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HEAVY ION PHYSICS

Multiparticle Production Processes from the Information Theory Point of View

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Abstract. We look at multiparticle production processes from the Information Theory point of view, both in its extensive and nonextensive versions. Examples of both symmetric (like pp or AA) and asymmetric (like pA) collisions are considered showing that some ways of description of experimental data used in the literature are of more general validity than usually anticipated.

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1. Introduction

The multiparticle production processes are usually first approached by means of statistical models [1] in order to make quick estimations of such parameters as temperature T or chemical potential μ of the hadronizing matter (with tacit assumptions that T and μ have the usual meaning in the realm of hadronic production). It means that "thermal-like" (i.e. exponential) form of relevant distribution (in transverse momentum p_T or in rapidity y) is used to fit data,

$$f(X) \sim \exp\left[-X/\Lambda\right] . \tag{1}$$

This formula occurred to be very robust, mainly because (cf., [2]) the $N-1$ unmeasured particles act as a *heath bath* which action on the observed particle is described by a single parameter Λ identified (for system in thermal equilibrium) with temperature T. Discrepancies from (1) observed in many places are then attributed to

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the fact that in reality such a "heath bath" is neither infinite nor homogeneous and one needs more parameters for its description. The minimal extension would be to regard Λ as being X-dependent, for example, $\Lambda \simeq \Lambda_0 + a \cdot X$ [3], what results in

$$f_q(X) \sim \exp_q\left[-X/\Lambda_0\right] \stackrel{\rm def}{=} \left[1-(1-q)X/\Lambda_0\right]^{1/(1-q)}, \qquad \text{where} \qquad q = 1 + a\,. \quad (2)$$

In the case when X is energy E and Λ is temperature T the coefficient a is just the inverse heat capacity, $a = 1/C_V$ [3]. Such a phenomenon leads to the so-called *nonextensivity* with q being the *nonextensivity parameter*. Notice that for $q \rightarrow 1$ (or for $a \rightarrow 0$) $f_q(X)$ becomes $f(X)$. We show here that Eqs. (1) and (2) originate in a natural way from information theory (IT) approach in its, respectively, extensive (based on Shannon entropy) and nonextensive (based on Tsallis entropy) forms. In what follows (Section 2) we shall briefly present this approach illustrating it (in Sections 3 and 4) with some fits to the existing data.

2. Information Theory and Multiparticle Production Processes

To introduce IT in the present context let us consider typical situation: experimentalists obtain some new and intriguing data. Immediately these data become subject of interest to theoreticians and in no time a number of distinctive and unique (as concerns assumptions and physical concepts) explanations is presented, which, disagreeing about physical concepts used, nevertheless all fit these data. The natural question arises: *which of the proposed models is the right one?* The answer is: *all of them* (to some extent). This is because experimental data are providing only limited amount of information and all models mentioned here are able to reproduce it. To quantify this reasoning one has to define, using IT,[a] the notion of information. Its extensive version is based on the Shannon information entropy,

$$S = -\sum_i p_i \, \ln p_i\,, \qquad (3)$$

where p_i denotes probability distribution of interest. The *least possible information*, corresponding to equal probability distribution of N states, $p_i = 1/N$, results in *maximal entropy*, $S = \ln N$. The opposite situation of *maximal information*, when only one state is relevant (i.e. $p_l = 1$ and $p_{i\neq l} = 0$) results in *minimal entropy*, $S = 0$. Denoting by $\langle R_k\rangle$ *a priori* information available on experiment, like conservation laws and results of measurements of some quantities R_k, one is thus seeking probability distribution $\{p_i\}$ such that: i) it tells us *the truth, the whole truth* about our experiment, i.e. in addition to being normalized it reproduces the known results:

$$\sum_i p_i = 1 \qquad \text{and} \qquad \sum_i p_i\, R_k(x_i) \;=\; \langle R_k\rangle\,, \qquad (4)$$

and ii) it tells us *nothing but the truth* about our experiment, i.e. it conveys *the least information* (only the information contained in this experiment).

To find such $\{p_i\}$ one has to *maximize* the Shannon entropy under the above constraints (therefore this approach is also known as MaxEnt method). The resultant distribution has familiar exponential shape

$$p_i \,=\, \frac{1}{Z}\cdot\exp\left[-\sum_k \lambda_k\cdot R_k(x_i)\right]\,. \tag{5}$$

Although it looks identical to the "thermal-like" (Boltzmann–Gibbs) formula (1) there are *no free parameters* here because Z is just normalization constant assuring that $\sum p_i = 1$ and λ_k are the corresponding Lagrange multipliers to be calculated from the constraint equations (4).[b]

