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# Local Field Effects and Multi-Peak Spectra in Coherent Scattering of Resonant Light in a Two-Level Medium

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**Abstract.** Scattering of a laser pulse is considered in a dense medium with account for local field effects. The interacting system of two-level atoms and quantized field modes is studied in the second Born approximation of the Bogolyubov–Born–Green–Kirkwood–Yvon hierarchy of equations for reduced density and correlation operators. The local field correction is derived consistently from the conventional interaction Hamiltonian and adjusted to presence of a dielectric host. Time-dependent spectra of scattered light are calculated both theoretically and numerically. Additional sidebands are shown to occur including the cases when the distance between the peaks is proportional to the Rabi frequency.

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## 1. Introduction

For a few decades it has been well established that in a dense ensemble the radiative response of each atom to an external field may strongly depend on its neighborhood. Cooperation of like light emitters via the radiation field produces the multiatomic emission phenomena such as superradiance and superfluorescence. At the same time the actual field that drives the atoms must meet the Lorentz local-field condition, i.e. be dependent on the light-induced macroscopic polarization. These effects are known to change the pattern of resonance fluorescence and be the mechanisms for optical bistabilities [1–3]. The current interest to this topic has been triggered by observations of intrinsic optical bistability [4, 5] and variations of the spontaneous decay rate [6] both explained by the local field effects among light emitters embedded into dielectric hosts. The modern theories show that such embedding

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appreciably enhances the local field nonlinearities. Besides, in an absorptive dielectric the dopant may experience cooperative, superradiant-like decay [3]. The latter is a purely intrinsic phenomenon but may be used to generate some effects associated with conventional, propagation-dependent superradiance.

As it was mentioned, cooperative atomic behavior was expected to modify the fluorescence spectra as compared to the single-atom case. The lines emitted by a collection of atoms may exhibit additional broadening and shifts [7,8]. However, the most dramatic signature of cooperativity could be appearance of extra sidebands [7–11]. This work aims to check for the possibility of these changes due to the intrinsic local-field cooperativity of two-level atoms in an absorptive host. We suppose the visible effects to be time-dependent in a nonlinear system so the choice of the calculation technique goes beyond the conventional methods of theoretical spectroscopy [12]. The local field contribution to atomic dynamics is most commonly studied phenomenologically in the semiclassical framework which cannot solve a spectroscopic problem alone. Our analysis is based on the Bogolyubov–Born–Green–Kirkwood–Yvon (BBGKY) hierarchy of equations for reduced density operators of atoms and field modes and atom–field correlation operators where the atomic cooperative effects are naturally described by the mean field terms. Knowing the field mode density operator allows prompt and handy calculation of the emission spectra.

## 2. Master Equation

The conventional Hamiltonian for a collection of atoms interacting via the electromagnetic field is

$$H = \sum_l H_A + \sum_{\mathbf{k},s} H_F + \sum_l \sum_{\mathbf{k},s} V_{AF} \,. \tag{1}$$

The free-atom, free-field and the interaction terms here are written in the electric dipole approximation and $1/\hbar$ units:

$$H_A = \omega_{21}\sigma_z \,, \qquad H_F = \omega_k \hat{a}^\dagger_{\mathbf{k},s}\hat{a}_{\mathbf{k},s} \,, \qquad V_{AF} = -(\hat{\mathbf{E}}_{\mathbf{k},s}, \hat{\mathbf{P}}_l) \,,$$

$$\hat{\mathbf{E}}_{\mathbf{k},s} = i\lambda_k \hat{\epsilon}_{\mathbf{k},s}(e^{i\mathbf{k}\cdot\mathbf{r}}\hat{a}_{\mathbf{k},s} - e^{-i\mathbf{k}\cdot\mathbf{r}}\hat{a}^\dagger_{\mathbf{k},s}) \,, \qquad \hat{\mathbf{P}}_l = \mathbf{d}_{21}(\sigma_+ + \sigma_-)\delta(\mathbf{r} - \mathbf{r}_l) \,. \tag{2}$$

where $\sigma_z$, $\sigma_+$, $\sigma_-$ are the atomic population inversion, raising and lowering operators for a two-level atom $l$ with transition frequency $\omega_{21}$ and coordinates $\mathbf{r}_l$. For the field part $\hat{a}^\dagger_{\mathbf{k},s}, \hat{a}_{\mathbf{k},s}$ are creation and annihilation operators of a photon with frequency $\omega_k$, wave vector $\mathbf{k}$ and unit polarization vector $\hat{\epsilon}_{\mathbf{k},s}$. The interaction is expressed in terms of operator $\hat{\mathbf{E}}_{\mathbf{k},s}$ for the field at point $\mathbf{r}$ and the atomic polarization operator $\hat{\mathbf{P}}_l$ determined by the dipole moment matrix element $\mathbf{d}_{21}$. $\lambda_k = \sqrt{2\pi\omega_k/\hbar W}$ is the coupling constant in the quantization volume $W$.

