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QUANTUM
ELECTRONICS

Decoherence Modes in Neutron Interferometry

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Abstract. We give a short introduction to the topics of decoherence, neutron interferometry and entanglement for single neutrons. We introduce two theoretical modes of decoherence for an entangled two qubit system via special Lindblad generators for the quantum master equation. The experimental realization of the decoherence modes is achieved within neutron interferometry where the decoherence is modeled by fluctuating magnetic fields in the interferometer.

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1. Introduction

Quantum systems have to be regarded as open systems due to the fact that any realistic system is subjected to a coupling to an uncontrollable external environment which influences it in a non-negligible way [1–3]. The phenomenon of decoherence arises which means that quantum correlations and interferences are destroyed in course of time. The reduced dynamics of open quantum systems can be described either by a quantum master equation with Lindblad generators [4, 5] or by a dynamical map represented with Kraus operators [6].

Neutron interferometry [7] is an almost ideal tool to investigate the evolution of a spin-$\frac{1}{2}$ system. In particular, when using a polarized beam we can create entanglement between different degrees of freedom, i.e. the spin and the path of the

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neutron. In this case it is physically rather non-contextuality than locality which is tested experimentally [8, 9].

The article is organized as follows. The next section, Sect. 2, introduces and discusses decoherence and how it can be described theoretically (Lindblad generators and Kraus operators). We continue in Sect. 3 with neutron interferometry where the entanglement of a single neutron is introduced. Sect. 4 is devoted to the discussion of two special decoherence modes for an entangled two-qubit system which can be realized within neutron interferometry via random magnetic fields which represent the environment [10]. In Sect. 5 the conclusions are formulated.

2. Decoherence

2.1. Open quantum systems — decoherence

In physical applications of quantum theory one has to take into account that there will never be a perfect isolation of a quantum system. Thus the concept of open quantum systems has to be introduced.

We suppose our system S of interest is coupled to an external environment E. This interaction causes on the one hand *dissipation* which is the energy flow between S and E. On the other hand we have *decoherence* which destroys the phase information in the system and thus the ability of the system to produce interferences is lost. Decoherence can serve as an explanation for the classicality of our world.

The unitary evolution U of the closed $S + E$ complex is governed by the total Hamiltonian H_{S+E} whereas the dynamics of the system S is given by a non-unitary evolution, the reduced dynamics

$$\rho_t = \mathrm{Tr}_E(\rho_{S+E}(t)) = \mathrm{Tr}_E(U\rho_{S+E}(0)U^\dagger)\,, \tag{1}$$

where the environmental degrees of freedom are traced out. The problem is that the environmental degrees of freedom are either not accessible or unknown. Therefore the reduced dynamics are better described by some effective evolution. There are two approaches for this: on the one hand the quantum master equation with Lindblad generators and on the other hand the method of Kraus operators. These two approaches are not independent of each other (see Ref. [11]).

2.2. Quantum master equation

The quantum master equation is an effective differential equation for the dynamics of the open system given by

$$\frac{\partial}{\partial t}\rho_t = -i[H, \rho_t] - \mathcal{D}(\rho_t)\,, \tag{2}$$

where the Liouville – von Neumann equation with the Hamiltonian H is modified by a non-unitary operator $\mathcal{D}(\rho_t)$. The most general form [4, 5] of the so-called *dissipator* is given by

$$\mathcal{D}(\rho_t) = \frac{1}{2}\sum_k \left(A_k^\dagger A_k \rho_t + \rho_t A_k^\dagger A_k - 2A_k \rho_t A_k^\dagger\right), \tag{3}$$

with the *Lindblad generators* A_k. Let us assume that the Lindblad generators are projection operators $A_k = \sqrt{\lambda} P_k$ satisfying $P_k^2 = P_k$ and $\sum_k P_k = \mathbb{1}$. This provides us with the simplified form of the dissipator

$$\mathcal{D}(\rho_t) = \lambda\left(\rho_t - \sum_k P_k \rho_t P_k\right). \tag{4}$$

The strength of the interaction is parameterized by the *decoherence parameter* λ.

2.3. Kraus operators

The other possibility to model the state change of an open system is to introduce the dynamical map

$$V(t) : \rho_0 \mapsto \rho_t = V(t)\rho_0\,. \tag{5}$$

The most general form of the complete positive dynamical map is given in terms of *Kraus operators* [6] defined by

$$V(t)\rho_0 = \sum_i M_i \rho_0 M_i^\dagger \qquad \text{with} \qquad \sum_i M_i^\dagger M_i = \mathbb{1}\,. \tag{6}$$

The Kraus operators are sometimes called jump operators.