It is worth to mention at this point [4] that the most probable multiplicity distribution $P(n)$ in the case when we know only the mean multiplicity $\langle n\rangle$ of distinguishable particles is geometrical distribution $P(n) = \langle n\rangle^n/(1+\langle n\rangle)^{(n+1)}$ (which is broad in the sense that its dispersion is $D(n)\sim\langle n\rangle$). Additional knowledge that all these particles are indistinguishable converts the above $P(n)$ into Poissonian form, $P(n) = \langle n\rangle^n\exp(-\langle n\rangle)/n!$, which is the narrow one in the sense that now its dispersion is $D(n)\sim\sqrt{\langle n\rangle}$. In between is situation in which we know that particles are grouped in k equally strongly emitting sources, in which case one gets Negative Binomial distribution [7][c]

$$P(n) = \Gamma(k+n)/\left[\Gamma(n+1)\Gamma(k)\right]\cdot\left(\frac{k}{\langle n\rangle}\right)^k \Big/ \left[1+\frac{k}{\langle n\rangle}\right]^{k+n}.$$

The other noticeable example provided in [8] is the use of IT to find the minimal set of assumptions needed to explain all multiparticle production data of that time. They were equally well described by models like multi-Regge, uncorrelated jet, thermodynamical, hydrodynamical etc., which, after closer scrutiny, turned out to share (in *explicit* or *implicit* manner), two basic assumptions: i) that only part $W = K\cdot\sqrt{s}$ $(0<K<1)$ of the initially allowed energy $\sqrt{s}$ is used for production of observed secondaries (located mostly in the center part of the phase space; in this way *inelasticity* K found its justification [12], it turns out that $K\sim 0.5$); ii) that transverse momenta of produced particles are limited and the resulting phase space is effectively one-dimensional (dominance of the longitudinal phase space). All other assumptions specific for a given model turned out to be spurious.[d]

Suppose now that some new data occur which disagree with the previously established form of $\{p_i\}$. In IT approach it simply signals that there is some additional information not yet accounted for. This can be done either by adding a new constraint (resulting in new λ_{k+1}, cf., for example [9], we shall not discuss it here) or by using some other form of IT, for example its nonextensive version (IT_q). The later is necessary step for systems which experience long range correlations, memory effects, which phase space has fractal character or which exhibit some intrinsic dynamical fluctuations of the otherwise intensive parameters (making them extensive ones, like T here). Such systems are truly *small* because the range of changes is of the order of their size. In this case the Shannon entropy (3) is no longer a good measure of information and should be replaced by some other measure. Out of the infinitely many possibilities [10] we shall choose the Tsallis entropy [11],

$$S_q \,=\, -\frac{1}{1-q}\sum_i\,(1-p_i^q) \quad \stackrel{q\to 1}{\Longrightarrow} \quad S_{q=1} = -\sum_i p_i\,\ln p_i\,, \tag{6}$$

which goes over to Shannon form (3) for $q\to 1$. The $\{p_i^{(q)}\}$ are obtained in the same way as before but with modified constraint equation:[e]

$$\sum_i\,[p_i]^q\;R_k(x_i)\;=\;\langle R_k^{(q)}\rangle\,. \tag{7}$$

This leads formally to the same formula for $p_i = p_i^{(q)}$ as in Eq. (5) but with $Z\to Z_q$ and $\exp(\dots)\to \exp_q(\dots)$. Because such an entropy is nonextensive, i.e. $S_{q(A+B)} = S_{qA} + S_{qB} + (1-q)S_{qA}\cdot S_{qB}$ [11], the whole approach became known as nonextensive (Tsallis) statistics. It should be stressed at this point that the nonextensivity parameter q cumulates action of all possible dynamical sources causing deviation from the usual Boltzmann–Gibbs statistics or Shannon entropy and as such can be considered as a useful phenomenological parameter allowing to describe data without deciding which of dynamical models leading to the same q is the right one [14]. However, in what follows we shall mainly address one possible sources of $q\neq 1$, namely intrinsic fluctuations present in the system represented by fluctuations of the $1/\Lambda$ parameter. This is because, as shown in [15], in this case

$$q\;=\;1\,\pm\,\left[\left\langle (1/\Lambda)^2\right\rangle\,-\,\langle 1/\Lambda\rangle^2\right]/\,\langle 1/\Lambda\rangle^2\,, \tag{8}$$

i.e. the parameter q is a measure of such fluctuations (with $\langle\dots\rangle$ denoting the respective averages over them.[f])

3. Confrontation with Experimental Data — Symmetric Case

We shall now confront these ideas with reality. At first we shall remind shortly main results of our recent investigations of hadronizations taking place in collisions of symmetric systems like pp and p$\bar{\text{p}}$ [12,18,19], heavy ions AA [5,20,21] or e^+e^- [22]. This will be followed by some new results on collisions of asymmetric systems exemplified by pA collisions. It must be stressed that what we are proposing is not a new model but rather sets of *least biased, most probable* distributions describing data by accounting for available information provided in terms of constraints emerging from conservation laws and from some previously known experimental facts (or from some assumed dynamical input, which is thus confronted with experimental data).