The system is described by the density operator $\rho$ which evolves in time according to the von Neumann equation:

$$i\frac{d}{dt}\rho - [H, \rho] = 0 \,. \tag{3}$$

Calculation of time-dependent spectra is sensitive to the operation of a detector, so the radiation loss during photon detection is later accounted for by introducing the operator

$$\mathcal{L}_{\mathbf{k},s}\ \rho = -i\eta_k/2(\hat{a}^\dagger_{\mathbf{k},s}\hat{a}_{\mathbf{k},s}\rho - 2\hat{a}_{\mathbf{k},s}\rho\hat{a}^\dagger_{\mathbf{k},s} + \rho\hat{a}^\dagger_{\mathbf{k},s}\hat{a}_{\mathbf{k},s})\,, \tag{4}$$

where $\eta_k$ characterizes the loss rate in each mode.

If initially no interaction is observed $\rho(t=0)$ can be written as a product of single-atom $\rho_A(0)$ and single-photon $\rho_F(0)$ density operators where $A$ and $F$ stand for different atoms and modes. Assuming the atoms are then driven by a classic (laser) field the initial conditions for each $\rho_F$ could be expressed via coherent states:

$$\rho_F(0) = |\alpha_{\mathbf{k},s}\rangle\langle\alpha_{\mathbf{k},s}| = D(\alpha_{\mathbf{k},s})|0_{\mathbf{k},s}\rangle\langle 0_{\mathbf{k},s}|D^+(\alpha_{\mathbf{k},s})\,, \tag{5}$$

with $D(\alpha_{\mathbf{k},s})$ denoting the coherent state operator, $|\alpha_{\mathbf{k},s}|^2$ being the average number of photons in the mode $(\mathbf{k}, s)$, and $|0_{\mathbf{k},s}\rangle$ describing the vacuum field state.

We substitute the von Neumann equation with the BBGKY hierarchy taken in the generalized second Born approximation avoiding the thermodynamic limit for single-atom and single-photon density operators and the atom–photon correlation operator $g_{AF}$:

$$i\frac{d}{dt}\rho_A - [H_A + \sum_{\mathbf{k},s}\mathcal{U}_A, \rho_A] = \sum_{\mathbf{k},s} Tr_F[V_{AF}, g_{AF}]\,, \tag{6}$$

$$i\frac{d}{dt}\rho_F - [H_F + \sum_{\mathbf{k},s}\mathcal{U}_F, \rho_F] - \mathcal{L}_{\mathbf{k},s}\ \rho_F = \sum_{l} Tr_A[V_{AF}, g_{AF}]\,, \tag{7}$$

$$i\frac{d}{dt}g_{AF} - [H_A + H_F + \sum_{\mathbf{k},s}\mathcal{U}_A + \sum_{l}\mathcal{U}_F, g_{AF}] - \mathcal{L}_{\mathbf{k},s}\ g_{AF} = [V_{AF} - \mathcal{U}_A - \mathcal{U}_F, \rho_A\rho_F]\,. \tag{8}$$

The summations in the left-hand side of the set along with the compensating terms in the right-hand side of Eq. (8) describe contributions of the mean fields. The mean field acting on an atom is by definition found from

$$\mathcal{U}_A = Tr_F V_{AF}\rho_F = -(\langle\hat{\mathbf{E}}_{\mathbf{k},s}\rangle, \hat{\mathbf{P}}_l)\,, \tag{9}$$

where

$$\langle\hat{\mathbf{E}}_{\mathbf{k},s}\rangle = Tr_F\hat{\mathbf{E}}_{\mathbf{k},s}\rho_F = i\lambda_k\hat{\epsilon}_{\mathbf{k},s}(e^{i\mathbf{k}\cdot\mathbf{r}}\beta_{\mathbf{k},s} - e^{-i\mathbf{k}\cdot\mathbf{r}}\beta^*_{\mathbf{k},s})\,. \tag{10}$$

