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Quantum Contextuality Induced by Spin-Path Entanglement in Single-Neutrons

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Abstract. Since Einstein, Podolsky and Rosen suggested in the thirties and nowadays known as an EPR-paradox, non-local correlations between sufficiently separated subsystems have been extensively discussed. From a theoretical point of view, such a non-locality can be interpreted as a consequence of the correlation between commuting observables. A more general concept, i.e. contextuality, compared to non-locality can be introduced to describe striking phenomena predicted by quantum theory. As the first example, a neutron interferometer experiment is presented, where the spin and the path degrees of freedom are used to exhibit the clear violation of a Bell-like inequality. Other aspects of the quantum contextuality is reported, e.g. a flavor of Kochen–Specker-like contradiction in neutron optical experiments. In addition, the quantum state tomography of the Bell states, which are used in the experiments, is shown.

Keywords: neutron, interferometer, contextuality, entanglement, Bell's inequality, Kochen–Specker theorem, state tomography
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1. Introduction

Since Einstein, Podolsky and Rosen (EPR) suggested that the quantum mechanics is incomplete due to its non-local correlations (nowadays known as the EPR-paradox)

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[1], quantum non-locality was one of the mysteries of quantum mechanics. Together with the Bell's inequalities [2], such a quantum non-locality has aroused considerable interests for many decades. Maybe due to its handling-achievability, most experimental tests of Bell's inequalities have been performed with correlated photon pairs [3]. It is appealing that the contradiction inherent in such non-local phenomena can easily be understood from our common experience by intuition. Within quantum terminology, this assumption of locality can be interpreted to result from independence of measurements of commuting observables due to the spatial separation of the measurements. A more general concept, i.e. contextuality, compared to non-locality is expected to describe striking phenomena predicted by quantum theory [4, 5]. Here, non-contextuality, a simple classical deduction for the quantum phenomena, implies that the result of a measurement is determined independently of the previous or simultaneous measurement of any set of mutually commuting observables [6].

In some cases, for perfect measurements non-contextual model logically contradicts quantum theory, as is first shown by Kochen and Specker in the thirties [4]. Since no experiment can be done in perfect circumstances, it is practically useful to consider the probabilistic model of non-contextuality. An inequality was introduced, which exhibits a violation by a factor of 2 [7].

Since its advent, neutron interferometric experiments have provided elegant demonstrations of the effects related to the fundamental aspects of quantum physics [8] for three decades. In particular, investigations of the property of spin-1/2 system, i.e. one of the simplest and the best-manipulatable quantum two-level system, have been carried out in a superior manner with the use of neutron interferometer. Recently, besides topological phase measurements [9–12], we have accomplished a polarized neutron optical experiment to demonstrate the violation of a Bell-like inequality in the study of the quantum contextuality [13]: the entanglement not between the particles but between the degrees of freedoms, e.g. the spin (internal) and the path (external) degrees of freedoms, is accomplished and interferometer experiments with enough high contrast, more than 71%, was achieved to show the violation of a Bell-like inequality.

2. Violation of a Bell-like Inequality

Here, we describe an experiment of single-neutron interferometry to show the violation of a Bell-like inequality [13]. In contrast to the conventional experiments with entangled particle pairs, the entanglement in our case is accomplished between different degrees of freedom of a single particle, i.e. spin and path. Observables of the spinor part commute with those of the spatial part, and this justifies the derivation of a Bell-like inequality equivalent to the rejection of the hypothesis of local realism [14]. The experiment consists of joint measurements of commuting observables for the spin and the path of single neutrons in an appropriately prepared non-factorizable so-called Bell state.

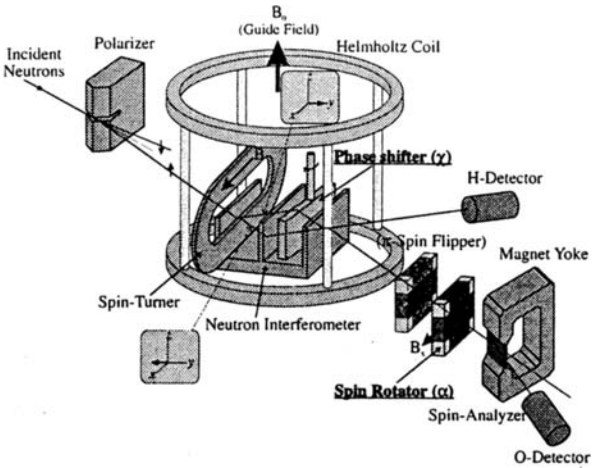

Fig. 1. Schematic view of the experimental setup to demonstrate the violation of a Bell-like inequality in single-neutron interferometry. The experiment consists of three sages: a preparation of the entangled state with the use of the spin-turner, a manipulation of the two parameters, a phase shift of χ and the spinor rotation angle of α together with a Heusler analyzer, and a detection