Hadronization means that some invariant energy M (assumed to be known) gives rise to a number N of secondaries (also assumed to be known) and question asked is: in what way these secondaries are distributed in the allowed phase space?

So far distributions in transverse momenta p_T were treated separately [5, 21] from distributions in rapidity [5, 12, 18–20], for which it was always assumed that mean transverse mass $\mu_T = \sqrt{m^2 + \langle p_T \rangle^2}$ was known, i.e. that we are working in the effectively 1-dimensional phase space.

Concerning p_T distributions, it has been shown that using q-statistics (with $q \simeq 1.05$ for AA collisions [21] to $q \simeq 1.1$ for $\bar{\rm p}$p collisions [5]) one can describe data in large domain of p_T, see, for example, Fig. 1. The characteristic feature is that now values of q are much smaller than those obtained fitting data in longitudinal phase space (cf. [5] for discussion). Following [15, 21] (see also [3, 16, 17]) we argue that $q > 1$ shows existence of the fluctuations of temperature in the hadronizing system mentioned at the beginning. In fact, the $q = 1.015$ for AA collisions [21] corresponds (according to (8)) to fluctuation of T of the order $\Delta T/T \simeq 0.12$, which do not vanish with increasing multiplicity [15]. These fluctuations exist in small parts of the hadronic system with respect to the whole system, they are not of the event-by-event type, for which $\Delta T/T \sim 0.06/\sqrt{N} \rightarrow 0$ for large N. It should be stressed at this point that such fluctuations are very interesting [23] because they provide a direct measure of the total heat capacity of the system, C:

$$\frac{\sigma^2(\beta)}{\langle \beta \rangle^2} \stackrel{\rm def}{=} \frac{1}{C} = q - 1\,. \tag{9}$$

In fact, because C grows with the collision volume V of reaction we expect that $q(\text{hadronic}) > q(\text{nuclear})$ which seems to be indeed observed (cf. also [14] where nuclear collisions data at different centralities providing direct access to volume V were analyzed).

When applied to rapidity distributions IT method leads to formula formally identical with formulas used in statistical models (cf., for example, like [24]),

$$f_N(y) \;=\; \frac{1}{Z} \cdot \exp\left[-\beta \cdot \cosh y\right]\,, \quad \text{where} \quad Z = \int_{-Y_M}^{Y_M} dy \exp\left[-\beta \cdot \cosh y\right]\,. \tag{10}$$

However, whereas in [24] (and in other similar models) $1/Z$ and β were just two free parameters, here Z is a normalization constant and $\beta = \beta(M, N, \mu_T)$ is obtained by solving constraint equation,[g]

$$\int_{-Y_M}^{Y_M} f_N(y) = 1 \qquad \text{and} \qquad \int_{-Y_M}^{Y_M} dy\left[\mu_T \cdot \cosh y\right] \cdot f_N(y) = \frac{M}{N}\,. \tag{11}$$

It means that the parameter β (inverse of the so-called *partition temperature* introduced in [24] type of models) is connected (via IT method used) with the *dynamical input* given by: the allowed energy M (usually taken as a fraction, $M = K\sqrt{s}$, of the total energy of reaction with parameter $K \in (0, 1)$ being the so-called *inelasticity* of the reaction), the number of secondaries produced N and the mean transverse mass μ_T. In asymmetric collisions to be discussed in the next section one would have to add also constraint imposed by momentum conservation (which is satisfied automatically in the case of symmetric collisions discussed at the moment).

As detailed in [4, 12], $f_N(y)$ changes from $f_{N=2}(y) = \frac{1}{2}[\delta(y-Y_M)+\delta(y+Y_M)]$ (with $\beta(N \to 2) \to -\infty$) via $f_{N_0} = \text{const.}$ (with $\beta(N_0 \simeq \ln(M/\mu_T)) \simeq 0$) to $f_{N\to N_{\max}} = \delta(y)$ (with $\beta(N_{\max} = M/\mu_T) = +\infty$). In other words, for small multiplicities β is *negative* (a feature alien to any statistical model!) and it becomes zero only for $N \to N_0$. At this multiplicity $f_N(y) = \text{const.}$, behavior known as *Feynman scaling.*[h] Its occurrence means that energy dependence of multiplicity follows that of the longitudinal phase space. For $N > N_0$ additional particles have to be located near the middle of phase space in order to minimize energy cost of their production. As result $\beta > 0$ now, in fact (see [4] for details) for some ranges of $\langle E\rangle = M/N$ quantity $\bar{\beta} = \beta\langle E\rangle$ remains approximately constant. Needless to say that for $N \to N_{\max}$ *all particles* have to stay as much as possible at the center and therefore $\beta(M, N \to N_{\max}) \to +\infty$, whereas $f_{N\to N_{\max}}(y) \to \delta(y)$.

