9, 10], infinitely thin conducting slabs, modeled by δ-potentials with time-dependent strength [11], or lossless homogeneous dielectrics with time-dependent permeability [12, 13]. Our goal is to give estimations of the number of photons which could be produced inside the cavity with a semiconductor time-dependent 'mirror', *taking into account the internal dissipation in the slab.*

2. Photon Generation in the Presence of Losses

An immediate consequence of the time variation of electromagnetic properties of the cavity walls is the time dependence of the eigenmode frequencies. Hence it follows the simple idea that one could understand the main features of the behavior of the quantum field in the cavity by considering a single selected mode, describing it as a quantum oscillator with 'instantaneous' time-dependent frequency [14,15]. Later on, it was justified (see, e.g. [16–18]) for three-dimensional cavities without an accidental degeneracy of the eigenmode frequency spectrum and for harmonical variations of the effective frequency (for numerical verifications of the accuracy of analytical approximations see [19]). We *assume* that even in the presence of dissipation and non-monochromatic periodical variations, the field problem still can be reduced approximately to the dynamics of the *single selected mode*, described in the classical limit as a harmonic oscillator with time-dependent *complex* frequency $\Omega(t) = \omega(t) - i\gamma(t)$, which can be found from the solution of the classical electrodynamical problem with the *instantaneous* geometry and material properties.

We use the model developed in [20–22]. It consists in the description of the dissipative quantum systems within the framework of the Heisenberg–Langevin operator equations. In the case concerned these equations can be written as

$$d\hat{x}/dt = \hat{p} - \gamma(t)\hat{x} + \hat{F}_x(t)\,, \qquad d\hat{p}/dt = -\gamma(t)\hat{p} - \omega^2(t)\hat{x} + \hat{F}_p(t)\,, \tag{1}$$

$$\langle \hat{F}_j(t)\hat{F}_k(t')\rangle = \delta(t-t')\chi_{jk}(t)\,, \qquad j,k = x,p\,, \tag{2}$$

$$\chi_{xp}(t) = -\chi_{px}(t) = i\gamma(t)\,, \qquad \chi_{pp}(t) = \chi_{xx}(t) = \gamma(t)G\,, \qquad G = 1 + 2\langle n\rangle_{th}\,. \tag{3}$$

Here $\hat{x}$ and $\hat{p}$ are dimensionless quadrature operators of the selected mode, normalized in such a way that the mean number of photons equals $\mathcal{N} = \frac{1}{2}\langle \hat{p}^2 + \hat{x}^2 - 1\rangle$ (in other words, in the subsequent formulas ω and γ are the frequency and damping coefficient normalized by the initial frequency ω_i); $\langle n\rangle_{th}$ is the equilibrium mean number of photons for the selected mode at the given temperature. The noise operators $\hat{F}_x(t)$ and $\hat{F}_p(t)$ do not commute between themselves, but they commute with $\hat{x}$ and $\hat{p}$. The choice of coefficients in (1) and (3) was justified in [20].

The solution of the system of equations (1) under the condition (2) can be expressed in terms of the function $\varepsilon(t)$, which satisfies the *classical equation of motion* of the harmonic oscillator with time-dependent frequency

$$\ddot{\varepsilon} + \omega^2(t)\,\varepsilon = 0 \tag{4}$$

and the initial condition $\varepsilon(t) = \exp(-it)$ for $t \to -\infty$. Note that $\varepsilon(t)$ does not depend on the damping coefficient $\gamma(t)$. If initially (at $t \to -\infty$) the field mode was in the thermal state, then the mean number of photons at the instant t equals [20]

$$\mathcal{N}(t) = Ge^{-2\Gamma(t)}\left\{\frac{1}{2}E_t + \int_{-\infty}^{t}\gamma(\tau)d\tau e^{2\Gamma(\tau)}\left(E_tE_\tau - \mathrm{Re}\left[\tilde{E}_t^*\tilde{E}_\tau\right]\right)\right\} - \frac{1}{2}, \tag{5}$$

$$\Gamma(t) = \int_{-\infty}^{t}\gamma(\tau)d\tau\,, \quad E_\tau = \frac{1}{2}\left[|\varepsilon(\tau)|^2 + |\dot{\varepsilon}(\tau)|^2\right], \quad \tilde{E}_\tau = \frac{1}{2}\left[\varepsilon^2(\tau) + \dot{\varepsilon}^2(\tau)\right].$$