A schematic view of the experimental setup is depicted in Fig. 1. The important parameters for the manipulation are the spinor rotation angle, α and the relative phase, χ, between the two beams. A maximum violation of the Bell-like inequality is expected for setting the spinor rotation angle α at 0, $\pi/2$, π, and $3\pi/2$. Typical sinusoidal intensity modulations, obtained by varying the phase shift χ, are shown in Fig. 2. We achieved to obtain enough high contrasts, more than 70.7%, to accomplish the experiment.

After fitting to sinusoidal dependence by the least squares method, the expectation values E_{obs} were determined. We obtained E_{obs} $(0,\ 0.79\pi)$, to be 0.542 ± 0.007 from the four intensities. In the same manner, we obtained $E_{\text{obs}}(0,\ 1.29\pi) = 0.488 \pm 0.012$, $E_{\text{obs}}(0.5\pi,\ 0.79\pi) = -0.538 \pm 0.006$, and $E_{\text{obs}}(0.5\pi,\ 1.29\pi) = 0.483 \pm 0.012$. The Bell-like inequality, S' was calculated to be

$$\begin{aligned} S' &\equiv E'(\alpha_1, \chi_1) + E'(\alpha_1, \chi_2) - E'(\alpha_2, \chi_1) + E'(\alpha_2, \chi_2) \\ &= 2.051 \pm 0.019 \ > \ 2 \end{aligned} \tag{1}$$

for α_1, $\alpha_2 = 0$, 0.50π, and χ_1, $\chi_2 = 0.79\pi$, 1.29π, respectively. This clearly shows a violation of the Bell-like inequality, which results from the quantum contextuality.

3. Peculiarity of Quantum Contextuality in Neutron Optical Experiment

Next, we describe another neutron interferometric experiment which was originally intended to exhibit a Kochen–Specker-like phenomenon and conclusively demonstrate a contradiction due to quantum contextuality [15]. A polarized incident beam is split into two beams in the interferometer and, after appropriate preparation and manipulation, three product observables are measured. Since no experiment exhibit

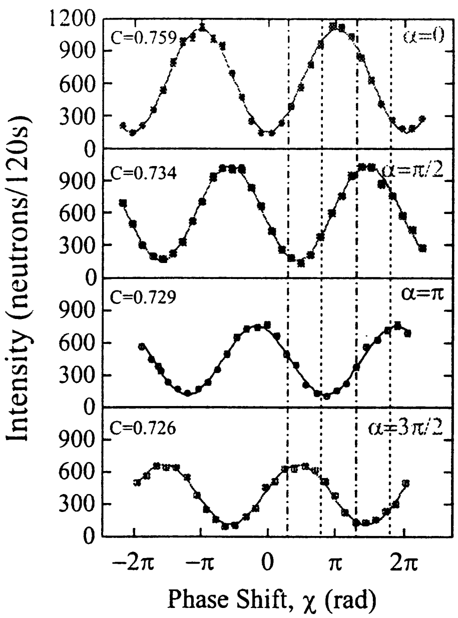

Fig. 2. Typical sinusoidal oscillations with spinor rotation angle $\alpha = 0$, $\pi/2$, π, and $3\pi/2$. Enough high contrasts were achieved. Expectation values, E_{obs}, were derived from the intensities of appropriate χ, where a maximum violation of the Bell-like inequality is expected. These values exhibited the final S' value of $2.051 \pm 0.019 > 2$: clear violation of the Bell-like inequality

perfect (anti-)correlations, inequalities are introduced to classify our experimental results and used for the analysis.

