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Dispersion Forces within the Framework of Macroscopic QED

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Abstract. Dispersion forces, which material objects in the ground state are subject to, originate from the Lorentz force with which the fluctuating, object-assisted electromagnetic vacuum acts on the fluctuating charge and current densities associated with the objects. We calculate them within the framework of macroscopic QED, considering magnetodielectric objects described in terms of spatially varying permittivities and permeabilities which are complex functions of frequency. The result enables us to give a unified approach to dispersion forces on both macroscopic and microscopic levels.

Keywords: dispersion forces, Lorentz-force approach, QED in linear causal media
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1. Introduction

As known, electromagnetic fields can exert forces on electrically neutral, unpolarized and unmagnetized material objects, provided that these are polarizable and/or magnetizable. Classically, it is the lack of precise knowledge of the state of the sources of a field what lets one resort to a probabilistic description of the field, so that, as a matter of principle, a classical field can be non-fluctuating. In practice, this would be the case when the sources, and thus the field, were under strict deterministic control. In quantum mechanics, the situation is quite different, as field fluctuations are present even if complete knowledge of the quantum state would be achieved; a strictly non-probabilistic regime simply does not exist. Similarly, polarization and magnetization of any material object are fluctuating quantities in quantum mechanics. As a result, the interaction of the fluctuating electromagnetic vacuum with the fluctuating polarization and magnetization of material objects in the ground state can give rise to non-vanishing Lorentz forces; these are commonly referred to as dispersion forces.

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In the following we will refer to dispersion forces acting between atoms, between atoms and bodies, and between bodies as van der Waals (vdW) forces, Casimir–Polder (CP) forces and Casimir forces, respectively. This terminology also reflects the fact that, although the three types of forces have the same physical origin, different methods to calculate them have been developed. The CP force that acts on an atom (Hamiltonian $\hat{H}_{\rm A}$) in an energy eigenstate $|a\rangle$ ($\hat{H}_{\rm A}|a\rangle = \hbar\omega_a|a\rangle$) at position $\mathbf{r}_{\rm A}$ in the presence of (linearly responding) macroscopic bodies is commonly regarded as being the negative gradient of the position-dependent part of the shift of the energy of the overall system, ΔE_a, with the atom being in the state $|a\rangle$ and the body-assisted electromagnetic field being in the ground state. The interaction of the atom with the field, which is responsible for the energy shift, is typically treated in the electric-dipole approximation, i.e. $\hat{H}_{\rm int} = -\hat{\mathbf{d}}\cdot\hat{\mathbf{E}}(\mathbf{r}_{\rm A})$ in the multipolar coupling scheme, and the energy shift is calculated in leading-order perturbation theory. In this way, one finds [1, 2]

$$\Delta E_a = -\frac{\mu_0}{\pi}\sum_b \mathcal{P}\int_0^\infty \mathrm{d}\omega\,\frac{\omega^2}{\omega_{ba}+\omega}\,\mathbf{d}_{ab}\cdot\mathrm{Im}\,\mathsf{G}(\mathbf{r}_{\rm A},\mathbf{r}_{\rm A},\omega)\cdot\mathbf{d}_{ba} \tag{1}$$

($\mathcal{P}$, principal value; $\omega_{ba}=\omega_b-\omega_a$), where $\mathsf{G}(\mathbf{r},\mathbf{r}',\omega)$ is the classical (retarded) Green tensor (in the frequency domain) for the electric field, which takes the presence of the macroscopic bodies into account. It can then be argued that, in order to obtain the CP potential $U_a(\mathbf{r}_{\rm A})$ as the position-dependent part of the energy shift, one may replace $\mathsf{G}(\mathbf{r}_{\rm A},\mathbf{r}_{\rm A},\omega)$ in Eq. (1) with $\mathsf{G}^{(\mathrm{S})}(\mathbf{r}_{\rm A},\mathbf{r}_{\rm A},\omega)$, where $\mathsf{G}^{(\mathrm{S})}(\mathbf{r},\mathbf{r}',\omega)$ is the scattering part of the Green tensor. Hence,

$$\Delta E_a \;\mapsto\; U_a(\mathbf{r}_{\rm A}) = U_a^{\rm or}(\mathbf{r}_{\rm A}) + U_a^{\rm r}(\mathbf{r}_{\rm A})\,, \tag{2}$$