The amplitude function $\beta_{\mathbf{k},s}$ is actually the eigenvalue of the annihilation operator and obeys the equation obtained straightforwardly from Eq. (7):

$$\frac{d}{dt}\beta_{\mathbf{k},s} = \frac{d}{dt}Tr_F\hat{a}_{\mathbf{k},s}\rho_F = -i(\omega_k - i\eta_k/2)\beta_{\mathbf{k},s} - \sum_m \lambda_k e^{-i\mathbf{k}\cdot\mathbf{r}_m}(\hat{\epsilon}_{\mathbf{k},s}, \langle\hat{\mathbf{P}}_m\rangle)\,. \tag{11}$$

Thus, one can see that $\langle\hat{\mathbf{E}}_{\mathbf{k},s}\rangle$ collects the external field perturbations as well as the fields created by the atoms of the ensemble. At the same time $\mathcal{U}_F$ describes the feedback of the atomic polarization to the field mode:

$$\mathcal{U}_F = Tr_A V_{AF}\rho_A = -(\hat{\mathbf{E}}_{\mathbf{k},s}, \langle\hat{\mathbf{P}}_l\rangle)\,, \qquad \langle\hat{\mathbf{P}}_l\rangle = Tr_A\hat{\mathbf{P}}_l\rho_A\delta(\mathbf{r} - \mathbf{r}_l)\,, \tag{12}$$

and $\exp(-i\mathcal{U}_F)$ is indeed a coherent state operator. Thus, according to [13] we can immediately write the unitary operator describing the solution for the homogenous Eq. (7):

$$U(t) = \exp(-i\sum_m \mathcal{U}_F)(t) = \exp(-i\phi)D(\beta_{\mathbf{k},s}), \tag{13}$$

The phase function here is found from $d\phi/dt = \mathrm{Im}(\beta^*_{\mathbf{k},s} d\beta_{\mathbf{k},s}/dt)$. From Eq. (5) it follows that $\beta_{\mathbf{k},s}(0) = \alpha_{\mathbf{k},s}$. Performing the unitary transformation $\rho = U\tilde{\rho}U^{-1}$ and neglecting the effects of multiple scattering (inhomogeneity in Eq. (7)) one can find that for the chosen initial conditions this picture yields $\tilde{\rho}_F = |0_{\mathbf{k},s}\rangle\langle 0_{\mathbf{k},s}|$. We can now build the formal solution for $\tilde{g}_{AF}$ and substitute it into Eq. (6). Having performed the integration over time and field modes we finally obtain [12]:

$$i\frac{d}{dt}\rho_A - \Big[H_A + \sum_{\mathbf{k},s}\mathcal{U}_A, \rho_A\Big] = \Gamma\rho_A - \Big[\sum_{\mathbf{k},s}\mathcal{U}^R_A, \rho_A\Big], \tag{14}$$

$$\mathcal{U}^R_A = -(\langle\hat{\mathbf{E}}^R_{\mathbf{k},s}\rangle, \hat{\mathbf{P}}_l), \qquad \langle\hat{\mathbf{E}}^R_{\mathbf{k},s}\rangle = i\int_0^t dt'\lambda_k^2 e^{-i\omega_k(t-t')}\hat{\epsilon}_{\mathbf{k},s}(\hat{\epsilon}_{\mathbf{k},s}, \langle\hat{\mathbf{P}}_l\rangle), \tag{15}$$

where $\Gamma\rho_A$ is the radiative relaxation operator and $\langle\hat{\mathbf{E}}^R_{\mathbf{k},s}\rangle$ is the reaction field of the atom [14] each one following from $V_{AF}$ and $\mathcal{U}_F$ in the right-hand side of Eq. (8), respectively. It now becomes apparent that the field which actually drives the atom is

$$\mathbf{E}_{\mathrm{loc}} = \sum_{\mathbf{k},s}(\langle\hat{\mathbf{E}}_{\mathbf{k},s}\rangle - \langle\hat{\mathbf{E}}^R_{\mathbf{k},s}\rangle). \tag{16}$$

Changing it back to the initial picture and dropping the relaxation terms, as we further investigate scattering of a short pulse, provides the final set of equations:

$$i\frac{d}{dt}\rho_A - [H_A - (\mathbf{E}_{\mathrm{loc}}, \hat{\mathbf{P}}_l), \rho_A] = 0, \qquad \rho_F = |\beta_{\mathbf{k},s}\rangle\langle\beta_{\mathbf{k},s}|, \tag{17}$$

plus Eq. (11) to find $\beta_{\mathbf{k},s}$. The first one is the Bloch equation for a two-level atom for which the local field is obtained explicitly in Section 3.