2.4. Examples

Two very often used examples of open systems are the phase damping channel and the depolarizing channel for one qubit.

The *phase damping channel* has no classical analog because it describes the loss of quantum information without loss of energy. Phase damping can happen, for example, due to random phase kicks or scattering processes occurring with a probability p. We can model this channel by the Kraus operators $M_0 = [1-\frac{1}{2}p]^{1/2}\,\mathbb{1}$,

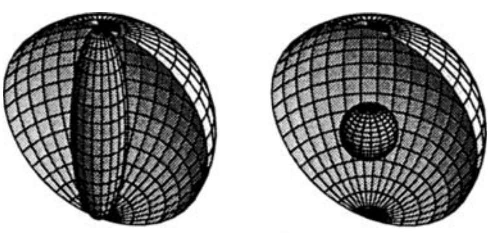

Fig. 1. The action of the phase damping channel and the depolarizing channel on the Bloch sphere for a certain value of p is shown

and $M_1 = [\frac{1}{2}p]^{1/2}\sigma_z$, which corresponds to a deformation of the Bloch sphere to an ellipsoid oriented along the z direction (see Fig. 1).

For the *depolarizing channel* the qubit gets totally mixed with a certain probability $\frac{3}{4}p$ due to the occurrence of three possible errors (spin flip σ_x, phase flip σ_z or both σ_y) and with $1 - \frac{3}{4}p$ the qubit remains unchanged. The corresponding Kraus operators are $M_0 = [1 - \frac{3}{4}p]^{1/2}\,\mathbb{1}$, $M_1 = [\frac{1}{4}p]^{1/2}\sigma_x$, $M_2 = [\frac{1}{4}p]^{1/2}\sigma_y$, and $M_3 = [\frac{1}{4}p]^{1/2}\sigma_z$. The Bloch sphere shrinks uniformly towards the totally mixed state (see Fig. 1).

3. Neutron Interferometry

Neutrons are massive particles ($m = 939.6$ MeV) with a finite lifetime of about 15 minutes. A perfect single silicon crystal interferometer for neutrons [7] (beam separation of about 5 cm), which is topologically identical to a Mach–Zehnder interferometer, operates with thermal neutrons with a wavelength of $\lambda = 2$ Å and a velocity of $v = 2000$ m/s. The time of flight across the interferometer is approximately $100\,\mu$s thus the neutrons can be considered stable in the interferometer.

The measured intensity of the interference fringes depends on the phase shift $\Delta\chi$ between the two paths of the interferometer and is given by $I_{\mathrm{O}} \sim 1 + \cos\Delta\chi$ for the so-called O-beam.

3.1. Entanglement and Bell inequality for neutrons

Entanglement is a feature which arises in a multi-partite Hilbert space. In the "classical" situation the bipartite Hilbert space under consideration, $\mathcal{H} = \mathcal{H}_{\mathsf{spin}} \otimes \mathcal{H}_{\mathsf{spin}}$, describes the spin degree of freedom of two particles, e.g. photons. There one can construct Bell inequalities to test the non-locality of the system [2].

One can also consider a Hilbert space of the form $\mathcal{H} = \mathcal{H}_{\mathsf{spin}} \otimes \mathcal{H}_{\mathsf{path}}$ where both degrees of freedom (spin and path) belong to the same particle. This corresponds, for example, to a neutron with certain spin states in an interferometer. As it turns out the construction of a Bell-like inequality is also possible but now one tests the *contextuality* of the system [8, 9].

In close analogy to the spin-$\frac{1}{2}$ case the maximally entangled state for a neutron is given by

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}\big(|\Downarrow\rangle \otimes |\mathrm{I}\rangle - |\Uparrow\rangle \otimes |\mathrm{II}\rangle\big)\,, \tag{7}$$

where $|\Uparrow, \Downarrow\rangle$ and $|\mathrm{I}, \mathrm{II}\rangle$ denotes spinor and spatial degrees of freedom, respectively. The entangled state of a neutron is created via a special kind of a spin flipper which is inserted in one path of the interferometer (see Fig. 2).