In our experiment, this total wave function $|\Psi\rangle$ is prepared in a Bell state $|\Psi\rangle = \frac{1}{\sqrt{2}}\{|\downarrow\rangle \otimes |I\rangle - |\uparrow\rangle \otimes |II\rangle\}$. We define Pauli-type operators for both the spin and the path degrees of freedom, e.g. $\hat{\sigma}_z^s = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$. The total wave function of neutrons $|\Psi\rangle$ satisfies the following eigenvalue equations: $\hat{\sigma}_l^s \hat{\sigma}_l^p |\Psi\rangle = -|\Psi\rangle$ for $l = x, y$. In addition, for a combination of product observables, one easily sees $(\hat{\sigma}_x^s \hat{\sigma}_y^p) \cdot (\hat{\sigma}_y^s \hat{\sigma}_x^p)|\Psi\rangle = -|\Psi\rangle$. Taking quantum non-contextuality into account, it follows from the above described eigenvalue equations that

$$\begin{cases} v[\hat{\sigma}_x^s] \cdot v[\hat{\sigma}_x^p] = -1\,, \quad v[\hat{\sigma}_y^s] \cdot v[\hat{\sigma}_y^p] = -1\,, \\ v[\hat{\sigma}_x^s \hat{\sigma}_y^p] \cdot v[\hat{\sigma}_y^s \hat{\sigma}_x^p] = v[\hat{\sigma}_x^s] \cdot v[\hat{\sigma}_y^p] \cdot v[\hat{\sigma}_y^s] \cdot v[\hat{\sigma}_x^p] = -1\,. \end{cases} \tag{2}$$

These equations cannot be satisfied simultaneously, since by multiplying the first two equations, one gets $v[\hat{\sigma}_x^s] \cdot v[\hat{\sigma}_y^p] \cdot v[\hat{\sigma}_y^s] \cdot v[\hat{\sigma}_x^p] = +1$, which conflicts with the last equation. The demonstration of the contradiction of a Kochen–Specker-like non-contextuality requires an experiment in perfect circumstances. No experiment, however, exhibits perfect correlation and/or anti-correlation in practice. Thus, we derive inequalities to analyze quantitatively our experimental results.

First, we define expectation values $E_l \equiv \langle \hat{\sigma}_l^s \hat{\sigma}_l^p \rangle$ $(l = x, y)$. They are rewritten in the form $E_l = p_l^+ - p_l^-$, where $p_l^\pm$ denote probabilities to obtain the value ± 1 and they satisfy a relation $p_l^+ + p_l^- = 1$. The four probabilities, obtained by two measurements of $\hat{X} \equiv \hat{\sigma}_x^s \hat{\sigma}_x^p$ and $\hat{Y} \equiv \hat{\sigma}_y^s \hat{\sigma}_y^p$, set a lower limit of the expectation value $E' \equiv \langle \hat{\sigma}_x^s \hat{\sigma}_y^p \cdot \hat{\sigma}_y^s \hat{\sigma}_x^p \rangle$ [15]:

$$E' \geq \{1 - (p_x^+ + p_y^+)\} - (p_x^+ + p_y^+) = -E_x - E_y - 1 \,. \tag{3}$$

This is the first inequality to be tested in the experiment.

In order to derive the second inequality, let us define a sum of product observables $\hat{C}$ as

$$\hat{C} \equiv \hat{I} - \hat{\sigma}_x^s \hat{\sigma}_x^p - \hat{\sigma}_y^s \hat{\sigma}_y^p - (\hat{\sigma}_x^s \hat{\sigma}_y^p) \cdot (\hat{\sigma}_y^s \hat{\sigma}_x^p) \,. \tag{4}$$

The non-contextual theories lead to [15]:

$$\langle C_{\rm NC} \rangle = |\langle \hat{I} \rangle - \langle \hat{\sigma}_x^s \hat{\sigma}_x^p \rangle - \langle \hat{\sigma}_y^s \hat{\sigma}_y^p \rangle - \langle \hat{\sigma}_x^s \hat{\sigma}_y^p \cdot \hat{\sigma}_y^s \hat{\sigma}_x^p \rangle| \leq 2 \,. \tag{5}$$

This is the second inequality which is considered as a testable consequence of the non-contextual theories. In contrast, the quantum theory predicts

$$\begin{aligned} C_{\rm QM} = \langle \Psi | \hat{C} | \Psi \rangle = 1 &- \langle \Psi | \hat{\sigma}_x^s \hat{\sigma}_x^p | \Psi \rangle - \langle \Psi | \hat{\sigma}_y^s \hat{\sigma}_y^p | \Psi \rangle \\ &- \langle \Psi | (\hat{\sigma}_x^s \hat{\sigma}_y^p \cdot \hat{\sigma}_y^s \hat{\sigma}_x^p) | \Psi \rangle = 4 \,. \end{aligned} \tag{6}$$

It is worth noting here that $C_{\rm QM}$ is twice as large as the higher limit of $C_{\rm NC}$.