For nonextensive approach $\exp(\ldots)$ must be replaced by $\exp_q(\ldots)_q$ and $N_0 \to N_0^{(q)} \simeq (2\ln N_{\max})^q$. As shown in [12] q acts now as a free parameter allowing for changing separately the shape and the height of $f_N^{(q)}$ (which were interlocked for the $q = 1$ case). For $q > 1$ one enhances tails of distribution reducing at the same time its height. For $q < 1$ the effect is opposite and additionally there is a kinematical condition, $1-(1-q)\beta_q\mu_T \cosh y \geq 0$, reducing in this case the allowed phase space.[i]

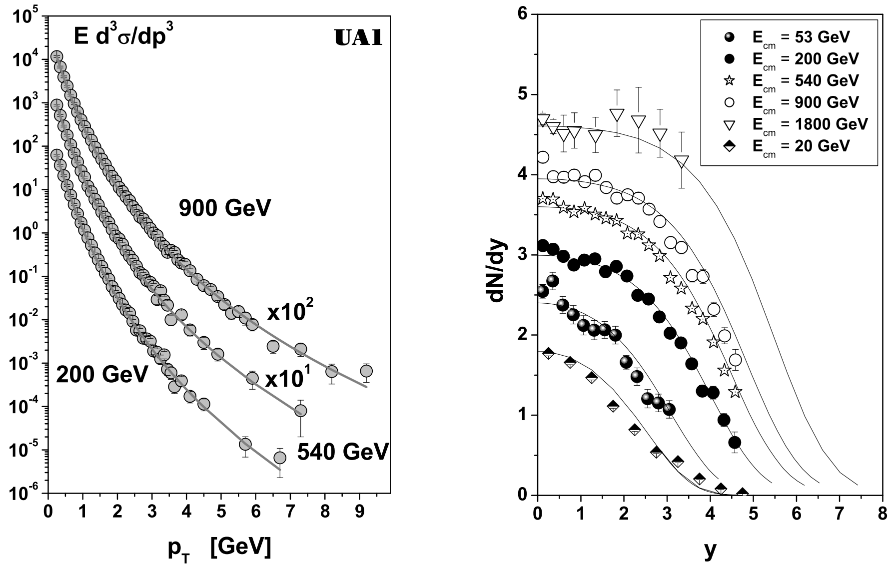

Fig. 1. Example of description of data on p_T spectra from UA1 experiment on $\bar{p}p$ collisions [5] (left panel) and data on rapidity spectra in pp and $\bar{p}p$ collisions [12] (right panel)

In [12] we have used $f_N^{(q)}$ to fit $\bar{\rm p}$p and pp data introducing inelasticity K as a free parameter to be deduced from data (cf. Fig. 1). We shall not discuss here the inelasticity issue (see [12] for details and further references) but concentrate on the q parameter, which turned out to follow *the same* energy dependence as the experimentally deduced parameter $1/k$ of the Negative Binomial (NB) multiplicity distribution [25]. This finding prompted us to argue that parameter q in this case is accounting for a new bit of information so far unaccounted for, namely for fluctuations in the multiplicity of produced secondaries (notice that in the constraint equation for β we were using in this case experimental values of the corresponding *mean multiplicities* only). In fact, it is known [7] that NB can be obtained from Poisson distribution once one allows for fluctuations in its mean value $\bar{n}$ of the gamma distribution type, namely

$$P(n) = \int_0^\infty d\bar{n}\frac{e^{-\bar{n}}\bar{n}^n}{n!}\cdot\frac{\gamma^k\bar{n}^{k-1}e^{-\gamma\bar{n}}}{\Gamma(k)} = \frac{\Gamma(k+n)}{\Gamma(1+n)\Gamma(k)}\cdot\frac{\gamma^k}{(\gamma+1)^{k+n}}\,, \qquad (12)$$

where

$$\gamma = \frac{k}{\langle\bar{n}\rangle} \qquad \text{and} \qquad \frac{1}{k} = \frac{\sigma^2(n_{ch})}{\langle n_{ch}\rangle^2} - \frac{1}{\langle n_{ch}\rangle}\,.$$

