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Collective Cooling of Atoms in a Ring Cavity

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Abstract. We study the cooling effect induced by a transversely pumped ring cavity on the motion of N linearly polarizable particles, all of which are trapped in the same cavity. We derive the friction tensor including cross friction terms and compare it to the friction coefficient of single-atom cavity cooling. We find that atoms that are not trapped by the cavity field are cooled independently with the same efficiency as a single untrapped atom. For atoms self-trapped in the cavity field, however, collective effects kick in. These result in an N-fold increase of the friction on the center-of-mass mode, however, all other modes of motion are practically not cooled by the cavity. This shows that cavity cooling works efficiently for many particles cooled collectively, and allows us to reach the regime where the particles are deeply trapped in harmonic wells. There it has to be complemented by other cooling methods, e.g. sideband cooling.

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1. Introduction

Radiative properties of atoms are substantially modified if, instead of being in free space, the field is enclosed in a resonator. *Cavity quantum electrodynamics* (CQED) leads to new phenomena and applications in the subject of manipulating the gross motion of neutral massive particles by optical means. One example is cavity cooling, a recently predicted [1] and demonstrated [2] method of efficiently cooling atoms by means of dynamically varying cavity fields. Cavity cooling can in principle be applied to an arbitrary particle with linear polarizability and is therefore a major research topic in several laboratories and theoretical groups today.

Cavity-induced forces on a dilute gas of atoms are distinguished by the inherent role of many-body effects. The evolution of the whole system of cavity field and

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atoms cannot be interpreted in terms of independent single-atom processes. The effect of one atom on the field is felt by other atoms in the resonator on the fast time-scale of the photon round trip time. There is then a cavity-mediated indirect interaction between remote atoms. This intriguing aspect of cavity QED, for instance can yield a collective behavior of the atoms [3, 4]. The status of current experiments related to the collective light forces in various types of resonators is reviewed in [5].

Central question to cavity cooling is whether the friction mechanism survives the indirect atom–atom coupling and whether it can be extended to a cloud of atoms. One might expect that the cooling relies on delicate correlations between an atom and the field mode dynamics, which is disturbed by the fluctuating motion of other atoms. This paper is devoted to this problem, and we determine the linear friction tensor for a many-particle system in the ring-cavity geometry.

2. Correlated Atom–Field Dynamics

We consider the system composed of a gas of N polarizable particles coupled to the electromagnetic field of a high-Q ring resonator. We assume a one-dimensional ring cavity with two degenerate counter-propagating modes, described by the plane-wave mode functions $f_1(x) = e^{ikx}$ and $f_2(x) = e^{-ikx}$, with coherent amplitudes α_1 and α_2. This geometry corresponds to the experimental setups in [6–8]. The particles are driven by a pump laser oriented perpendicular to the cavity axis. The electric field in the cavity is then given by $E(x) = f_1(x)\alpha_1 + f_2(x)\alpha_2 + \eta_t/g$, where the last term corresponds to the pumping field with amplitude η_t that is assumed to be constant along the resonator axis 'x', and g is the atom–mode coupling constant. For simplicity, we consider the system only in one spatial dimension: the atoms are supposed to be confined near the resonator axis by e.g. a strong dipole trap.

The interaction is in the dispersive regime, i.e. the pump laser is very far detuned from all the resonance frequencies of the gas particles: no real excitations take place. In this limit the atoms can be treated as linearly polarizable particles described by the single atom light shift U_0 related to the atomic properties by $U_0 = -\omega_C \chi'/\mathcal{V}$, where χ' is the real part of the linear polarizability and $\mathcal{V}$ is the cavity mode volume. The atoms redistribute photons by coherent scattering between the two modes and the pump field. This process feeds the cavity modes with an effective amplitude $\eta = U_0\eta_t/g$. The dynamics of the cavity modes with the scatterers at x_j, $j = 1...N$ is given by the following differential equations [9]:

$$\frac{d}{dt}\,\boldsymbol{\alpha} = \mathbf{A}\,\boldsymbol{\alpha} - i\eta \sum_{j=1}^{N} \mathbf{f}^*(x_j)\,, \tag{1}$$

using a formal vector notation for the mode amplitudes $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)^T$, for the mode functions $\mathbf{f}(x) = (f_1(x), f_2(x))^T$ and for the coupling matrix

$$\mathbf{A} = \begin{pmatrix} i(\Delta_C - NU_0) - \kappa & -iNU_0\sigma \\ -iNU_0\sigma^* & i(\Delta_C - NU_0) - \kappa \end{pmatrix}. \tag{2}$$