The experiment was carried out in a similar condition as the former experiment: a schematic view of the experimental setup is shown in Fig. 1. In the first and the second measurements, the values of $E_x = \langle \hat{\sigma}_x^s \hat{\sigma}_x^p \rangle$ and $E_y = \langle \hat{\sigma}_y^s \hat{\sigma}_y^p \rangle$ were determined. With the use of the spin analyzer, neutrons are selected according to their spinor property, and χ was varied to obtain sinusoidal oscillations. In the third experiment, the value of $E_z = \langle \hat{\sigma}_z^s \hat{\sigma}_z^p \rangle$ was determined by the use of only one beam whereas the other was stopped with a beam-stopper and the polarization analysis was done by varying α

The maximum violation of the inequalities are expected for the measurements for $E_x \equiv \langle \hat{\sigma}_x^s \hat{\sigma}_x^p \rangle$, $E_y \equiv \langle \hat{\sigma}_y^s \hat{\sigma}_y^p \rangle$, and $E' \equiv \langle (\hat{\sigma}_x^s \hat{\sigma}_y^p) \cdot (\hat{\sigma}_y^s \hat{\sigma}_x^p) \rangle$, in our experiment $E_z \equiv \langle \hat{\sigma}_z^s \hat{\sigma}_z^p \rangle$. Typical intensity modulations are shown in Fig. 3. The expectation

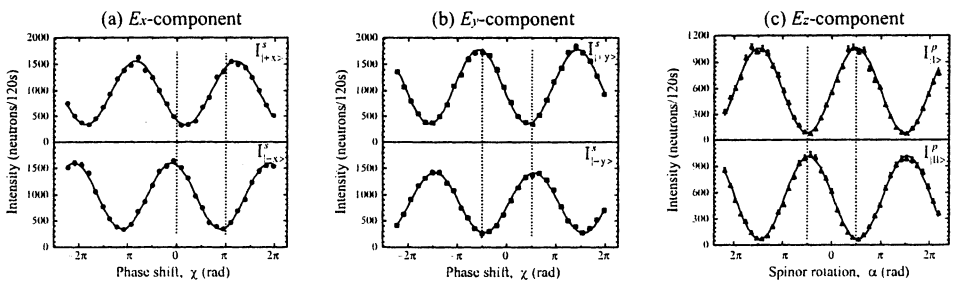

Fig. 3. Typical intensity modulations for various spin analysis. Expectation values, E_{obs}, were derived from the intensities of appropriate, where a maximum violation of the inequality is expected. The final results exhibit clear violation of the prediction by the non-contextual theory

values were calculated from the intensities of appropriate α and χ values. We obtained $E_x = -0.610 \pm 0.008$, $E_y = -0.667 \pm 0.008$, and $E' = -0.861 \pm 0.010$.

First, a contradiction between standard quantum mechanics and the non-contextual theory is attained by a fraction as large as 63% with the use of neutrons in our experiment: $E_x \cdot E_y = 0.407 > -0.861 = E_z$. Next, from the two values E_x and E_y, the non-contextual theory demands $E' \geq -E_x - E_y - 1 = 0.277 \pm 0.011$. In contrast, the measured value -0.861 ± 0.010 is evidently smaller than this lower limit. Finally, C' is calculated to be $C' = 1 - E_x - E_y - E_z = 3.138 \pm 0.015 > 2$. This again clearly shows the violation of the inequality derived by the non-contextual theory.

4. Quantum State Tomography of Neutron's Bell State

At last, we describe a neutron optical experiment to carry out a quantum state tomography of neutron's Bell state [16]. Neutron interferometric experiments exhibit some striking phenomena of quantum non-contextuality, as described above. There, a so-called Bell state, a product state of a spinor and a spatial wave function, plays a very important role. In order to characterize the neutron's Bell state, a quantum state tomography was accomplished. Here, we describe a preliminary result of the experiments.

A theory was proposed and an associated measurement was demonstrated to determine the state, the measurement of density matrices, of a pair of quantum two-level systems, e.g. qubits [17]. This strategy is applicable not only to a multi-particle entangled system, but also to a multi-entangled, i.e. between the degrees of freedoms, system in a single-particle. Namely, a tomographic reconstruction of a neutron's Bell state is accomplished, where the density matrix is determined from a set of measured quantities.