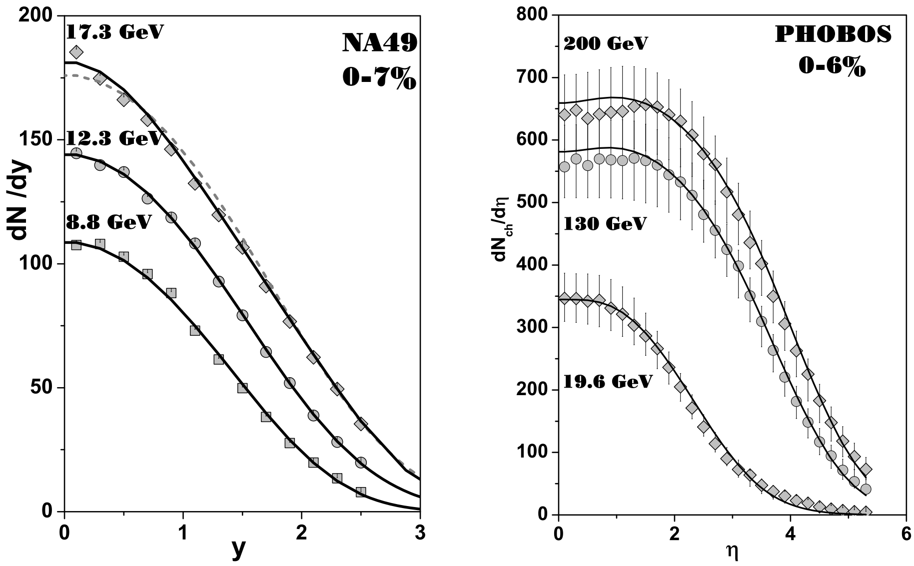

Fig. 2. Example of description of data on rapidity spectra from NA49 (left panel) for negatively charged pions produced in central PbPb collisions at different energies [5]; the best fit for 17.3 GeV is with additional (assumed) information of existence of two extensive sources located at $y = \pm\Delta y = 0.83$ in rapidity (see text for details). Right panel: fits to PHOBOS data for the most central Au+Au collisions [20]

Assuming now that these fluctuations contribute to nonextensivity defined by the parameter q, i.e. that $D(\bar{n}) = q - 1$ one gets

$$q = 1 + \frac{1}{k}, \tag{13}$$

what we do observe (cf. also [19]).

In Fig. 2 we show comparison with AA data at SPS (NA49) [5] and RHIC (PHOBOS) [20] energies. The NA49 data can be fitted with $q = 1.2$, 1.16 and 1.04 going from top to bottom and so far we do not have plausible explanation for these number. However, it should be stressed that at highest energy two-component extensive source is preferable. The PHOBOS data with $q = 1.27$, 1.26 and 1.29 going from top to bottom. Extensive fits do not work here at all. The first clear discrepancy has been found when fitting e^+e^- data (see [22]) where dn/dy with clear minimum for $y = 0$ cannot be reproduced in such a simple ITq approach. It is thus obvious that there is some new information we did not accounted for. It looks like we have two sources here separated in rapidity (two jets of QCD analysis) but of no statistical origin (rather connected with cascading process) [22].

4. Confrontation with Experimental Data — Asymmetric Case

Let us now proceed to a more complicated case of asymmetric collisions exemplified by pA processes [26–28]. In this case: i) both colliding objects are different and have different masses (one can therefore expect that energy transfer to the central region from each of them is not necessary the same as it was assumed before); ii) the pA collisions introduce a new element of uncertainty, the effective size of the target, i.e. the number of nucleons, ν, from the nucleus A with which the impinging nucleon is interacting and the way this interaction proceeds.[j] The number ν can be estimated from measurement of the number of "gray tracks" [29], therefore it will be included to our input information. We shall analyze data [26] in which attempt was made to isolate dN_ν/dy for $\nu = 1$, 2, 3, 4 and data [27] where only the mean $\bar{\nu}$ number of collision is known together with $dN_{\bar{\nu}}/dy$. The *a priori* available invariant energy $\sqrt{s_\nu}$ in such a case is equal to (we neglect all nuclear binding and Fermi motion effects): $s_\nu = \nu s + (\nu-1)^2 m^2$ where $s = 2m^2 + 2m[p^2_{\rm LAB} + m^2]^{1/2}$ is the invariant energy squared for NN collisions.

