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# Non-Cyclic Geometric Phases in Mixed State Neutron Polarimetry

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**Abstract.** We have measured the non-cyclic geometric phase acquired during a unitary SU(2) evolution of a neutron spinor wave function for the mixed state case. The data is in good agreement with theory, verifying the predicted values for the mixed state geometric phase.

*Keywords:* mixed state, spinor, geometric phase, neutron polarimetry
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## 1. Introduction

During the last twenty years the concept of geometric phase, first discovered by Berry in 1984 [1], has given rise to a variety of research fields. Both, theoretical and experimental approach have developed from its first verification, for photons in 1986 [2] and later for neutrons [3], to a highly sophisticated theory. Originating from the geometric phase factor, acquired by transporting a quantal system round a circuit, generalizations such as non-adiabatic [4], non-cyclic [5], including the Pancharatnam relative phase [6], off-diagonal evolutions [7–9], as well as the mixed state case [10–12], have been established within the scope of this concept. Furthermore it has become apparent, that the Berry phase provides a proper basis for a diversity of fundamental quantum mechanic experiments [13, 14].

At the beginning of Section 2 the concept of geometric phase measurement is introduced in respect to the current setup. In [15] we reported on a neutron polarimetric experiment for measuring the Pancharatnam relative phase, implementing a method described by Wagh and Rakhecha in [16], considering the mixed state case put forward by Larsson and Sjöqvist [17]. The main modifications of this setup, now providing higher stability for geometric phase measurement, are explained in detail in Section 2.1. Finally in Section 2.2 the results of the mixed state geometric phase measurement are presented followed by a short outlook.

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## 2. Geometric Phase Measurement

Generally the spinor part of a neutron state vector can be seen as a superposition of 'up' and 'down' spin states referring to an arbitrary quantization axis. In the described setup a phase shift is implemented by rotations around the $x$ axis which then consequently is chosen as basis. A schematic view of the setup is shown in Fig. 1. The incident beam, polarized in the $|+z\rangle$ direction, now written as

$$|+z\rangle = \frac{1}{\sqrt{2}}[|+x\rangle + |-x\rangle] \tag{1}$$

passes two spin rotators (DC coils), with their magnetic fields with same absolute value but opposite directions ($\hat{x}$ and $-\hat{x}$).

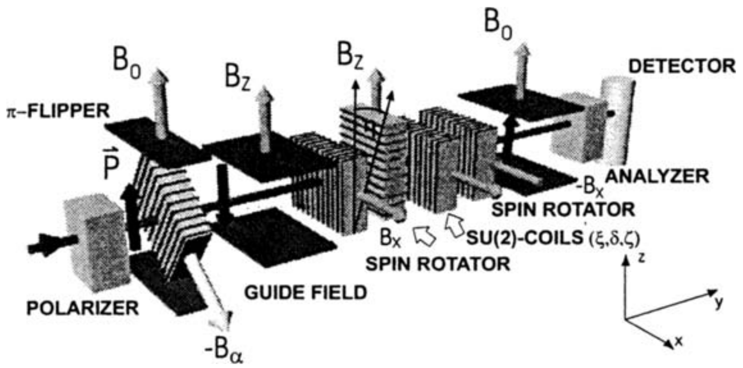


**Fig. 1.** Schematic sketch of the neutron polarimetric setup for mixed state geometric phase measurement, including evolution of the polarization vector

According to the spin rotation formalism using an unitary operator $\hat{U}$ denoted as

$$\hat{U}(\vec{\alpha}) = \exp\left(-i\frac{\hat{\vec{\sigma}}\cdot\vec{\alpha}}{2}\right) = \mathbb{1}\cos\frac{\alpha}{2} - i\left(\hat{\vec{\sigma}}\cdot\frac{\vec{\alpha}}{\alpha}\right)\sin\frac{\alpha}{2}, \tag{2}$$

the polarization vector $\vec{P}$ is rotated by an angle $\eta$ after the first coil and returns back to its initial orientation when passing the second coil. In order to obtain a Pancharatnam phase caused by a unitary SU(2) evolution given by

$$\hat{U}_{\mathrm{SU(2)}}(\xi,\delta,\zeta) = \begin{pmatrix} e^{i\delta}\cos\xi & -e^{-i\zeta}\sin\xi \\ e^{i\zeta}\sin\xi & e^{-i\delta}\cos\xi \end{pmatrix} \tag{3}$$

additional coils (SU(2) coils) are displaced between the two DC coils. Hence a continuous variation of the magnetic fields in the DC coils, resulting in a diversity of angles $\eta$, automatically yields the desired oscillations of the intensity required to calculate the Pancharatnam phase.