$$U_a^{\rm or}(\mathbf{r}_{\rm A}) = -\frac{1}{\pi\varepsilon_0 c^2}\sum_b\int_0^\infty \mathrm{d}\xi\,\frac{\omega_{ab}\xi^2}{\omega_{ab}^2+\xi^2}\,\mathbf{d}_{ab}\cdot\mathsf{G}^{(\mathrm{S})}(\mathbf{r}_{\rm A},\mathbf{r}_{\rm A},i\xi)\cdot\mathbf{d}_{ba}\,, \tag{3}$$

$$U_a^{\rm r}(\mathbf{r}_{\rm A}) = -\frac{1}{\varepsilon_0 c^2}\sum_b\Theta(\omega_{ab})\omega_{ab}^2\mathbf{d}_{ab}\cdot\mathrm{Re}\,\mathsf{G}^{(\mathrm{S})}(\mathbf{r}_{\rm A},\mathbf{r}_{\rm A},\omega_{ab})\cdot\mathbf{d}_{ba}\,, \tag{4}$$

where $U_a(\mathbf{r}_{\rm A})$ has been decomposed into an off-resonant part $U_a^{\rm or}(\mathbf{r}_{\rm A})$ and a resonant part $U_a^{\rm r}(\mathbf{r}_{\rm A})$, by taking into account the analytic properties of the Green tensor as a function of complex ω, and considering explicitly the singularities excluded by the principal-value integration in Eq. (1).

Let us restrict our attention to ground-state atoms. (Forces on excited atoms lead to dynamical problems in general [2]). In this case, there are of course no resonant contributions, as only upward transitions are possible [$\omega_{ab}<0$ in Eq. (4)]. Thus, on identifying the (isotropic) ground-state polarizability of an atom as

$$\alpha(\omega) = \lim_{\epsilon\to0}\frac{2}{\hbar}\sum_b\frac{\omega_{b1}}{\omega_{b1}^2-\omega^2-i\omega\epsilon}\,|\mathbf{d}_{1b}|^2, \tag{5}$$

we may write the CP potential of a ground-state atom in the form of (see, e.g. Refs. [1–6])

$$U(\mathbf{r}_A) = \frac{\hbar}{2\pi\varepsilon_0 c^2}\int_0^\infty d\xi\, \xi^2\, \alpha(i\xi)\mathrm{Tr}\big[\mathsf{G}^{(S)}(\mathbf{r}_A,\mathbf{r}_A,i\xi)\big], \tag{6}$$

from which the force acting on the atom follows as

$$\mathbf{F}(\mathbf{r}_A) = -\boldsymbol{\nabla} U(\mathbf{r}_A)\,. \tag{7}$$

Now consider, instead of the force on a single ground-state atom, the force on a collection of ground-state atoms distributed with a (coarse-grained) number density $\eta(\mathbf{r})$ inside a space region of volume V_M. When the mutual interaction of the atoms can be disregarded, it is permissible to simply add up the CP forces on the individual atoms to obtain the force acting on the collection of atoms due to their interaction with the bodies outside the volume V_M, i.e.

$$\mathbf{F} = \int_{V_M} d^3r\, \eta(\mathbf{r})\mathbf{F}(\mathbf{r}) = -\frac{\hbar}{2\pi\varepsilon_0 c^2}\int_{V_M} d^3r \int_0^\infty d\xi\, \xi^2 \eta(\mathbf{r})\alpha(i\xi)\boldsymbol{\nabla}\mathrm{Tr}\,\mathsf{G}^{(S)}(\mathbf{r},\mathbf{r},i\xi)\,. \tag{8}$$

Since the collection of atoms can be regarded as constituting a weakly dielectric body of susceptibility $\chi_M(\mathbf{r},i\xi)$,

$$\eta(\mathbf{r})\alpha(i\xi) \mapsto \varepsilon_0\chi_M(\mathbf{r},i\xi)\,, \tag{9}$$

Eq. (8) gives the Casimir force acting on such a body. Note that special cases of this formula were already used by Lifshitz [7] in the study of Casimir forces between dielectric plates. The question is how Eq. (8) can be generalized to an arbitrary ground-state body whose susceptibility $\chi_M(\mathbf{r},i\xi)$ is not necessarily small. An answer to this and related questions can be given by means of the Lorentz-force approach to dispersion forces, as developed in Refs. [8,9].