The assumed knowledge of ν must be supplemented by the known dependence of total mean multiplicity of secondaries produced in the central region, $\bar{N}_\nu$, on the mean number of inelastic collisions $\langle\nu\rangle$, [26, 27]:

$$\bar{N}_\nu = \frac{1}{2}(1 + \langle\nu\rangle)\bar{N} \tag{14}$$

($\bar{N}$ is multiplicity of particles produced in NN collisions at the same energy $\sqrt{s_{\nu=1}}$).[k] To be able to apply IT methods we must additionally decide whether pA collision is more like a two-body collision between a kind of "tube" of mass $m_\nu = \nu m$ containing ν collectively acting nucleons and single nucleon of mass m (cf. Fig. 3(a) and

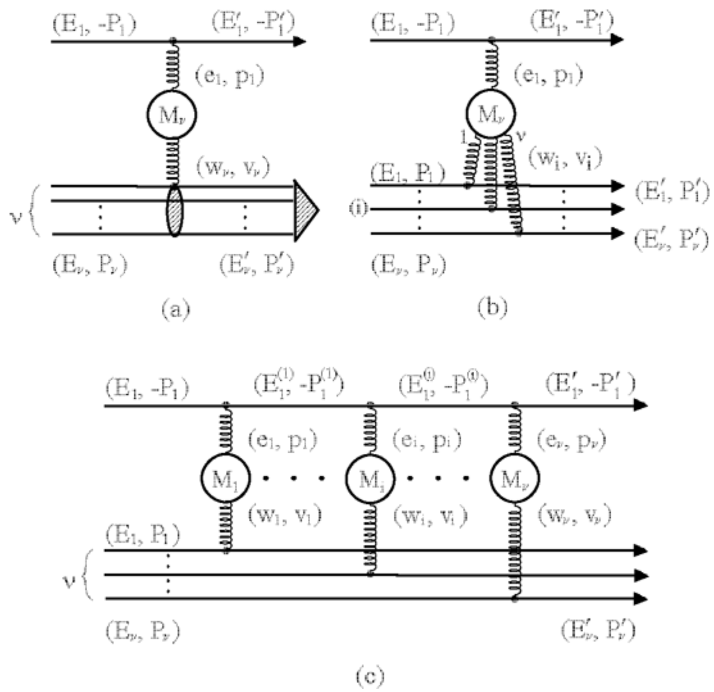

Fig. 3. Schematic views of tube (a) and sequential (c) models of the pA collisions. The case (b) is essentially a particular realization of the tube type of model (a)

(b)), or whether it is rather a sequence of ν consecutive collisions of the impinging nucleon with ν nuclear nucleons (cf. Fig. 3c).

In the first case we have situation similar to considered before with the following formula for the rapidity distribution of secondaries produced in two-body $p(\nu N)$ (in the CM frame of the NN system):

$$\frac{dN_\nu}{dy} = f_\nu(y) = \frac{1}{Z_\nu} \cdot \exp\left(-\lambda_\nu \mu_T \sinh y - \beta_\nu \mu_T \cosh y\right) , \tag{15}$$

$$Z_\nu = \int_{Y_{\min}^{(\nu)}}^{Y_{\max}^{(\nu)}} dy \, \exp\left(-\lambda_\nu \mu_T \sinh y - \beta_\nu \mu_T \cosh y\right) . \tag{16}$$

We have now two Lagrange multipliers, λ_ν and β_ν, which are given by the corresponding energy and momentum conservation constraints:

$$\int_{Y_{\min}^{(\nu)}}^{Y_{\max}^{(\nu)}} dy \, e^{-\mu_T[\lambda_\nu \sinh y - \beta_\nu \cosh y]} \begin{Bmatrix} \cosh y \\ \sinh y \end{Bmatrix} = \frac{Z_\nu}{N_\nu \mu_T} \begin{Bmatrix} W_\nu = (\nu R + K)\frac{1}{2}\sqrt{s} \\ P_\nu = -(\nu R - K)\frac{1}{2}\sqrt{s - 4m^2} \end{Bmatrix} . \tag{17}$$

The energy transfer from the projectile nucleon, characterized by inelasticity K, is allowed to differ from energy transfers from the nuclear nucleons (cf. Fig. 3b) characterized by inelasticities R_i (for simplicity we shall assume $R_i = R$ in what follows). The invariant mass M_ν of hadronizing system is now equal to

$$M_\nu \;=\; \sqrt{W_\nu^2 - P_\nu^2} \;=\; \sqrt{KR\nu s \,+\, (\nu R - K)^2 m^2} \quad \stackrel{R=K}{\Longrightarrow} \quad K\sqrt{s_\nu}\,, \qquad (18)$$

whereas the (longitudinal) phase space in which particles are produced is given by:

$$Y^{(\nu)}_{\rm max} = Y_{\nu m} - \delta_\nu \qquad \text{and} \qquad Y^{(\nu)}_{\rm min} = -Y_{\nu m} - \delta_\nu\,, \qquad (19)$$

($Y_{\nu m}$ is the same as Y_M before but calculated for M_ν) where δ_ν being the rapidity shift between the NN and $(\nu N) - N$ center-of-mass frames:

$$\tanh\delta_\nu = -\frac{P_\nu}{W_\nu} \quad \Longrightarrow \quad \delta_\nu \simeq \frac{1}{2}\ln\left[\frac{R}{K}\cdot\nu\right] \stackrel{R=K}{\Longrightarrow} \frac{1}{2}\ln\nu\,. \qquad (20)$$