The Bell-like inequality for neutrons is given by

$$S(\chi, \chi', \xi, \xi') = \big|E(\chi, \xi) - E(\chi, \xi')\big| + \big|E(\chi', \xi) + E(\chi', \xi')\big| \leq 2\,, \tag{8}$$

where $E(\chi, \xi)$ denotes the expectation value of a joint measurement of the path and the spin observable. The experiments show a clear violation of the Bell-like

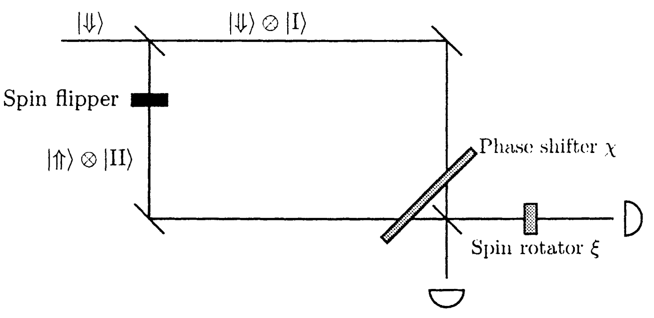

Fig. 2. Schematical setup of the neutron interferometer where an entangled state of a neutron is created via the spin flipper in one path. The path degree of freedom is parameterized with the phase shifter χ and the spinor degree of freedom is measured with the spin rotator ξ

inequality with a value of $S_{\text{exp}} = 2.051 \pm 0.019$ [9]. The small amount of the violation, which is nevertheless significant, is because of the reduced contrast in the interferometer due to the additional spin flipper.

4. Decoherence Modes

4.1. Theory

In the following we want to consider two different modes of decoherence for an entangled system composed of two spin-$\frac{1}{2}$ particles, see Ref. [10]. We start with the quantum master equation where the dissipator is given by the simplified form, Eq. (4),

$$\frac{\partial}{\partial t}\rho_t = -\mathcal{D}(\rho_t) = -\lambda\big(\rho_t - \sum_k P_k \rho_t P_k\big)\,, \tag{9}$$

where we neglect dynamical effects induced by the Hamiltonian H. We choose four projection operators as Lindblad generators such that: in the first case (A) the operators P_k project onto the eigenstates of the Hamilton operators H of the undisturbed system and in the second case (B) the projections are onto rotated states.

Case (A) can be called "$E \otimes E$" because the projection operator is just the product of the projections onto the eigenstates of the subsystems $P_k = P_k^{(1)} \otimes P_k^{(2)}$. The solution for this case with the initial condition of the maximally mixed antisymmetric Bell state $\rho_0 = |\Psi^-\rangle\langle\Psi^-|$ is given by

$$\rho_0 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \rho_t^{(A)} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -e^{-\lambda t} & 0 \\ 0 & -e^{-\lambda t} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{10}$$

For case (B) the projection operators are rotated in the first subspace according to $\widetilde{P_k} = UP_k^{(1)}U^\dagger \otimes P_k^{(2)}$ where the unitary rotation U just creates an equal superposition of the subspace-eigenstates. This justifies the name "$R \otimes E$". The solution of the master equation with the same initial condition ρ_0 is given by

$$\rho_t^{(B)} = \frac{1}{4}\begin{pmatrix} 1-e^{-\lambda t} & 0 & 0 & 0 \\ 0 & 1+e^{-\lambda t} & -2e^{-\lambda t} & 0 \\ 0 & -2e^{-\lambda t} & 1+e^{-\lambda t} & 0 \\ 0 & 0 & 0 & 1-e^{-\lambda t} \end{pmatrix}. \tag{11}$$

We can calculate the Kraus operators for both modes. They are given by $\mathbb{1}\otimes\mathbb{1}$, $\mathbb{1}\otimes\sigma_z$, $\sigma_z\otimes\mathbb{1}$, $\sigma_z\otimes\sigma_z$ for mode $E\otimes E$ and by $\mathbb{1}\otimes\mathbb{1}$, $\mathbb{1}\otimes\sigma_z$, $\sigma_x\otimes\mathbb{1}$, $\sigma_x\otimes\sigma_z$ for mode $R\otimes E$ (with some scaling factors in front, see Ref. [10]).

4.2. Decoherence via random magnetic fields

We want to test the above discussed modes of decoherence within neutron interferometry. Thus we have to find a source of decoherence which allows for experimental control. This can be done by using randomly fluctuating magnetic fields.

The action of a magnetic field $\vec{B} = B\vec{n}$ on the spin of a neutron is determined by a unitary rotation $U(\alpha) = e^{i\frac{\alpha}{2}\vec{n}\cdot\vec{\sigma}}$ where $\vec{n}$ determines the direction of the field and the rotation angle $\alpha = 2\mu_B Bt$ is given by the length and the duration of the field. For example, a spin-up state in z direction $|\Uparrow\rangle$ enters a magnetic field oriented along z direction which results just in an ordinary phase shift, $|\Uparrow\rangle \longrightarrow U(\alpha)|\Uparrow\rangle = e^{i\frac{\alpha}{2}\sigma_z}|\Uparrow\rangle = e^{+i\frac{\alpha}{2}}|\Uparrow\rangle$. If the field is oriented along x direction we get a superposition of spin-up and spin-down state, $|\Uparrow\rangle \longrightarrow U(\alpha)|\Uparrow\rangle = e^{i\frac{\alpha}{2}\sigma_x}|\Uparrow\rangle = \cos\frac{\alpha}{2}|\Uparrow\rangle + i\sin\frac{\alpha}{2}|\Downarrow\rangle$.