## 3. Local Field Correction

The set of equations derived in the previous section contains the infinite number of equations for functions $\beta_{\mathbf{k},s}$. However, to solve a spectroscopic problem one needs to find the density operator $\rho_A(\mathbf{r}_l)$ determined by the field strength at its location. For the sake of simplicity we put $\eta_k = 0$ in this section. Thus, the formal integration of Eq. (11) with the initial conditions (5) yields

$$\beta_{\mathbf{k},s} = \alpha_{\mathbf{k},s}e^{-i\omega_k t} - \sum_m \int_0^t dt'\lambda_k e^{-i\mathbf{k}\cdot\mathbf{r}_m - i\omega_k(t-t')}\hat{\epsilon}_{\mathbf{k},s}(\hat{\epsilon}_{\mathbf{k},s}, \langle\hat{\mathbf{P}}_m\rangle). \tag{18}$$

After we substitute this result into Eq. (16) and change the summation over field modes and atoms for the respective integrals we get

$$\mathbf{E}_{\rm loc}(\mathbf{r},t) = \mathbf{E}_0(\mathbf{r},t) - \int_0^t dt' \int_{V-\delta} d\mathbf{r}_m n(\mathbf{r}_m) \int \frac{\omega_k d\mathbf{k}}{(2\pi)^2}\, e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_m)} \sin(\omega_k(t-t'))\hat{\epsilon}_{\mathbf{k},s}(\hat{\epsilon}_{\mathbf{k},s}, \langle\hat{\mathbf{P}}_m\rangle)\,. \quad (19)$$

The notation $v-\delta$ in the lower limit of the spatial integral reflects the fact that subtraction of $\langle\hat{\mathbf{E}}^R_{\mathbf{k},s}\rangle$ in Eq. (16) in practice eliminates atom $l$ from the summation. $\delta$ is here a small volume around $\mathbf{r}_l$. The solution for the field can be written in a shorter from:

$$\mathbf{E}_{\rm loc}(\mathbf{r},t) = \mathbf{E}_0(\mathbf{r},t) - \int_0^t dt' \int_{V-\delta} d\mathbf{r}_m \mathbf{G}(\mathbf{r}-\mathbf{r}_m, t-t')\ \mathbf{P}(\mathbf{r}_m,t')\,. \quad (20)$$

Here $\mathbf{P} = n(\mathbf{r}_m)\langle\hat{\mathbf{P}}_l\rangle$ is the atomic polarization and $\mathbf{G}(\mathbf{r}-\mathbf{r}_m, t-t')$ is the Green's function of the electromagnetic field. However, it is known that the complete Green's function has a singularity at $\mathbf{r}=0$ which can be explicitly shown by taking the Fourier transform [15,16]:

$$\mathbf{G}(\mathbf{r},\omega_k) = \frac{4\pi\omega_k^2}{c^2}\frac{e^{ikr}}{r}\left(R(ikr)\mathbf{I} + Q(ikr)\frac{\mathbf{r}\times\mathbf{r}}{r^2}\right) - \frac{4\pi}{3}\delta(\mathbf{r})\,,$$
$$R(z) = 1 - \frac{1}{z} + \frac{1}{z^2}\,, \qquad Q(z) = -1 + \frac{3}{z} - \frac{3}{z^2}\,. \quad (21)$$

Remembering the full Green's function in Eq. (20) and separating spatial integrations over the small and entire volumes gives

$$\mathbf{E}_{\rm loc}(\mathbf{r},t) - \frac{4\pi}{3}\mathbf{P} = \mathbf{E}_0(\mathbf{r},t) - \int_0^t dt' \int_v d\mathbf{r}_m \mathbf{G}(\mathbf{r}-\mathbf{r}_m, t-t')\ \mathbf{P}(\mathbf{r}_m,t')\,. \quad (22)$$