2. Lorentz Force

Let us consider macroscopic QED in a linearly, locally and causally responding medium with given (complex) permittivity $\varepsilon(\mathbf{r},\omega)$ and permeability $\mu(\mathbf{r},\omega)$. Then, if the current density that enters the macroscopic Maxwell equations is

$$\hat{\mathbf{j}}_N(\mathbf{r}) = \int_0^\infty d\omega\, \hat{\underline{\mathbf{j}}}_N(\mathbf{r},\omega) + \mathrm{H.c.}\,, \tag{10}$$

the source-quantity representations of the electric and induction fields

$$\hat{\mathbf{E}}(\mathbf{r}) = \int_0^\infty d\omega\, \hat{\underline{\mathbf{E}}}(\mathbf{r},\omega) + \mathrm{H.c.}\,, \qquad \hat{\mathbf{B}}(\mathbf{r}) = \int_0^\infty d\omega\, \hat{\underline{\mathbf{B}}}(\mathbf{r},\omega) + \mathrm{H.c.} \tag{11}$$

read as

$$\hat{\underline{\mathbf{E}}}(\mathbf{r},\omega) = i\mu_0\omega \int d^3r'\, \mathsf{G}(\mathbf{r},\mathbf{r}',\omega)\cdot \hat{\underline{\mathbf{j}}}_N(\mathbf{r}',\omega)\,, \tag{12}$$

$$\underline{\hat{\mathbf{B}}}(\mathbf{r},\omega) = \mu_0 \boldsymbol{\nabla} \times \int \mathrm{d}^3 r' \, \mathsf{G}(\mathbf{r},\mathbf{r}',\omega) \cdot \underline{\hat{\mathbf{j}}}_{\mathrm{N}}(\mathbf{r}',\omega) \,, \tag{13}$$

where the retarded Green tensor $\mathsf{G}(\mathbf{r},\mathbf{r}',\omega)$ corresponds to the prescribed medium. In Eqs. (12) and (13), it is assumed that the medium covers the entire space so that solutions of the homogeneous Maxwell equations do not appear. Free-space regions can be introduced by performing the limits $\varepsilon \to 1$ and $\mu \to 1$, but not before the end of the actual calculations.

Because of the polarization and/or magnetization currents attributed to the medium, the *total* charge and current densities are given by

$$\hat{\rho}(\mathbf{r}) = \int_0^\infty \mathrm{d}\omega \, \underline{\hat{\rho}}(\mathbf{r},\omega) + \mathrm{H.\,c.}\,, \qquad \hat{\mathbf{j}}(\mathbf{r}) = \int_0^\infty \mathrm{d}\omega \, \underline{\hat{\mathbf{j}}}(\mathbf{r},\omega) + \mathrm{H.\,c.}\,, \tag{14}$$

where

$$\begin{aligned} \underline{\hat{\rho}}(\mathbf{r},\omega) &= -\varepsilon_0 \boldsymbol{\nabla} \cdot \Big\{ [\varepsilon(\mathbf{r},\omega) - 1] \underline{\hat{\mathbf{E}}}(\mathbf{r},\omega) \Big\} + \underline{\hat{\rho}}_{\mathrm{N}}(\mathbf{r},\omega) \\ &= \frac{i\omega}{c^2} \boldsymbol{\nabla} \cdot \int \mathrm{d}^3 r' \, \mathsf{G}(\mathbf{r},\mathbf{r}',\omega) \cdot \underline{\hat{\mathbf{j}}}_{\mathrm{N}}(\mathbf{r}',\omega) \,, \end{aligned} \tag{15}$$