The apparently asymmetric form of rapidity distribution as given by Eq. (15) is, however, an artifact connected with our choice of the reference frame. Changing variables in Eqs. (15)–(17) from y to $\tilde{y} = y + \delta_\nu$, i.e. proceeding from the CMS of NN to the CMS of $(\nu N) - N$, one gets similar distribution as before (i.e. depending only on one Lagrange multiplier $\tilde{\beta}_\nu$):

$$f_\nu(y) \Rightarrow f_\nu(y = \tilde{y} - \delta_\nu) = \frac{1}{\tilde{Z}_\nu}\cdot e^{-\tilde{\beta}_\nu\mu_T\cosh\tilde{y}}\,, \qquad \tilde{Z}_\nu = \int_{-Y_{\nu m}}^{Y_{\nu m}} d\tilde{y}\, e^{-\tilde{\beta}_\nu\mu_T\cosh\tilde{y}}\,, \quad (21)$$

$$\int_{-Y_{\nu m}}^{Y_{\nu m}} d\tilde{y}\cosh\tilde{y}\, e^{-\tilde{\beta}_\nu\mu_T\cosh\tilde{y}} = \frac{Z_\nu}{N_\nu\mu_T}\cdot M_\nu\,, \qquad (22)$$

where $M_\nu = M_\nu(K,R)$. It means that in such an approach we always can find frame in which rapidity distributions f_ν are *symmetric function* given by (21). In Fig. 4 we show examples of fits to some available data (notice that data display clear asymmetric character which cannot be reproduced by the method used here).

In the second case pA scattering is assumed to proceed via sequence of the ν two-body processes, cf. Fig. 3c. The resultant rapidity distribution $f_\nu(y)$ is then composed of "elementary" distribution functions describing collisions of the impinging nucleon with the subsequent nucleons in the target nucleus:

$$f_\nu(y) = \sum_{i=1}^{\nu} f_i(y)\,. \qquad (23)$$

Contrary to (14), this formula stresses not the fact that in the pA reaction one has $\nu+1$ participating nucleons but that one has here ν consecutive collisions treated as "elementary" ones. This can be visualized rewriting (14) as $\bar{N}_\nu = \bar{N}_1 + (\nu-1)\bar{N}_2/2$ where $\bar{N}_{1,2}$ are such that $2\bar{N}_1 + (\nu-1)\bar{N}_2 = (\nu+1)\bar{N}$. We have thus ν "elementary"

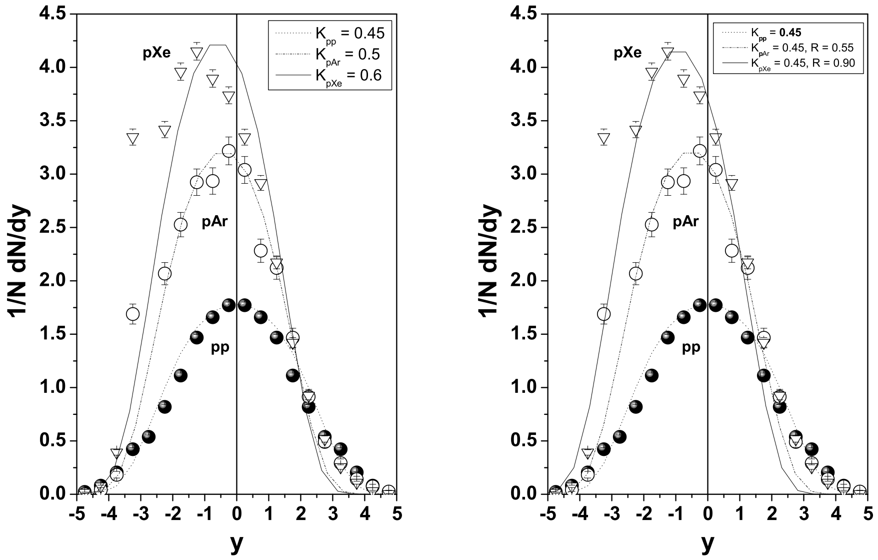

Fig. 4. Example of fits to pp, pAr and pXe data [27] by means of the collective tube model approach as given by Eq. (21) with $R = K$ (left panel) and with $R \neq K$ for pA collisions (right panel)