$\mathbf{E}_0(\mathbf{r},t)$ is by definition the solution of the homogeneous wave equation. At the same time, the last term here represents a specific solution of the inhomogeneous wave equation. Consequently, the right-had-side terms together form the macroscopic field, so we come to the well-known relation describing the difference between the acting field and the macroscopic average field:

$$\mathbf{E}_{\rm loc}(\mathbf{r},t) = \mathbf{E}_M(\mathbf{r},t) + \frac{4\pi}{3}\mathbf{P}\,. \quad (23)$$

For the resonant atoms imbedded into a dielectric host this relation is modified as following:

$$\mathbf{E}_{\rm loc}(\mathbf{r},t) = \mathbf{E}_M(\mathbf{r},t) + \frac{4\pi}{3\varepsilon(\omega_k)}\mathbf{P}\,. \quad (24)$$

with $\varepsilon(\omega_k)$ denoting the complex permittivity. Finally, the operator describing the field contribution to the dynamics of an atom may be written as

$$V_A = -(\mathbf{E}_{\rm loc}, \hat{\mathbf{P}}_l) = V_M + V_P\,,$$

$$V_M = -R(t)e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega_0 t}\sigma_+ + \text{H.c.}\,, \qquad V_P = -\frac{\gamma}{2}\frac{\pi n(\mathbf{r})}{k_{21}^3}(\tilde{\varepsilon}' + i\tilde{\varepsilon}'')\sigma_+\rho_{21} + \text{H.c.}\,, \quad (25)$$

$$\tilde{\varepsilon}' = \text{Re}(\varepsilon(\omega_{21}))/|\varepsilon(\omega_{21})|^2\,, \qquad \tilde{\varepsilon}'' = -\text{Im}(\varepsilon(\omega_{21}))/|\varepsilon(\omega_{21})|^2\,,$$

where $R(t) = (\mathbf{d}_{21}, \mathbf{E}_M)/2\hbar$ is the Rabi frequency, $\rho_{12}$ is the matrix element of the single-atom density operator, $\mathbf{k}_0$ and $\omega_0$ are the carrier wave vector and frequency, $\gamma$ is the spontaneous decay rate, and $k_{21} = \omega_{21}/c$. The real part of $V_P$ is responsible for the renormalization of the resonance frequency while its imaginary part produces intrinsic collective relaxation similar to superradiation. The complex operator was used in [17, 18]. However, the general relation (23) was not obtained consistently within the kinetic equations except for the limit of the thin layer geometry [17]. Besides, the approaches used previously did not allow a straightforward calculation of spectral characteristics.

## 4. Time-Dependent Spectra of Scattered Light from a Dot Sample

The spectral intensity absorbed by a detector can be easily found from Eq. (7) if estimated as irreversible loss of radiation:

$$I_{\mathbf{k},s}(t) = \hbar\omega_k Tr_F\{-i\mathcal{L}_{\mathbf{k},s}\rho_F\hat{a}^\dagger_{\mathbf{k},s}\hat{a}_{\mathbf{k},s}\} = \eta_k\hbar\omega_k|\beta_{\mathbf{k},s}|^2\,. \quad (26)$$

Therefore, finding the fluorescence spectrum produced by a short pulse is reduced to solving Eq. (11) with the atomic density operators found from Eqs. (17). However, despite the lack of the incoherent component in Eq. (26) we can expect various frequencies of the scattered light. The aim of this work is to check for possible occurrence of additional spectral lines due to the local field correction. To do so we chose the dielectric host to satisfy $\tilde{\varepsilon}'' \gg \tilde{\varepsilon}'$ so the superradiant part of $V_P$ is dominant. In our derivation of the approximate analytical solution to Eq. (26) we also select a simple case of a dot sample with dimensions smaller than the resonant wavelength so the spatial dependencies are negligible. Further, for the sake of simplicity, this sample is irradiated by a short rectangular laser pulse with the carrier frequency in exact resonance with the atomic transition. In the rotating wave approximation, i.e. operating in terms of $\nu_k = \omega_k - \omega_0$, the set of equations to describe this model is

$$\frac{d}{dt}\rho_A = -i[V_M + V_P, \rho_A]\,, \quad (27)$$

$$\frac{d}{dt}\beta_{\mathbf{k},s} = -i(\nu_k - i\eta_k/2)\beta_{\mathbf{k},s} - N\lambda_k d_{12}\rho_{21}\,. \quad (28)$$