$[\underline{\hat{\rho}}_{\mathrm{N}}(\mathbf{r},\omega) = (i\omega)^{-1} \boldsymbol{\nabla} \cdot \underline{\hat{\mathbf{j}}}_{\mathrm{N}}(\mathbf{r},\omega)]$ and

$$\begin{aligned} \underline{\hat{\mathbf{j}}}(\mathbf{r},\omega) &= -i\omega\varepsilon_0 [\varepsilon(\mathbf{r},\omega) - 1] \underline{\hat{\mathbf{E}}}(\mathbf{r},\omega) + \boldsymbol{\nabla} \times \{\mu_0^{-1} [1 - \mu^{-1}(\mathbf{r},\omega)] \underline{\hat{\mathbf{B}}}(\mathbf{r},\omega)\} + \underline{\hat{\mathbf{j}}}_{\mathrm{N}}(\mathbf{r},\omega) \\ &= \left(\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times -\frac{\omega^2}{c^2} \right) \int \mathrm{d}^3 r' \, \mathsf{G}(\mathbf{r},\mathbf{r}',\omega) \cdot \underline{\hat{\mathbf{j}}}_{\mathrm{N}}(\mathbf{r}',\omega) \,. \end{aligned} \tag{16}$$

As we have not yet specified the current density $\hat{\mathbf{j}}_{\mathrm{N}}(\mathbf{r})$ in any way, the above formulas are generally valid so far, and they are valid both in classical and in quantum electrodynamics. In any case, it is clear that knowledge of the correlation function $\langle \underline{\hat{\mathbf{j}}}_{\mathrm{N}}(\mathbf{r},\omega) \underline{\hat{\mathbf{j}}}_{\mathrm{N}}^{\dagger}(\mathbf{r}',\omega') \rangle$, where the angle brackets denote classical and/or quantum averaging, is sufficient to compute the correlation functions $\langle \underline{\hat{\rho}}(\mathbf{r},\omega) \underline{\hat{\mathbf{E}}}^{\dagger}(\mathbf{r}',\omega') \rangle$, $\langle \underline{\hat{\rho}}^{\dagger}(\mathbf{r},\omega), \underline{\hat{\mathbf{E}}}(\mathbf{r}',\omega') \rangle$, $\langle \underline{\hat{\mathbf{j}}}(\mathbf{r},\omega) \underline{\hat{\mathbf{B}}}^{\dagger}(\mathbf{r}',\omega') \rangle$ and $\langle \underline{\hat{\mathbf{j}}}^{\dagger}(\mathbf{r},\omega) \underline{\hat{\mathbf{B}}}(\mathbf{r}',\omega') \rangle$, from which the (slowly varying part of the) Lorentz force density follows as

$$\begin{aligned} \mathbf{f}_{\mathrm{L}}(\mathbf{r}) = \int_0^\infty \mathrm{d}\omega \int_0^\infty \mathrm{d}\omega' \, \Big[&\langle \underline{\hat{\rho}}(\mathbf{r},\omega) \underline{\hat{\mathbf{E}}}^{\dagger}(\mathbf{r}',\omega') \rangle + \langle \underline{\hat{\mathbf{j}}}(\mathbf{r},\omega) \times \underline{\hat{\mathbf{B}}}^{\dagger}(\mathbf{r}',\omega') \rangle \\ &+ \langle \underline{\hat{\rho}}^{\dagger}(\mathbf{r},\omega) \underline{\hat{\mathbf{E}}}(\mathbf{r}',\omega') \rangle + \langle \underline{\hat{\mathbf{j}}}^{\dagger}(\mathbf{r},\omega) \times \underline{\hat{\mathbf{B}}}(\mathbf{r}',\omega') \rangle \Big]_{\mathbf{r}' \to \mathbf{r}} \,, \end{aligned} \tag{17}$$

where the limit $\mathbf{r}' \to \mathbf{r}$ must be understood in such a way that divergent self-forces, which would be formally present even in a uniform (bulk) medium, are omitted. The force on the matter in a volume V_{M} is then given by the volume integral

$$\mathbf{F}_{\mathrm{L}} = \int_{V_{\mathrm{M}}} \mathrm{d}^3 r \, \mathbf{f}_{\mathrm{L}}(\mathbf{r}) \,, \tag{18}$$

which can be rewritten as the surface integral

$$\mathbf{F}_{\mathrm{L}} = \int_{\partial V_{\mathrm{M}}} \mathrm{d}\mathbf{a} \cdot \mathsf{T}(\mathbf{r}) \,, \tag{19}$$

where $\mathsf{T}(\mathbf{r})$ is (the expectation value of) Maxwell's stress tensor (as opposed to Minkowski's stress tensor), which is (formally) identical with the stress tensor in microscopic electrodynamics. Note that in going from Eq. (18) to Eq. (19), a term resulting from the (slowly varying part of the) Poynting vector has been omitted, which is valid under stationary conditions. If $\hat{\mathbf{j}}_{\mathrm{N}}(\mathbf{r})$ can be regarded as being a classical current density producing classical radiation, $\hat{\mathbf{j}}_{\mathrm{N}}(\mathbf{r}) \mapsto \mathbf{j}_{\mathrm{class}}(\mathbf{r},t)$, then the Lorentz force computed in this way gives the classical radiation force that acts on the material inside the chosen space region of volume V_{M} (see also Ref. [10]).