Formula (5) is *exact* for arbitrary functions $\omega(t)$ and $\gamma(t)$. However, we are interested here in the special case when the functions $\omega(t)$ and $\gamma(t)$ have the form of *periodical* pulses, separated by intervals of time with $\omega = 1$ and $\gamma = 0$ (we neglect the damping of the field between pulses, supposing that the quality factor of the cavity is high enough). Moreover, the relative change of the frequency $\omega(t)$ during pulses is very small: $\omega(t) = \omega_0[1 + \chi(t)]$ with $|\chi| \ll 1$. Under these conditions, the integral in (5) was calculated in [20–22]. The maximal number of photons which can be created after $n \gg 1$ pulses equals

$$\mathcal{N}_n \approx \frac{G\nu}{4(\nu - \Lambda)}\exp[2n(\nu - \Lambda)] + \frac{G-1}{2}\,, \tag{6}$$

where

$$\Lambda = \int_{t_i}^{t_f}\gamma(\tau)d\tau\,, \qquad \nu = \left|\int_{t_i}^{t_f}\omega_0\chi(t)e^{-2i\omega_0 t}dt\right|, \tag{7}$$

(t_i and t_f are the initial and final time moments of any pulse). Formula (6) holds, if the ratio $\nu/(\nu - \Lambda)$ is not too big and if the periodicity of pulses T satisfies the *resonance condition* $T = T_{\mathrm{res}} = \frac{1}{2}T_0\,(m - \varphi/\pi)$, where T_0 is the period of oscillations in the selected field mode and $\varphi = \omega_0\int_{t_i}^{t_f}\chi(t)dt$. Under realistic conditions, the parameters ν and Λ are very small.

3. Complex Frequency Shift of the Cavity Mode

We consider a cylindrical cavity with an arbitrary cross section and the axis parallel to the x direction, supposing that the main part of the cavity is empty, except for a thin slab of a semiconductor material. Thus we write the dielectric function $\epsilon(x)$ as $\epsilon(x) \equiv 1$ for $-L < x < 0$ and $\epsilon(x) \neq 1$ for $0 < x < D$, where D is the thickness of the slab and L is the cavity length. We assume that $D \ll L$ and the dielectric permeability depends only on the longitudinal space variable x.

For the TE modes, Maxwell's equations give rise to the usual three-dimensional Helmholtz equation $\Delta\mathbf{E} + (\Omega/c)^2\epsilon(x)\mathbf{E} = 0$ for the monochromatic component of the electric vector $\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})\exp(-i\Omega t)$. In this case one can factorize any scalar component of the electric field as $E(x,\mathbf{r}_\perp) = \psi(x)\Phi(\mathbf{r}_\perp)$, where the function $\Phi(\mathbf{r}_\perp)$ obeys the two-dimensional Helmholtz equation $\Delta_\perp\Phi + k_\perp^2\Phi = 0$. Consequently, the problem is reduced to solving the one-dimensional Helmholtz equation $\psi'' + \left[(\Omega/c)^2\epsilon(x) - k_\perp^2\right]\psi = 0$. Its solution in the domain $-L < x < 0$, satisfying the boundary condition $\psi(-L) = 0$, is $\psi(x) = F_1\sin[k(x+L)]$, where the constant coefficient k is related to the field eigenfrequency Ω and the corresponding wavelength in vacuum λ as $\Omega = c(k^2 + k_\perp^2)^{1/2}$, $\lambda = 2\pi(k^2 + k_\perp^2)^{-1/2}$. The conditions of continuity of the function $\psi(x)$ and its derivative at $x = 0$ result in the transcendental equation for the wave number k of the form $\tan(kL) = k\psi_+(0;k)/\psi_+'(0;k)$, where $\psi_+(x;k)$ is the solution of the Helmholtz equation in the domain $0 < x < D$, satisfying the boundary condition $\psi_+(D) = 0$. In the case of thin slab with $D \ll \lambda \sim L$, the value of k must be close to π/L (for the lowest mode of the cavity). Thus we can write $k = (1+\xi)\pi/L$ with $|\xi| \ll 1$ and replace $\tan(\pi\xi)$ simply by $\pi\xi$. With the same accuracy we can identify k with π/L in the right-hand side of the equation. Thus we arrive at the formula $\xi = \eta\Delta R(0)$, where the function $R(\tilde{x}) = \tilde{\psi}_+(\tilde{x})/\tilde{\psi}_+'(\tilde{x})$ of the dimensionless variable $\tilde{x} = x/D$ satisfies the boundary condition $R(1) = 0$ and the first-order nonlinear *generalized Riccati equation* for $0 < \tilde{x} < 1$,

$$\frac{dR}{d\tilde{x}} = 1 + \pi^2\Delta^2\left[\epsilon(\tilde{x}) - 1 + \eta^2\right]R^2\,, \qquad \eta = \frac{\lambda}{2L} < 1\,, \quad \Delta = \frac{2D}{\lambda} \ll 1\,. \tag{8}$$