Now suppose each neutron in the interferometer feels a different strength of the magnetic field which results in different rotation angles for the neutrons. If we integrate over the whole distribution of rotation angles we get a non-unitary evolution of the whole ensemble of neutrons passing through the interferometer

$$\rho \longrightarrow \rho' = \int \underbrace{U(\alpha)\,\rho\, U^\dagger(\alpha)}_{\rho(\alpha)}\; P(\alpha)d\alpha\,. \tag{12}$$

We suppose the distribution $P(\alpha)$ to be a Gaussian with standard deviation σ.

4.3. Experimental realization

The experimental realization of the decoherence modes discussed in Sect. 4.1 can be done within neutron interferometry. Figure 3 shows the experimental setup for mode $E\otimes E$.

The incoming polarized neutron beam $|\Downarrow\rangle$ is split at the first plate of the interferometer. In path I the neutron experiences an arbitrarily fluctuating magnetic field $B_z^{(\mathrm{I})}(\alpha)$ oriented along z direction and operating with a rotation angle α.

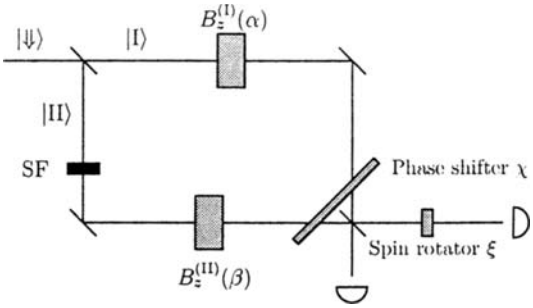

Fig. 3. Schematical setup decoherence experiment for mode $E \otimes E$, case (A). In both paths there are arbitrarily fluctuating magnetic fields, $B_z^{(I)}(\alpha)$ and $B_z^{(II)}(\beta)$, oriented in z direction

In path II there is the additional spin flipper which creates the entanglement and another fluctuating magnetic field $B_z^{(II)}(\beta)$ oriented in z direction with the rotation angle β. Both magnetic fields act independently but we assume that the distributions of both fields have the same deviation σ.

The action of the magnetic fields can be seen as a conditioned operation depending on the spin state

$$|\psi_{\text{spin}}\rangle \otimes |\text{I}\rangle \longrightarrow U(\alpha)|\psi_{\text{spin}}\rangle \otimes |\text{I}\rangle\,, \qquad |\psi_{\text{spin}}\rangle \otimes |\text{II}\rangle \longrightarrow U(\beta)|\psi_{\text{spin}}\rangle \otimes |\text{II}\rangle\,, \quad (13)$$

where $U(\alpha)$ and $U(\beta)$ denote the unitary rotations corresponding to the magnetic fields. Applying this conditioned operation to each neutron in the initial state (7) and performing the integration over all possible rotation angles we get the state of the neutron ensemble after passing the interferometer

$$\rho' = \int \rho(\alpha,\beta)\, P(\alpha)\, P(\beta) d\alpha\, d\beta = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -e^{-\frac{1}{4}\sigma^2} & 0 \\ 0 & -e^{-\frac{1}{4}\sigma^2} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (14)$$

This state can be compared with the theoretically predicted decoherent state $\rho_t^{(A)}$, Eq. (10), which immediately leads to the conclusion

$$\lambda t = \tfrac{1}{4}\sigma^2\,. \quad (15)$$

This means the width of the distribution σ of the random fields determines the strength of the decoherence which is given by λt.

The experimental realization for mode $R \otimes E$ will not be discussed here because of its analogy to mode $E \otimes E$; details can be found in Ref. [10].

5. Conclusions

We have presented a theoretical model of decoherence for an entangled system which can be tested experimentally within neutron interferometry. The important result, Eq. (15), relates a theoretical parameter — the decoherence parameter λ — to an experimental parameter, the deviation σ. The time t corresponds to the time of flight through the magnetic fields and is fixed in the experiments. Thus it is possible to confirm experimentally the predicted exponential decrease $e^{-\lambda t}$ of the off-diagonal elements of the density matrix $\rho_t^{(A)}$, Eq. (10), due to decoherence by measuring the density matrix elements, e.g. via state tomography [12, 13], for different distributions with different deviations σ.

The experimental realization is work in progress.

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