Using Eqs. (1)–(3) the intensity $I$ at the detector for a pure state is calculated as

$$\begin{aligned} I &= |\langle z_+|\ \exp\left(-i\tfrac{1}{2}\vec{\sigma}\hat{x}\eta\right)\cdot \hat{U}_{\mathrm{SU(2)}}(\xi,\delta,\zeta)\cdot \exp\left[-i\tfrac{1}{2}\vec{\sigma}\hat{x}(2\pi n-\eta)\right]|z_+\rangle|^2 \\ &= \cos^2\xi\cos^2\delta+\sin^2\xi\sin^2(\zeta+\eta) \end{aligned} \tag{4}$$

which is the same result for the oscillating intensity as developed in [16].

Therefore, the Pancharatnam phase $\Phi = \arg\langle +z|\hat{\mathrm{U}}_0|+z\rangle = \delta + \arg\cos\xi$ can be calculated modulo $\pi$ from

$$\Phi = \arccos\left(\sqrt{\frac{I_{\mathrm{min}}}{1-I_{\mathrm{max}}+I_{\mathrm{min}}}}\right). \tag{5}$$

For each SU(2) transformation several maxima and minima were measured by implementing a continuous variation of $\eta$. The values for $I_{\mathrm{max}}$ and $I_{\mathrm{min}}$ are calculated as an average of these maxima and minima.

The mixed state case is described by a density matrix in form of

$$\hat{\rho} = \tfrac{1}{2}(\hat{\mathbb{1}} + r\hat{\sigma}_z)\,, \tag{6}$$

with $r$ being the degree of polarization $r$ along the positive $z$ axis and the mixed state intensity given by

$$I^{\varrho} = \tfrac{1}{2}(1-r) + rI \tag{7}$$

with $\frac{1}{2}(1+r)$ as the probability of finding the system in the up state and $\frac{1}{2}(1-r)$ for the down state. For $r=1$, Eq. (7) is reduced to the pure state intensity. Using Eqs. (5) and (7) one obtains

$$\Phi = \arctan\left(r\tan(\delta+\arg\cos\xi)\right) \tag{8}$$

finally yielding the Pancharatnam phase for the mixed state case

$$\Phi = \arccos\left(\sqrt{\frac{\left(I^{\varrho}_{\mathrm{min}}-\frac{1}{2}(1-r)\right)/r}{r\left(\frac{1}{2}(1+r)-I^{\varrho}_{\mathrm{max}}\right)+\left(I^{\varrho}_{\mathrm{min}}-\frac{1}{2}(1-r)\right)/r}}\right), \tag{9}$$

which again does not differ from the calculated phases in [17].

For the implementation of the mixed states an additional spin flipper is attached to the polarizer. Hence the setup can be traversed by the neutron beam weather with the spin flipper turned on or turned off. Consequently for every angle $\eta$ two intensities are measured which are denoted as $I_{\mathrm{off}}$ for spin flipper off and $I_{\mathrm{on}}$ for spin flipper on. The density matrix for the mixed states is calculated as a weighted sum of $I_{\mathrm{off}}$ and $I_{\mathrm{on}}$ referring to a certain degree of polarization $r$. With a suitable combination of these two intensities an arbitrary degree of polarization can be calculated. Two guide fields, located in front and behind the two SU(2) coils, are used to suppress depolarization which leads to a loss of visibility.

The Pancharatnam relative phase can be identified with the non-cyclic geometric phase if the evolution is parallel transporting, which can be realized by a sequence of unitary transformations along great circles on the Bloch sphere. This can be achieved by an arrangement of several orthogonal wired coils, implementing rotation axis along the $\hat{x}$ and $\hat{z}$ direction and therefore defining the SU(2) parameters $(\xi, \delta, \zeta)$ introduced in Eq. (3). If $\hat{U}_{SU(2)}(\xi, \delta, \zeta)$ is parallel transporting the Pancharatnam phase is given as $\Phi = -\frac{1}{2}\Omega$, with $\Omega$ being the solid angle on the sphere enclosed by transport path and its shortest geodesic closure (see Fig. 2) and equals the non-cyclic geometric phase.