The solution $\rho_A^0(t)$ for Eq. (27) with the Rabi frequency $R(t) = R$, $0 < t < T$ and $V_P(\rho_A = 0)$ is very well known. The next approximation $\rho_A^1(t)$ with $V_P(\rho_A = \rho_A^0)$ is found easily as well:

$$\rho_A^1(t) = \frac{1}{2}\begin{pmatrix} 1 - \sin\Omega(t) & i\sin\Omega(t) \\ -i\sin\Omega(t) & 1 + \cos\Omega(t) \end{pmatrix}, \quad (29)$$

$$\Omega(t) = 2Rt + B(1 - \cos(2Rt)), \qquad B = \frac{\pi n(\mathbf{r})\tilde{\varepsilon}''}{k_{21}^3}\frac{\gamma}{4R},$$

where $B$ is the dimensionless parameter that describes the density of the light emitters and, in order for the approximation above to be valid, should meet the condition $B \ll 1$. Now we neglect the signal retardation to find $\beta_{\mathbf{k},s}$ from Eq. (28). Leaving the residual luminescence out of our consideration we restrict the time integration limit $\tau$ to the pulse duration by putting $\tau = 2RT$ for observations at $t > T$ and $\tau = 2Rt$ at $t < T$. Under these assumptions the solution is

$$\beta_{\mathbf{k},s}(t) = M_k e^{-i\zeta_k 2Rt} \int_0^{\tau} d2Rt' e^{i\zeta_k 2Rt'} \sin\Omega(t'), \tag{30}$$

where

$$M_k = i\frac{N\lambda_k d_{21}}{4R}, \qquad \zeta_k = \frac{\nu_k - i\eta_k/2}{2R}.$$

Using the Bessel-function expansions [19]

$$\cos(B\cos\varphi) = \sum_{j=-\infty}^{\infty} (-1)^j J_{|2j|}(B) e^{i2j\varphi},$$
$$\sin(B\cos\varphi) = \sum_{j=-\infty}^{\infty} (-1)^j J_{|2j+1|}(B) e^{i(2j+1)\varphi}. \tag{31}$$

in the integration of Eq. (30) we find that

$$I_{\mathbf{k},s}(t) = N^2 G_k e^{-\eta_k \tau} \sum_{j=-\infty}^{\infty} \frac{1 + e^{-\eta_k T} - 2e^{-\eta_k T/2}\cos((\nu_k + 2jR)T)}{(\nu_k + 2jR)^2 + \nu_k^2/4} |Y_j|^2, \tag{32}$$

$$Y_{2m+1} = \left(J_{|2m|}e^{iB} + J_{|2m+2|}(B)e^{-iB}\right), \quad Y_{2m} = \left(J_{|2m|+1}(B)e^{iB} - J_{|2m|+3}(B)e^{-iB}\right),$$
$$Y_{-2m} = Y_{2m}^*, \qquad Y_0 = 2J_1(B)\cos(B),$$

where $\tau = 0$ for $t < T$ and $\tau = t - T$ for $t > T$. The prefactor $G_k = \eta_k \hbar\omega_k(\lambda_k d_{21})^2$. The spectrum of scattered light is comprised of a series of peaks multiple to the Rabi frequency with intensities proportional to $|Y_j|^2$. The structure of the numerator function is better understood if taken in the limit of either long, compared to detection time, or short pulse, i.e. $\eta_k T \gg 1$ or $\eta_k T \ll 1$, respectively.

In the first case the contribution from the exponential terms is negligible and the spectrum becomes a series of equidistant sidebands with intensities determined by $|Y_j|^2$ only. From the asymptotic expansion $J_j(B) \sim 1/\sqrt{2\pi k}(eB/2j)^j$ it follows that intensities of the peaks complementary to the Mollow triplet decrease dramatically with greater $j$. As it is shown in Fig. 1 this result is in good agreement with precise numerical calculations of Eqs. (17) and (11). Figure 1 shows the spectra for densities beyond the approximation (29) where the sidebands tend to gather around