3. Dispersion Force

As already mentioned in Sec. 1, the dispersion force is obtained if $\hat{\mathbf{j}}_{\mathrm{N}}(\mathbf{r})$ is identified with the noise current density attributed to the polarization and magnetization of the material. Let us restrict our attention to the zero-temperature limit, i.e. let us assume that the overall system is in its ground state. (The generalization to thermal states is straightforward.) From macroscopic QED in dispersing and absorbing linear media [11,12] it can be shown that the relevant current correlation function reads as

$$\begin{aligned}\left\langle \hat{\underline{\mathbf{j}}}_{\mathrm{N}}(\mathbf{r},\omega)\hat{\underline{\mathbf{j}}}_{\mathrm{N}}^{\dagger}(\mathbf{r}',\omega')\right\rangle = \frac{\hbar}{\mu_0\pi}\,\delta(\omega-\omega')\bigg\{ & \frac{\omega^2}{c^2}\,\mathrm{Im}\,\varepsilon(\mathbf{r},\omega)\mathsf{I}\delta(\mathbf{r}-\mathbf{r}') \\ & - \boldsymbol{\nabla}\times[\mathrm{Im}\,\mu^{-1}(\mathbf{r},\omega)\mathsf{I}\delta(\mathbf{r}-\mathbf{r}')]\times\overleftarrow{\boldsymbol{\nabla}}'\bigg\},\end{aligned} \tag{20}$$

(I, unit tensor). Combining Eqs. (12), (13), (15), (16) and (20), and making use of standard properties of the Green tensor, one can then show that

$$\left\langle \hat{\underline{\rho}}(\mathbf{r},\omega)\hat{\underline{\mathbf{E}}}^{\dagger}(\mathbf{r}',\omega')\right\rangle = \frac{\hbar}{\pi}\frac{\omega^2}{c^2}\,\delta(\omega-\omega')\boldsymbol{\nabla}\cdot\mathrm{Im}\,\mathsf{G}(\mathbf{r},\mathbf{r}',\omega) \tag{21}$$

and

$$\left\langle \hat{\underline{\mathbf{j}}}(\mathbf{r},\omega)\hat{\underline{\mathbf{B}}}^{\dagger}(\mathbf{r}',\omega')\right\rangle = -\frac{\hbar}{\pi}\,\delta(\omega-\omega')\left(\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times-\frac{\omega^2}{c^2}\right)\mathrm{Im}\,\mathsf{G}(\mathbf{r},\mathbf{r}',\omega)\times\overleftarrow{\boldsymbol{\nabla}}' \tag{22}$$

[note that $\langle \hat{\underline{\rho}}^{\dagger}(\mathbf{r},\omega)\hat{\underline{\mathbf{E}}}(\mathbf{r}',\omega')\rangle = \langle \hat{\underline{\mathbf{j}}}^{\dagger}(\mathbf{r},\omega)\hat{\underline{\mathbf{B}}}(\mathbf{r}',\omega')\rangle = 0$ in the ground state]. Insertion of Eqs. (21) and (22) in Eq. (17) eventually yields the dispersion force density, from which, according to Eq. (18), the dispersion force acting on the magnetodielectric material inside the chosen space region can be computed.