The diagonal terms include the detuning between the pump laser and the cavity modes, $\Delta_C = \omega - \omega_C$, the cavity decay rate κ, and the shift of the cavity resonance due to the forward scattering of the photons with an amount of U_0 per atom. The total frequency shift due to the polarized gas is NU_0. The off-diagonal terms give the coupling between the cavity modes that stems from the stimulated back reflection off the gas. This process is heavily dependent on the positions of the atoms, through the complex parameter $\sigma = \frac{1}{N}\sum_j e^{-2ikx_j}$ describing spatial order. Similar to the Debye–Waller factor, $|\sigma|$ is 1 if the gas forms a perfect lattice with period of an integer multiple of $\lambda/2$, and less than 1 for a non-perfect lattice; for a homogeneous gas, it is $|\sigma| \propto 1/\sqrt{N}$. The phase of σ gives x_0, the "position of the lattice" modulo $\lambda/2$, with the definition $\sigma = |\sigma|\, e^{-2ikx_0}$. The importance of this parameter in a ring cavity, and the conditions of thermodynamic stability for various spatial distributions are discussed in a recent paper [10].

The correlated dynamics of the field and the atoms emerges when we consider the force exerted by the cavity field on each atom. If the pumping field is far detuned from the atomic resonance, scattering into other than the cavity modes is negligible therefore the light force acting on the atoms is solely the dipole force [9]:

$$\dot{p}_i = 4\hbar k U_0 \,\mathrm{Im}[e^{2ikx_i}\alpha_1\alpha_2^*] + 2\hbar k\,\mathrm{Im}[\eta\,(e^{ikx_i}\alpha_1 - e^{-ikx_i}\alpha_2)]\,. \tag{3}$$

The dipole force originates from the coherent redistribution of photons between the optical modes. The first term is due to the photon scattering between the two cavity modes, and the second term comes from the scattering between the pumping mode and each cavity mode.

3. Friction on Slowly Moving Atoms

Moving atoms can be cooled down through the photon loss channel of a high-Q cavity via the dipole force Eq. (3). Friction could be explained on the ground of the coupled atom–field dynamics considering that the influence of the atomic motion on the cavity field appears with the characteristic time lag $1/\kappa$. The back action of the field on the atom is non-adiabatic, hence a friction force can effectively occur for certain parameter settings [1,9].

We intend to reveal the linear velocity dependence of the force in the regime of small atomic velocities ($kv \ll \kappa$), when the atoms move much less than a wavelength during the characteristic time $1/\kappa$ of the cavity field dynamics. In this limit we expand the mode amplitudes $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)^T$ up to first order in each v_j, the velocity of the jth atom:

$$\boldsymbol{\alpha} = \boldsymbol{\alpha}^{(0)} + \sum_{j=1}^{N} v_j\, \boldsymbol{\alpha}_j^{(1)}. \tag{4}$$

The field amplitudes depend on the position of the atoms therefore the total time derivatives are given by the usual expression:

$$\frac{d}{dt}\boldsymbol{\alpha}(t, x_1, \ldots, x_N) = \frac{\partial}{\partial t}\boldsymbol{\alpha} + \sum_{j=1}^{N} v_j \frac{\partial}{\partial x_j}\boldsymbol{\alpha}\,. \tag{5}$$

We substitute Eq. (5) and Eq. (4) into Eq. (1), to obtain the field dynamics up to first order in the small velocities v_j. The quasi-stationary solution of the field amplitudes can be obtained by setting the partial time derivatives in Eq. (5) to 0. Since in the "slow atomic motion" limit $\kappa^{-1} \ll (kv)^{-1}$, the field follows the motion of the atoms almost adiabatically. The quasi-stationary value $\boldsymbol{\alpha}^{(0)}$ is the adiabatic approximation, whereas the quasi-stationary value of $\boldsymbol{\alpha}^{(1)}$ furnishes the first order correction to the adiabaticity, and is the key to obtaining the friction force:

$$\boldsymbol{\alpha}^{(0)} = i\eta\, \mathbf{A}^{-1} \sum_i \mathbf{f}^*(x_i)\,, \tag{6}$$

$$\boldsymbol{\alpha}_j^{(1)} = \mathbf{A}^{-1} \frac{\partial}{\partial x_j}\boldsymbol{\alpha}^{(0)} = i\eta \mathbf{A}^{-1} \frac{\partial}{\partial x_j} \mathbf{A}^{-1} \sum_l \mathbf{f}^*(x_l)\,, \tag{7}$$

where the inverse of the coupling matrix $\mathbf{A}$ reads:

$$\mathbf{A}^{-1} = \frac{1}{D} \begin{pmatrix} i(\Delta_C - NU_0) - \kappa & iNU_0\sigma \\ iNU_0\sigma^* & i(\Delta_C - NU_0) - \kappa \end{pmatrix}, \tag{8}$$

and D is the determinant of $\mathbf{A}$:

$$D = \det \mathbf{A} = (i(\Delta_C - NU_0) - \kappa)^2 + N^2 U_0^2 |\sigma|^2. \tag{9}$$