The small relative shift of the resonance frequency can be expressed as

$$\chi_\Omega \equiv [\Omega - \omega_0]/\omega_0 = \eta^2(\xi - \xi_0) = \eta^3\Delta\left[R(0) - R_0(0)\right],$$

where ξ_0 corresponds to the non-excited semiconductor with $\epsilon(\tilde{x}) = \epsilon_1 = \text{const}$. In this case Eq. (8) has an exact solution, which shows that for $\epsilon_1 \sim 10$ (typical values for semiconductors), $R_0(0) \approx -1$ with an accuracy better than 0.01. When the semiconductor slab is illuminated by the laser pulse, the absolute value of the function $\epsilon(\tilde{x}) = \epsilon_1 + i\epsilon_2(\tilde{x})$ can attain very big values, so that $\pi^2\Delta^2|\epsilon(\tilde{x})| \gg 1$ in some region $0 < \tilde{x} < \tilde{x}_0$ near the surface of the slab. Obviously, to create an effective 'plasma mirror' one needs the material with $\tilde{x}_0 \sim (\alpha D)^{-1} \ll 1$, where α is the absorption coefficient of the laser radiation. In the region $0 < \tilde{x} < \tilde{x}_0$ we can neglect the first term 1 in the right-hand side of Eq. (8), as well as the term $1 - \eta^2$. The simplified equation can be integrated immediately, resulting the following interpolation formula for the complex frequency shift:

$$\chi_\Omega = \eta^3\Delta\frac{E}{E-1}\,, \qquad E = (\pi\Delta)^2\int_0^\infty \tilde{\epsilon}(\tilde{x})d\tilde{x}\,. \tag{9}$$

Here $\tilde{\epsilon}(\tilde{x})$ means the *change of dielectric function* caused by the laser excitation. The upper limit of integration in (9) is extended formally to infinity, because the function $\tilde{\epsilon}(\tilde{x})$ quickly goes to zero outside the interval $(0, \tilde{x}_0)$.

In Ref. [21] we supposed that the creation of carriers in the semiconductor slab influences the imaginary part $\epsilon_2(\tilde{x})$ of the dielectric function only. However, this assumption can be questioned in the case of high frequencies and a big interval between collisions τ_c. For example, the simple Drude model gives the complex mobility at frequency ω in the form $b(\omega) = e\tau_c/[m(1 + i\omega\tau_c)]$, where e is the electron charge and m the effective mass. Thus we can write

$$E = (i + \mu)A\,, \qquad A(t) = (4\pi^2\Delta|eb|/c)\int_0^\infty n(x,t)dx\,, \tag{10}$$

where $n(x,t)$ is the volume concentration of electron–hole pairs created inside the slab, $c = f_0\lambda$ is the velocity of light, b is the total mobility of carriers for each electron–hole pair (in the CGS units) and μ is the dimensionless parameter characterizing the ratio $\tilde{\epsilon}_1/\tilde{\epsilon}_2$ in the photo-excited slab. For the Drude model, $\mu = \omega\tau_c$. Then we can express the real and imaginary parts of the complex frequency shift as

$$\chi = \frac{\eta^3\Delta\left[A^2(1+\mu^2) - \mu A\right]}{A^2(1+\mu^2) - 2\mu A + 1}\,, \qquad \gamma = \frac{\eta^3\Delta A}{A^2(1+\mu^2) - 2\mu A + 1}\,. \tag{11}$$

Equation (11) shows that big values of χ can be achieved asymptotically for $A \gg 1$. If $\mu \ll 1$, then the maximal value of γ is achieved for $A = 1$, and $\gamma_{\max}$ is only twice smaller than $\chi_{\max}$, so that the influence of damping by no means can be neglected. However, the situation can be different for $\mu \gg 1$, because in this case $\gamma_{\max}$ is suppressed as μ^{-3}, and one can expect that the negative contribution of coefficient Λ to the photon generation rate (6) can be reduced in the same proportion.