The experiment was carried out in a similar condition as the first experiment (see Fig. 1). A Bell-like state, $|\Psi_+\rangle = \frac{1}{\sqrt{2}}\{|\rightarrow\rangle \otimes |\mathrm{I}\rangle + |\leftarrow\rangle \otimes |\mathrm{II}\rangle\}$, was produced. The spin analysis was tuned to measure S_{+x}, S_{+y}, and $S_{\pm z}$ for all measurements. Then, in the first and the second measurements, the relative phase χ was adjusted to 0 and $\pi/2$: we obtained 8 count rates for joint measurements. In the third measurement, one of the beams in the interferometer was blocked one after the other: we obtained other 8 count rates for joint measurements. From these 16 count rates, 16 parameters are directly determined. Typical result of the quantum state tomography of a neutron's Bell-like state is shown in Fig. 4. A characteristic of the Bell state, i.e. four poles for the real part of the density matrix, is clearly seen.

By turning the interferometer slightly, it was easily possible to tune the incident polarization for $|\uparrow\rangle$ and $|\downarrow\rangle$: in turn, one obtains two Bell-like states, $|\Psi_+\rangle$ and $|\Phi_+\rangle$. Such experiments have already been done and fidelities of 0.79 and 0.75 were measured. In addition to these Bell-like states, we are interested in the state estimation of a (separable but useful) entangled state of neutrons, $\frac{1}{\sqrt{2}}\{|\uparrow\rangle \otimes |\mathrm{I}\rangle +$

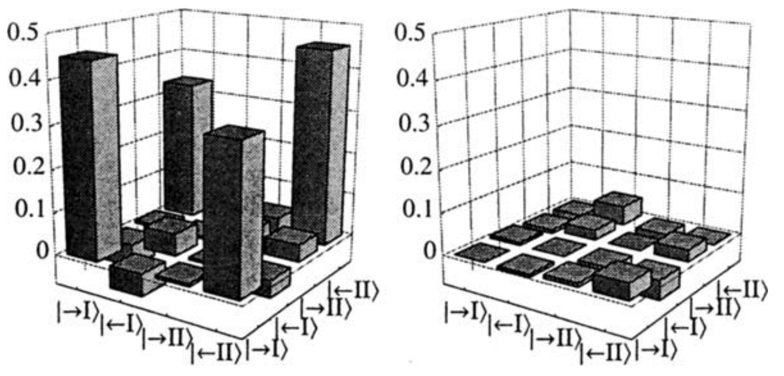

Fig. 4. Typical result of graphical representation of the quantum state tomography of a neutron's Bell-like state, $|\Psi_+\rangle = \frac{1}{\sqrt{2}}\{|\rightarrow\rangle \otimes |\mathrm{I}\rangle + |\leftarrow\rangle \otimes |\mathrm{II}\rangle\}$. Left: real part, and right: imaginary part of the density matrix

$|\uparrow\rangle \otimes |\mathrm{II}\rangle\}$, which was directly produced from the $|\uparrow\rangle$ polarized beam (without spinor manipulation). Here also the four poles for the real part of the density matrix were clearly seen and the fidelity of 0.91 was measured. We are going to use this entangled state of higher fidelity to investigate decoherence phenomena in quantum mechanics [18].

5. Concluding Remarks

We have accomplished neutron-interferometric experiments to exhibit some characteristics of quantum contextuality induced by the spin-path entanglement. The first example is concerning a Bell-like inequality: clear violation of the inequality was demonstrated ($S' = 2.051 \pm 0.019 > 2$) with the use of the entanglements not between the particles but between the degrees of freedoms of single-neutrons. The second one is originally intended to exhibit a Kochen–Specker-like phenomenon and conclusively demonstrate, not a statistical violation but a logical contradiction due to quantum contextuality. The obtained results showed a contradiction between standard quantum mechanics and the non-contextual theory by a fraction as large as 63%. In addition, inequalities are introduced in order to analyze quantitatively the results, which are again clearly violated: $E_x \cdot E_y = 0.407 > -0.861 = E_z$ and $C' = 1 - E_x - E_y - E_z = 3.138 \pm 0.015 > 2$. Finally, quantum state tomographies for two Bell-like states and another entangled state were successively accomplished. Characteristics of the Bell state, i.e. four poles for the real part of the density matrix, are clearly seen. We have obtained fidelities of 0.79 and 0.75 for two Bell-like states. In addition, another tomographical measurement of a (separable but useful) entangled state of neutrons, which was directly produced without spinor manipulation in the interferometer, was revealed to have four poles again and high fidelity of 0.91. We are going to utilize this state for further investigations of quantum mechanical phenomena, e.g. decoherence of an entangled system.

Acknowledgments

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