We express the friction force in the following form,

$$F_i^{(1)} = -\sum_{j=1}^{N} \beta_{ij}\, v_j \tag{10}$$

that defines the friction tensor β_{ij} describing the friction force on the ith atom due to the motion of the jth one. Substituting the velocity dependent mode amplitudes Eq. (4) into the expression of the force Eq. (3), and keeping just the leading order in velocity one can end up with the formal expression for the friction tensor:

$$\beta_{ij} = -4\hbar k U_0 \operatorname{Im}\left\{ (\boldsymbol{\alpha}^{(0)})^T \begin{pmatrix} 0 & e^{2ikx_i} \\ -e^{-2ikx_i} & 0 \end{pmatrix} (\boldsymbol{\alpha}_j^{(1)})^* \right\} \tag{11}$$

$$- 2\hbar k \operatorname{Im}\left\{ \eta\, (e^{ikx_i}, -e^{-ikx_i}) \boldsymbol{\alpha}_j^{(1)} \right\}.$$

Investigating the properties of this friction tensor reveals important results on the cavity cooling of a large ensemble of atoms by a ring cavity.

4. Collective Cooling

The cooling process of many atoms in a driven single-mode standing-wave cavity was numerically simulated and a linear slowing down with increasing number of atoms was obtained in [11]. A closer inspection of the data revealed that the atom number influenced the cavity photon number in a way which led to the decrease of the cooling force. With parameters ensuring a photon number closely constant, the cooling efficiency was predicted not to change. The possibility of collective cooling has been found for a modified geometry in [3]: there, instead of driving the cavity, the atoms are directly illuminated from the side. However, no analytical derivation has been provided to confirm and discuss this effect. For well trapped particles oscillating closely harmonically, a simplified model has been solved [12]. In the present article we calculate the friction force on many atoms in a transversely pumped ring cavity without any assumption on their spatial distribution.

4.1. Single atom friction

For reference, let us first derive the friction coefficient for a single slowly moving atom in the cavity. This also supplies us the diagonal elements of the friction tensor of the N-atom case. We are interested in the limit where the coupling between the atom and the resonator modes is small, meaning that $|U_0| \ll \kappa$. To leading order in the small parameter U_0/κ, we find that the friction coefficient, averaged over one wavelength, is given by:

$$\beta = \frac{8\hbar k^2 |\eta|^2 \kappa(-\Delta_C)}{(\kappa^2 + \Delta_C^2)^2} \,. \tag{12}$$

Remembering that $\eta = U_0 \eta_t / g$, the friction constant is proportional to U_0^2. Similar results have been obtained for a symmetrically pumped ring cavity [14], and also for other geometries [13].

The friction coefficient β, of Eq. (12), is depicted in Fig. 1 in the limit $|U_0| \ll \kappa$ as a function of the cavity detuning Δ_C, in units of $8\hbar k^2 \eta^2 / \kappa^2$. The result shown in Fig. 1 supports a simple view of the cooling mechanism. When $\Delta_C < 0$, i.e. $\omega < \omega_C$, the frequency of the photons scattered from the pump to the cavity is converted slightly upwards at the expense of the kinetic energy of the atom — this results in a cooling force on the atoms. For blue detuning with respect to the cavity, $\Delta_C > 0$, pump photons lose energy when scattered into the cavity, and therefore the scattering atoms are heated. Note, however, that this simple interpretation of the cooling mechanism can be invoked only in the limit of large atomic detuning. We also remark that $\beta \Delta_C < 0$ is also valid if the first-order corrections to Eq. (12) in U_0/κ are taken into account.

As seen in Fig. 1, a good choice of the cavity detuning that warrants effective single-atom cooling is $\Delta_C \approx -\kappa$. Let us now compare the friction obtained with that choice to free-space Doppler cooling. Substitution into Eq. (12) gives:

$$\Delta_C = -\kappa \quad \Longrightarrow \quad \beta = 2\hbar k^2 \frac{|\eta|^2}{\kappa^2} = 2\hbar k^2 P_e \frac{g^2}{\kappa^2} \,. \tag{13}$$

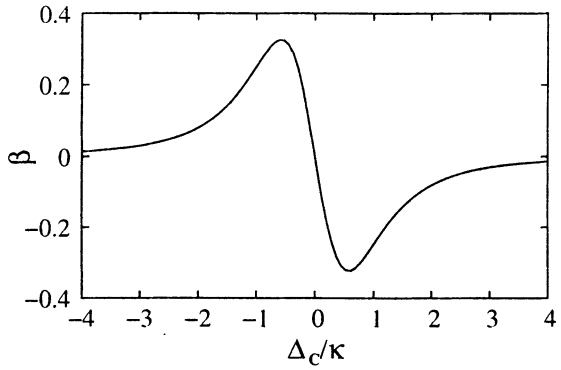

Fig. 1. The averaged one atom friction as a function of Δ_C/κ under the assumption $|U_0| \ll \kappa$. β is in arbitrary units

Neglecting a constant factor we obtain friction g^2/κ^2 times larger than the friction force of the Doppler-cooling in free space, $2\hbar k^2 P_e$, where P_e is the mean excitation of the atom. The factor g^2/κ^2 can be much larger than 1 in the so called strong coupling regime, where the cooling process is thus accelerated owing to the resonator.