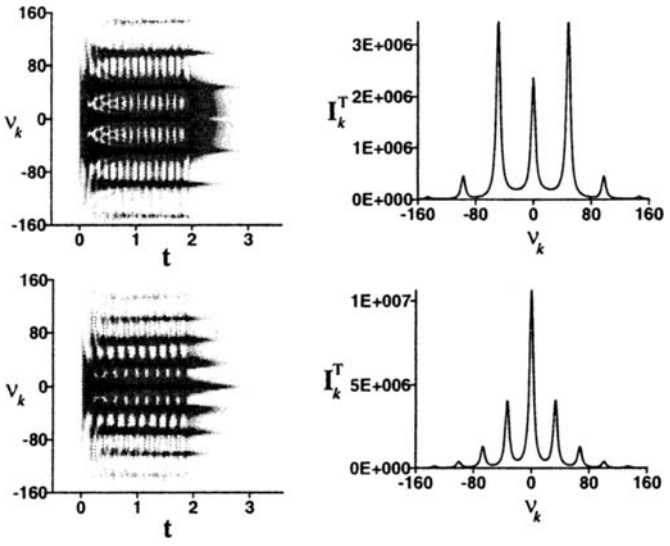


**Fig. 1.** Intensity spectra $I_{\mathbf{k},s}$ (left) and time-integrated spectra $I^T_{\mathbf{k},s}$ (right) of scattered light. $\nu_k$ is expressed in $100\gamma$ units, intensity magnitudes are given in relative units proportional to $N^2$: $\omega_{21} - \omega_0 = 0$, $R = \pi 10^3\gamma$, $\eta_k = 500\gamma$, $T = 0.019\gamma^{-1}$, $B = 0.64$ (top), $B = 0.85$ (bottom)

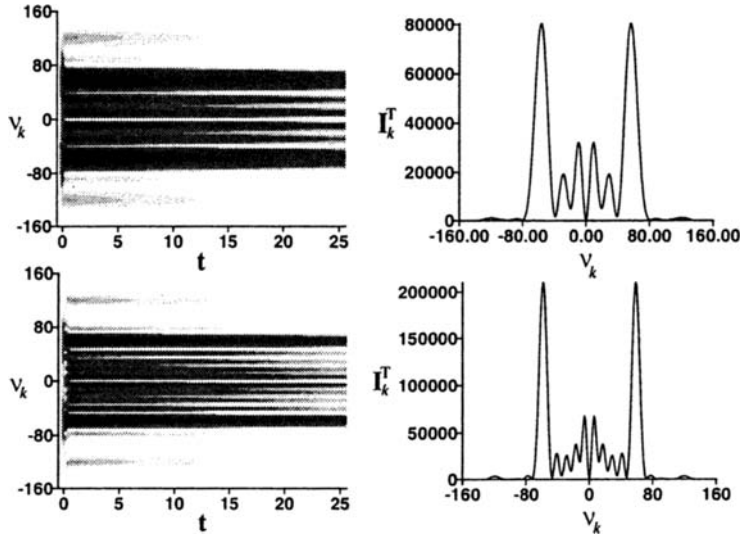


**Fig. 2.** The same as in Fig. 1. Magnitudes are given in relative units proportional to $N^2$: $\omega_{21} - \omega_0 = 0$, $R = \pi 10^3\gamma$, $\eta_k = 10\gamma$, $B = 0.28$, $T = 0.003\gamma^{-1}$ (top), $T = 0.005\gamma^{-1}$ (bottom)

the resonance. However, the intensity ratios remain similar to those estimated by the analysis. The central peak is highly pronounced in the plot as it now depicts the residual luminescence. The rib-like pattern in the time-dependent spectral diagram demonstrates the Rabi oscillations in contrast to the short pulse picture shown in Fig. 2.

In this limit the numerator describes modulation of peak intensities $\sim|Y_j|^2\times$ $[1-\cos((\nu_k+2jR)T)]$ which gives a good qualitative explanation of the profiles obtained numerically for $6\pi$ and $10\pi$ pulses. Nevertheless, one can see in Fig. 2 that the sidebands are actually displaced towards the center. Consequently, the multi-peak pattern can be generated by the intrinsic cooperativity and is best seen at moderate densities of the light emitters.

## 5. Conclusions

We have studied the scattering of resonant light in a dense medium with account for local field effects. The set of equations describing the atomic dynamics and spectral intensity of scattered light was derived from the second Born approximation of BBGKY hierarchy of equations for reduced density and correlation operators. The local field correction terms were obtained consistently from the conventional interaction Hamiltonian. Scattering of a short laser pulse by a quasi-dot sample has been investigated both theoretically and numerically. Multi-peak spectra are found to occur in the transient regime including the cases when the distance between the peaks is multiple to the Rabi frequency. Relative intensities are computable and well observable for neighboring peaks around the resonance.

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