Let us consider, for instance, an isolated dielectric body of volume $V_{\rm M}$ and susceptibility $\chi_{\rm M}(\mathbf{r},\omega)$ in the presence of arbitrary magnetodielectric bodies, which are well separated from the dielectric body. In this case, further evaluation of Eq. (18) leads to the following formula for the dispersion force on the dielectric body:

$$\mathbf{F} = -\frac{\hbar}{2\pi c^2}\int_{V_{\rm M}} \mathrm{d}^3r \int_0^\infty \mathrm{d}\xi\,\xi^2 \chi_{\rm M}(\mathbf{r},i\xi)\boldsymbol{\nabla}\mathrm{Tr}\,[\mathsf{G}_{\rm M}(\mathbf{r},\mathbf{r}',i\xi)]_{\mathbf{r}'\to\mathbf{r}}\,, \tag{23}$$

where $\mathsf{G}_{\rm M}(\mathbf{r},\mathbf{r}',i\xi)$ is the Green tensor of the system that includes the dielectric body. When the dielectric body is not an isolated body but a part of some larger body (again in the presence of arbitrary magnetodielectric bodies), Eq. (23) must be supplemented with a surface integral,

$$\mathbf{F} = -\frac{\hbar}{2\pi c^2}\int_0^\infty \mathrm{d}\xi\,\xi^2 \bigg\{ \int_{V_{\rm M}} \mathrm{d}^3r\,\chi_{\rm M}(\mathbf{r},i\xi)\boldsymbol{\nabla}\mathrm{Tr}\,[\mathsf{G}_{\rm M}(\mathbf{r},\mathbf{r}',i\xi)]_{\mathbf{r}'\to\mathbf{r}} \\ -2\int_{\partial V_{\rm M}} \mathrm{d}\mathbf{a}\cdot\chi_{\rm M}(\mathbf{r},i\xi)[\mathsf{G}_{\rm M}(\mathbf{r},\mathbf{r}',i\xi)]_{\mathbf{r}'\to\mathbf{r}}\bigg\}, \tag{24}$$

which may be regarded as reflecting the screening effect due to the residual part of the body.

At this point it should be mentioned that if Minkowski's stress tensor were used to calculate the force on a dielectric body, Eq. (24) would be replaced with

$$\mathbf{F}^{(\rm Mink)} = \frac{\hbar}{2\pi c^2}\int_0^\infty \mathrm{d}\xi\,\xi^2 \int_{V_{\rm M}} \mathrm{d}^3r\,[\boldsymbol{\nabla}\chi_{\rm M}(\mathbf{r},i\xi)]\,\mathrm{Tr}\,[\mathsf{G}_{\rm M}(\mathbf{r},\mathbf{r}',i\xi)]_{\mathbf{r}'\to\mathbf{r}}\,. \tag{25}$$

Although both Eq. (24) and (25) properly reduce to Eq. (23) when the dielectric body is an isolated one, they differ by a surface integral in the case where the body is some part of a larger body. In the latter case, Minkowski's tensor is hence expected to lead to incorrect and even self-contradictory results [9, 13]. It should be pointed out that the differences between the Lorentz-force approach to dispersion forces and approaches based on Minkowski's tensor or related quantities are not necessarily small. For instance, the ground-state Lorentz force (per unit area) that acts on an almost perfectly reflecting planar plate in a planar dielectric cavity bounded by almost perfectly reflecting walls reads

$$F = \frac{\hbar c\pi^2}{240}\,\frac{1}{\sqrt{\varepsilon}}\left(\frac{2}{3}+\frac{1}{3\varepsilon}\right)\left(\frac{1}{d_{\rm R}^4}-\frac{1}{d_{\rm L}^4}\right), \tag{26}$$

provided the distances $d_{\rm L}$ and $d_{\rm R}$ of the plate to the left and right cavity walls, respectively, are sufficiently large. [Note that, as a consequence of these simplifying assumptions, only the static value $\varepsilon=\varepsilon(\omega\to 0)$ of the permittivity of the cavity medium appears in the formula.] In contrast, the corresponding result on the basis of Minkowski's stress tensor is [14]

$$F^{(\mathrm{Mink})} = \frac{\hbar c \pi^2}{240} \frac{1}{\sqrt{\varepsilon}} \left(\frac{1}{d_{\mathrm{R}}^4} - \frac{1}{d_{\mathrm{L}}^4} \right), \tag{27}$$

which can indeed noticeably differ from Eq. (26).