4. Evaluation of the Photon Generation Rate

The dependence $n(x,t)$ can be found from equations which take into account, besides the photo-absorption, the effect of diffusion and different recombination processes. We use the simplified version of the equation introduced in [23, 24],

$$\partial n/\partial t = \nabla\cdot(Y\nabla n) + (\alpha\zeta/E_s)I(t)e^{-\alpha x} - \beta_1 n\,, \qquad Y\partial n/\partial x|_{x=0} = Rn(0)\,. \tag{12}$$

Here Y is the coefficient of ambipolar diffusion, α is the absorption coefficient of the laser radiation inside the layer, E_s is the energy gap of the semiconductor (which is close to the energy of laser photons), $I(t)$ is time-dependent intensity of the laser pulse which enters the slab, $\zeta \le 1$ is the efficiency of the photo-electron conversion, β_1 is the trap-assisted recombination coefficient and R is the surface recombination velocity. The simplification made consists in omitting the nonlinear terms in the right-hand side of (12). This assumption can be justified for materials with high concentration of impurities, which result in very small recombination times, of the order of $T_r \sim 20 \div 30\,\mathrm{ps}$. Since Eq. (12) is *linear*, it can be solved analytically [25]. The maximal rate of photon generation is achieved for a small surface recombination (as one could expect), so we consider here the case $R = 0$.

Supposing that the duration of laser pulse is much less than the recombination time, we approximate the function $I(t)$ by the delta function: $I(t) = (W/S)\delta(t)$, where W is the total energy of the laser pulse and S is the area of the surface of the semiconductor slab (we assume that the energy is distributed uniformly over this area). Then the time-dependent function $A(t)$ in equation (11) turns out to be equal to $A(\tau) = A_0 \exp(-\tau/Z)$ (we assume that $n(x,t) = 0$ for $t < 0$), where

$$\tau = \omega_0 t\,, \qquad Z = \frac{\omega_0}{\beta_1} = \frac{2\pi T_r}{T_0}\,, \qquad A_0 = \frac{4\pi^2 |eb| \zeta W \Delta}{(cE_s S)}\,. \tag{13}$$

According to (6), the rate of photon generation is determined by the difference

$$\nu - \Lambda = \eta^3 \Delta\, F(A_0, Z, \mu)\,, \qquad F(A_0, Z, \mu) = \tilde{\nu} - \tilde{\Lambda}\,. \tag{14}$$

The numerical analysis of function $F(A_0, Z, \mu)$ in the case $\mu = 0$ was done in [21]. If we fix the number of photons to be created after n pulses, then the total energy of all pulses is proportional to the quantity $\mathcal{J} = A_0 S/[\eta^3 \Delta^2 F(A_0, Z, \mu)]$. It was found that the minimum of $\mathcal{J}$ for the fixed values of η and Δ is achieved for $A_0 \approx 10$ and $Z \approx 0.3$ (this value corresponds to the recombination time $T_r = 20\,\mathrm{ps}$ for $f_0 = 2.5\,\mathrm{GHz}$, which is quite realistic from the point of view of the available technology). For the rectangular cavity with the excited lowest TE_{110} mode, the ratio S/η^3 is proportional (for the fixed resonance frequency) to the function [22] $f(\eta) = \eta^3\sqrt{1-\eta^2}$, whose minimum is achieved for $\eta_0 = \sqrt{3}/2 = 0.866$, when $f(\eta_0) = 0.325$. However, this minimum is rather flat, and even the value $\eta_1 = \sqrt{2}/2 \approx 0.7$ is still admissible, because $f(\eta_1) = 0.25$. In this case the shape of the cavity wall, which is perpendicular to the electric field vector, is close to a quadrat (but it must be slightly different from the quadrat, in order to avoid excitation of other field modes [18]). For the optimal choice of parameters A_0, Z, η, the number of photons created after n pulses is given by a simple formula $\mathcal{N}_n \approx G\exp(n\Delta/5)$.