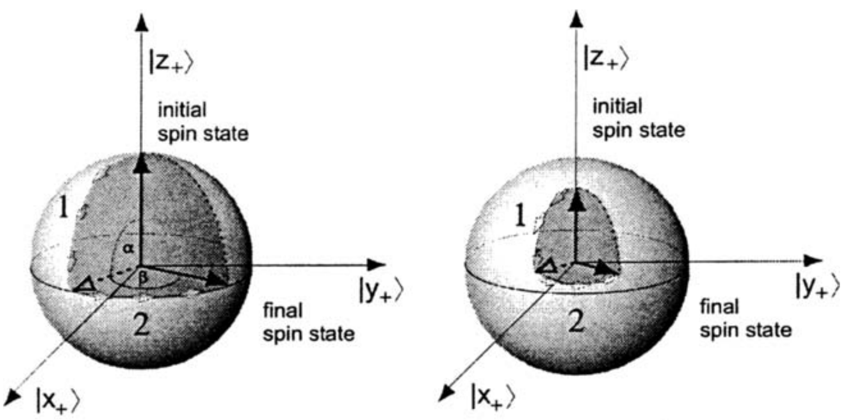


**Fig. 2.** Path on the Bloch sphere corresponding to the parameter set $(\xi, \delta, \zeta) = (\frac{\pi}{4}, \frac{\pi}{4}, -\frac{3\pi}{4}) \equiv \alpha = \beta = \pi/2$, for $r = 1$ (left) and $r < 1$ (right)

## *2.1. Modification of the setup*

We have already measured the Pancharatnam relative phase, which is reported in [15], this experiment considered the mixed state case as well. However, it is possible to perform the phase measurement by implementing a different, more sophisticated and robust version of the setup.

First, the extra phase-shift $\eta$ can be achieved not only by translating the pair of $\pi/2$-flippers at distance $L_0$ in a homogenous magnetic field, but also by substituting this variation of distances $L$ and $L'$ that correspond to the angles $\eta$ and $2\pi n - \eta$ by rotations through the same angles in two DC coils situated directly in front and behind the SU(2) coils. Inaccuracies due to the manual translation of the flipper pair at constant distance $L_0 = L + L'$ and magnetic field deviations of the capacious guide field are the major difficulties in performing the measurements described in [15]. The distance $n \cdot L_0$ was chosen to implement an integer multiple of a full rotation through $2\pi$.

Suppose that the coils are designed equally and connected in series with the poles exchanged at the second coil, i.e. the direction of the current and therefore of the field in the second coil is inverted at constant absolute value. A variation of the coil current automatically yields the phase-shift $\eta$ that causes the oscillations required to calculate the Pancharatnam phase $\Phi$ without the need to move components of the setup. Generally the spinor part of a neutrons state vector can be denoted as a superposition of 'up' and 'down' states referring to the chosen quantization axis, which was the $z$ axis in the case of the Wagh setup [16]. In the modified setup without the $\pi/2$-flippers, where the additional phase shift is implemented by rotations around the x-axis, it is appropriate to change to a description of the state vector by the superposition of 'up' and 'down' referring to the $x$ axis, i.e. $|+x\rangle$ and $|-x\rangle$.

Furthermore, it is not necessary to rotate the polarization into the $x$,$y$ plane. All the physical predictions of [16] remain correct for the implementation of $\eta$ simply by rotations in the $y$,$z$ plane. Thus, the pair of $\pi/2$-flippers can be omitted, avoiding again loss of intensity and accuracy, caused by absorption and limited adjustment ability.

## 2.2. Results

The experiments were carried out on the tangential beam tube of the 250 kW TRIGA research reactor of the Atomic Institute of the Austrian Universities, Vienna. The out coming neutron beam is monochromatized by a mosaic crystals made of pyrolitic graphite, selecting a wavelengths of 1.65 Å. A typical sinusoidal oscillation, defined by a given SU(2) parameter set, is depicted in Fig. 3. This oscillation was obtained by tuning the current in the DC coils.

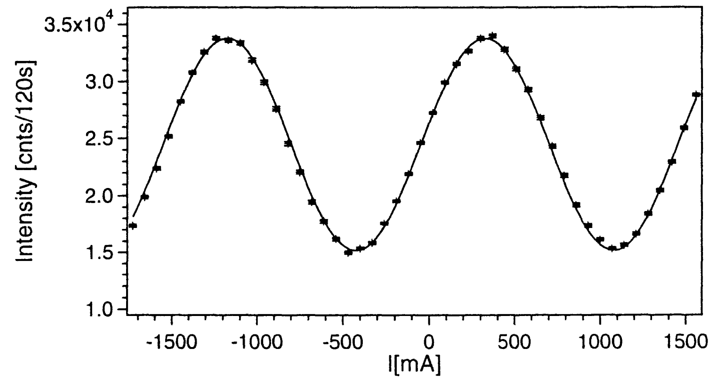


**Fig. 3.** Intensity fringes of the SU(2) parameter set $(\xi, \delta, \zeta) = (\frac{\pi}{4}, \frac{\pi}{4}, -\frac{3\pi}{4})$ caused by the variation of the magnetic field in the DC coils