4.2. Collective friction for many atoms

Loading many atoms into the cavity, they determine together the total cavity field, therefore the friction force on any single atom is influenced by the motion of the other atoms. Cross friction terms occur as expressed by Eq. (10). The linear friction force on the ith atom as a function of the velocity of the jth atom is defined by the elements of the friction tensor β_{ij} that is given by Eq. (11).

For more than one atom the determinant D of the coupling matrix becomes dependent on the position of the atoms. However, performing our calculations in the regime $|U_0| \ll \kappa$ and further assuming that NU_0/κ is still a small parameter (even collective coupling is weak), the determinant D would depend only on the atomic positions in second order of NU_0/κ that could be neglected.

Up to the lowest order of U_0/κ and NU_0/κ the friction tensor has the following form:

$$\beta_{ij} = \frac{8\hbar k^2 \eta^2 \kappa(-\Delta_C)}{(\kappa^2 + \Delta_C^2)^2} \cos\, k(x_i - x_j)\,, \tag{14}$$

which is the main result of this paper. The elements depend on the positions as the cosine function of the distance between the corresponding atoms i and j. One can easily deduce the following important consequences.

- The diagonal elements β_{ii} up to this order are the same as the one atom friction coefficient, see Eq. (12), that is depicted in Fig. 1.

- The matrix β_{ij} is symmetric and real, thus it has an orthogonal eigenvector system, and furthermore it is a positive semidefinite matrix, so it has non-negative eigenvalues. This last property ensures that none of the eigenmodes of the atomic motion would be heated.

- The trace of the friction tensor is N times the one atom friction coefficient, so in average the one atom friction is recovered. However, as we see later, for deeply trapped atoms this average is not representative: the center-of-mass mode is cooled N times more efficiently than a single atom, all other modes are cooled only poorly.

The result of Eq. (14) leads to further conclusions concerning the effectiveness of cavity cooling in a ring cavity. Atoms in a thermal cloud *not trapped by the cavity field* move with random velocities with respect to each other. This already decreases the importance of cross-friction. Furthermore, note that as the atoms move freely, it is justified to average Eq. (14) over the positions of the atoms. Since the off-diagonal terms of the friction coefficient matrix are proportional to cos $k(x_i - x_j)$, these disappear after averaging. Thus, untrapped particles cool independently with the rate of the single atom friction coefficient. Cavity cooling is suitable to remove the kinetic energy of the atoms until their motion is synchronized, allowing them to radiate energy into the cavity in phase and be self-trapped in the dipole potential wells (see Refs. [4,10] for an analytical treatment of this interesting phase transition).

Once the atoms are *trapped by the cavity field*, they are localized in a lattice, such that $x_j = x_0 + n_j\lambda + \xi_j$, with n_j integer and $|\xi_j| \ll \lambda$ for every $j = 1, \ldots, N$. The parameter x_0 fixes the position of the lattice, and the order parameter is $|\sigma| \approx 1$. Now all elements of β_{ij} are equal to the single atom friction coefficient β (but not to the position-averaged formula of Eq. (12) up to first order in the ξ_j. Such a square matrix with all elements identical can be written into the form of a dyadic product, $\boldsymbol{\beta} = \beta\,\mathbf{d}^T \circ \mathbf{d}$, where $\mathbf{d}$ is the N-dimensional vector consisting of 1's: $\mathbf{d} = (1, \ldots, 1)$. The only nonzero eigenvalue of the matrix corresponds to this eigenmode. Physically, this means that only the center-of-mass $X = \sum_i x_i$ of the atom cloud is subjected to friction, albeit to a very strong one, with coefficient $N\beta$. All the other modes are undamped (but not heated) as the atoms are more and more localized in the lattice.

5. Conclusions

On studying the friction tensor of a many-particle system in a transversely pumped ring cavity, we found that cavity cooling remains efficient until the particles are strongly trapped in the harmonic oscillation regime. This cooling mechanism can therefore be used for ensembles to reach the strongly bound vibrational regime where further cooling can be provided, e.g. by sideband cooling method [15,16].

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