For the semiconductor slab of thickness $2\,\mathrm{mm}$ ($\Delta = 1/30$) we need about 1400 pulses to create 10^4 photons from the initial vacuum state of field ($G = 1$). Taking a realistic value of the mobility of carriers $b \sim 3\,\mathrm{m}^2/(\mathrm{V{\cdot}s})$ and $E_s = 1.4\,\mathrm{eV}$ (as for GaAs), we find that the value $A_0 = 10$ corresponds to the energy density of each laser pulse $W/S \approx 10^{-5}\,\mathrm{J/cm}^2$ and the effective surface concentration of carriers immediately after the pulse about $5 \cdot 10^{13}\,\mathrm{cm}^{-2}$. For the illuminated area $10 \times 2\,\mathrm{cm}^2$, we obtain $W \approx 0.2\,\mathrm{mJ}$ and the total energy of all pulses $0.3\,\mathrm{J}$. The necessary number of pulses and total energy can be significantly reduced in the case of initial thermal state with nonzero temperature [22, 26], due to the factor G in formula (6). For example, taking the temperature $T = 14\,\mathrm{K}$, we have $G \approx 240$, so that the same number 10^4 photons can obtained after only 800 pulses with the total energy $0.2\,\mathrm{J}$.

In Fig. 1 we show, what can happen if the parameter μ in formula (11) is different from zero. The left-hand side plot gives the dependence $F(\mu)$ for $Z = 0.3$

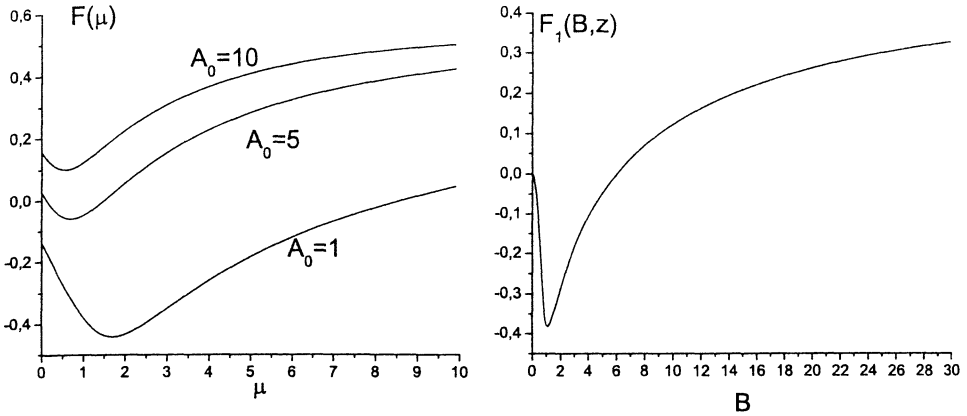

Fig. 1. Left: The amplification coefficient F versus the parameter μ for the fixed value $Z = 0.3$ and different values of parameter A_0. Right: The dependence of the amplification coefficient F on the parameter B (17) for $Z = 0.3$ and $\mu = 1$

and three fixed values of A_0. Since variations of parameter μ are accompanied by variations of mobility (which is proportional to the mean time between collisions τ_c), the coefficient A in formulas (10) and (11) can depend on μ. For $\mu = \omega\tau_c$ we have

$$\tilde{\nu} = 2BZ\mu^2 \left| \int_0^\infty dx\, e^{-2iZx} \frac{2B\exp(-2x) - \exp(-x)}{1+\mu^2+4B\mu^2\left[B\exp(-2x)-\exp(-x)\right]} \right| , \quad (15)$$

$$\tilde{\Lambda} = 2BZ\mu \int_0^\infty dx \frac{\exp(-x)}{1+\mu^2+4B\mu^2\left[B\exp(-2x)-\exp(-x)\right]} , \quad (16)$$

where the new dimensionless parameter B is defined according to the relations

$$A_0 = \frac{2\mu B}{1+\mu^2} , \qquad B = \frac{\pi\Delta\zeta e^2 W\lambda}{E_g S c^2 m_{\rm eff}} , \quad (17)$$

$m_{\rm eff}$ being the effective mass of carriers. The right-hand side plot in Fig. 1 shows the dependence of the amplification coefficient on B for $\mu = 1$, when the real part of conductivity assumes the maximal possible value in the Drude approximation. We see that reasonable values of the amplification coefficient can be obtained for $B > B_* = 10$. Taking the same values of parameters as above and $m_{\rm eff} = 0.1 m_e$, we obtain for the corresponding energy density of single pulse the value $W_*/S \sim 7 \cdot 10^{-7}\,{\rm J/cm^2}$, which is smaller than the estimation for $\mu = 0$ by more than one order of magnitude. The value $\mu = 1$ corresponds to an extremely high mobility $b_1 = e/(2m\omega) \sim 50\,{\rm m^2/V{\cdot}s}$, which, perhaps, cannot be achieved. However, this example shows that looking for semiconductor materials with specific properties one could reduce the necessary energy of laser pulses to a sufficiently low level.

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