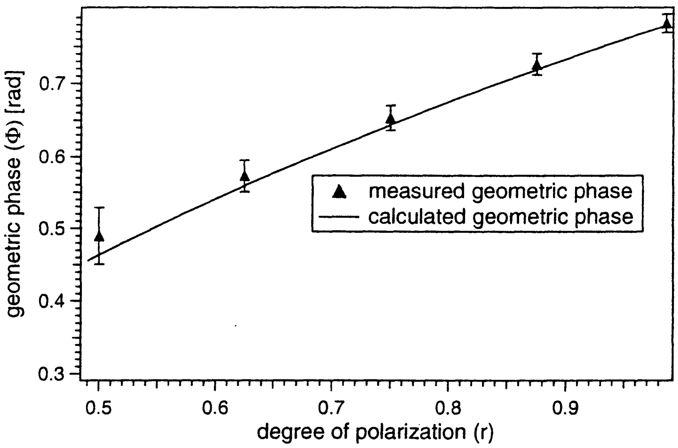


**Fig. 4.** Results for geometric phase corresponding to the parameter set $(\xi, \delta, \zeta) = (\frac{\pi}{4}, \frac{\pi}{4}, -\frac{3\pi}{4}) \equiv \alpha = \beta = \pi/2$, for a diversity of polarization degrees

For $r = 1$ the geometric phase equals $\pi/4$ for the given parameters $((\xi, \delta, \zeta) = (\frac{\pi}{4}, \frac{\pi}{4}, -\frac{3\pi}{4}) \equiv \alpha = \beta = \pi/2)$. However, in practice it is impossible to obtain $r = 1$, our initial degree of polarization was determined as $0.981 \pm 0.0027$. This value and the flipping ratio of the $\pi$-flipper ($48.043 \pm 1.5968$) are taken in consideration for all further calculations. The decline of the geometric phase versus the degree of polarization in respect to the calculated distribution, given by Eq. (8), is depicted in Fig. 4. The measured values and the theoretical predictions show a high correlation over a wide range of the polarization degree.

## 3. Future Perspectives

We are well aware of the fact that the above described method for creating a neutron beam in a mixed state does not fulfill all criteria that are considered crucial with respect to depolarization. There is no element of randomness in the mixing procedure since the relative frequencies for finding the system in either one of the pure states $|+z\rangle$ or $|-z\rangle$ are well known.

Nevertheless the states are based on measurements on the ensemble undergoing SU(2) transformation and can be described by a density operator $\hat{\rho}$ of the form of Eq. (6), thereby fulfilling the predictions developed by Sjöqvist et al. [10].

Hence we are now focusing on different methods for the creation of the mixed states. For instance placement of a randomly magnetized material in the beam trajectory, where the beam is completely depolarized up to the degree of covering of the beam cross section. Alternatively we are considering a slightly detuned RF flipper, thereby arbitrarily diminishing the degree of polarization.

## 4. Conclusions

The geometric phase acquired during a unitary SU(2) evolution of a spin-1/2 state was measured for the mixed state case using a neutron polarimetric setup. The results are in good agreement with theory, verifying the predicted values for the mixed state geometric phase.

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## References

1. M.V. Berry, *Proc. R. Soc. Lond. A* **392** (1984) 45.
2. A. Tomita et al., *Phys. Rev. Lett.* **57** (1986) 937.
3. T. Bitter and D. Dubbers, *Phys. Rev. Lett.* **59** (1987) 251.
4. Y. Aharonov and J.S. Anandan, *Phys. Rev. Lett.* **58** (1987) 1593.
5. J. Samuel and R. Bhandari, *Phys. Rev. Lett.* **60** (1988) 2339.
6. S. Pancharatnam, *Proc. Indian Acad. Sci. A* **44** (1956) 247.
7. Y. Hasegawa et al., *Phys. Rev. Lett.* **87** (2001) 070401.
8. N. Manini and F. Pistolesi, *Phys. Rev. Lett.* **85** (2000) 3067.
9. Y. Hasegawa et al., *Phys. Rev. A* **65** (2002) 05211.
10. E. Sjöqvist et al., *Phys. Rev. Lett.* **85** (2000) 2845.
11. S. Filipp and E. Sjöqvist, *Phys. Rev. A* **68** (2003) 042112.
12. S. Filipp and E. Sjöqvist, *Phys. Rev. Lett.* **90** (2003) 050403.
13. Y. Hasegawa and G. Badurek, *Phys. Rev. A* **59** (1999) 4614.
14. G. Badurek et al., *Phys. Rev. Lett.* **71** (1993) 307.
15. J. Klepp et al., *Phys. Lett. A* **342** (2005) 48.
16. A.G. Wagh and V.C. Rakhecha, *Phys. Lett. A* **197** (1995) 112.
17. P. Larsson and E. Sjöqvist, *Phys. Lett. A* **315** (2003) 12.