Let us return to Eq. (23) and assume that the (isolated) dielectric body is well described by a susceptibility of Clausius–Mossotti type, $\chi_{\mathrm{M}}(\mathbf{r},\omega) = \varepsilon_0^{-1}\eta(\mathbf{r})\alpha(\omega)/[1-\eta(\mathbf{r})\alpha(\omega)/(3\varepsilon_0)]$, so that the force on the body becomes

$$\mathbf{F} = -\frac{\hbar\mu_0}{2\pi} \int_{V_{\mathrm{M}}} \mathrm{d}^3 r \int_0^\infty \mathrm{d}\xi\, \xi^2 \eta(\mathbf{r}) \alpha(i\xi) \left[1 + \tfrac{1}{3}\chi_{\mathrm{M}}(\mathbf{r}, i\xi)\right] \boldsymbol{\nabla} \mathrm{Tr}\left[\mathsf{G}_{\mathrm{M}}(\mathbf{r}, \mathbf{r}', i\xi)\right]_{\mathbf{r}' \to \mathbf{r}} . \tag{28}$$

In the case of weakly dielectric material, the leading-order contribution to the force is obviously

$$\mathbf{F} = -\frac{\hbar\mu_0}{2\pi} \int_{V_{\mathrm{M}}} \mathrm{d}^3 r \int_0^\infty \mathrm{d}\xi\, \xi^2 \eta(\mathbf{r}) \alpha(i\xi) \boldsymbol{\nabla} \mathrm{Tr}\, \mathsf{G}^{(\mathrm{S})}(\mathbf{r}, \mathbf{r}, i\xi), \tag{29}$$

which is nothing but Eq. (8). Needless to say that performing in Eq. (29) the limit $V_{\mathrm{M}} \to 0$, $\eta \to \infty$ in such a manner that $V_{\mathrm{M}}\eta \to 1$ leads to the CP force acting on a single atom, as given by Eq. (7) together with Eq. (6). Note that Eq. (8) has been the outcome of microscopic considerations, whereas Eq. (29) has been derived from a macroscopic treatment. Moreover, Eq. (28) [or, more generally, Eqs. (23) and (24)] also contain, as limiting cases, well-known expressions for vdW interactions, which are commonly derived by treating the interaction between the atoms on a microscopic level. To see this, one may resort to the Dyson-type integral equation obeyed by the Green tensor $\mathsf{G}_{\mathrm{M}}(\mathbf{r}, \mathbf{r}', \omega)$ and the iterative (Born series) solution of this equation. Specifically, let us consider two ground-state atoms and allow for the presence of magnetodielectric bodies. The two-atom vdW force acting on a ground-state atom [polarizability $\alpha_1(i\xi)$] at position $\mathbf{r}_1$ due to the presence of another ground-state atom [polarizability $\alpha_2(i\xi)$] at position $\mathbf{r}_2$ is obtained from the first Born approximation of the Green tensor as

$$\mathbf{F}_{12}^{(\mathrm{vdW})} = \frac{\hbar\mu_0^2}{2\pi} \int_0^\infty \mathrm{d}\xi\, \xi^4 \alpha_1(i\xi) \alpha_2(i\xi) \boldsymbol{\nabla}_1 \mathrm{Tr}\left[\mathsf{G}(\mathbf{r}_1, \mathbf{r}_2, i\xi) \cdot \mathsf{G}(\mathbf{r}_2, \mathbf{r}_1, i\xi)\right], \tag{30}$$

in full agreement with Refs. [15–17].

4. Summary

We have shown how dispersion forces on ground-state objects can be calculated within a unified and conceptually transparent macroscopic framework, which is based on QED in linear, causal media. In this context, we have identified the dispersion force on an object as the Lorentz force with which the fluctuating electromagnetic vacuum acts on the charge and currents densities associated with the induced (by the fluctuating electromagnetic vacuum) polarization and magnetization as well as the noise polarization and magnetization of the object. As a result, we have presented very general formulas for dispersion forces—formulas which apply to macro- and micro-objects and even to single atoms. In particular, they have enabled us to derive, in a 'top-down' manner, from the Casimir force acting on a dielectric body in the zero-temperature limit the CP force acting on a ground-state atom, which is usually derived microscopically from the ground-state energy shift calculated in leading-order perturbation theory, with the atom–field interaction being treated in electric-dipole approximation. Similarly, the formulas also contain, as limiting case, the well-known vdW interaction between ground-state atoms, so that the Lorentz-force approach may indeed be said to provide a unifying basis for the description of dispersion forces within the framework of QED in media.

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