collisions with a possibility that the first can differ from the remain $\nu - 1$ (in general they all are different). As result, the elementary inelasticities (i.e. inelasticities in subsequent collisions) are not necessarily the same. In this way one approaches as near as possible the notion of independent production for which one demands the *a priori* knowledge of energy of each reaction. Also here we shall differentiate between the fractions of energies contributed to each f_i by the nuclear nucleons, $R_{i=1,\dots,\nu}$, and fractions of energies contributed by the impinging nucleon at each collision, $K_{i=1,\dots,\nu}$. In this case the general form of the energy–momenta (e_i, p_i) and (w_i, v_i) flowing to the blob M_i in Fig. 3c is given by:

$$e_i = \frac{1}{2} K_i \prod_{j=1}^{i-1} (1 - K_j) \sqrt{s}\,,$$

$$p_i = \frac{1}{2}\left[\prod_{j=1}^{i-1} (1 - K_j)\, s - 4m^2\right]^{1/2} - \frac{1}{2}\left[\prod_{j=1}^{i} (1 - K_j)\, s - 4m^2\right]^{1/2}, \qquad (24)$$

$$w_i = \frac{1}{2} R_i \sqrt{s}\,, \qquad v_i = \frac{1}{2}\sqrt{s - 4m^2} - \frac{1}{2}\sqrt{(1 - R_i)\cdot s - 4m^2}\,, \qquad (25)$$

with $M_i^2 = \left[(e_i + w_i)\,;(p_i + v_i)\right]^2$. Notice that the real fraction of energy deposited by impinging nucleon in, say, second collision, is equal to $K^{(2)} = K_2 \cdot (1 - K_1)$, in general, $K^{(i)} = K_i \cdot \prod_{j=1}^{i-1}(1-K_j)$. We now have

$$f_i(y) = \frac{1}{Z_i} e^{-\beta_i \mu_T \cosh y}, \tag{26}$$

$$Z_i = \int_{-Y_i}^{Y_i} dy e^{-\beta_i \mu_T \cosh y}, \qquad \int_{-Y_i}^{Y_i} dy \cosh y e^{-\beta_i \mu_T \cosh y} = \frac{Z_i}{N_i \mu_T} \cdot M_i, \tag{27}$$

with Y_i calculated in the same way as Y_M before with M_i and n_i replacing W and N, respectively. These distributions will be centered (in the CM frame of NN) at

$$y_i \simeq \frac{1}{2} \ln \left[\frac{K_i}{R_i} \prod_{j=1}^{i-1} (1-K_j) \right] \quad \overset{R_i = K_i = K}{\Longrightarrow} \quad \propto \frac{(i-1)}{2} \ln(1-K), \tag{28}$$

i.e. the expected shift in rapidity is now *linear* in the number of collisions, not logarithmic one as in Eq. (20). However, for our limited values of ν this difference is practically not visible. The last input needed to apply IT is some estimation of multiplicities N_i. This is the most uncertain point here and can be done in many model-dependent ways. Here we simply assumed that $N_i = a \cdot M_i^c$ (with a and c being free parameters) and made use the fact that $N = \sum_{j=1}^{\nu} N_i = a \cdot \sum_{j=1}^{\nu} M_j^c$ to eliminate parameter a and to write $N_i = N/[1 + \sum_{j \neq i}^{\nu} (M_j/M_i)^c]$, where $c = 0.556$ is obtained from reproducing the observed multiplicity N (actually, logarithmic dependence $N_i = a + b \cdot \ln M_i$ would work equally well). In order not to introduce too much freedom with the choices of K_i we were proceeding in the following way: K_1 was fixed by the $\nu = 1$ data and then used as $K_{i=1}$, then K_2 was fixed by $\nu = 2$ data and used as $K_{i=2}$ and so on up to K_4, which was fixed by $\nu = 4$ data.[l] On the other hand, the fractions of energy deposited by the consecutive target nucleons participating in the collision was kept the same for all of them and equal $R = R_\nu$.

Notice that in the sequential model we have clearly smaller consecutive energy transfers (as given by $K_{i>1}$) from the nucleon traversing the nucleus (this follows observation made in [30]). On the other hand, both approaches lead to essentially the same results. However, only sequential model is potentially able to reproduce the visible asymmetry seen in data for larger ν because "elementary" f_i enter with different weights and cover slightly different regions of phase space. However, at present energies the effect is not as large as expected and sequential model leads to essentially the same results as tube model.[m] To account for the shift in the position of maximum of rapidity distribution in the tube model we have to allow for $K \neq R$, i.e. for different energy depositions from each of the projectiles. However, even then we cannot produce the asymmetry of rapidity distribution seen in data, in particular we are not able to correctly describe the part of distribution connected with the impinging proton. It should be stressed that using in this case the nonextensive approach ITq would not help because also in this case the obtained distributions are symmetric in the rest frame of the hadronizing system. There is clearly difference between the impinging proton and nuclear parts of the phase space. We conclude therefore that, according to the rules of information theory approach, data [26, 27] still bear some additional information, not identified so far.

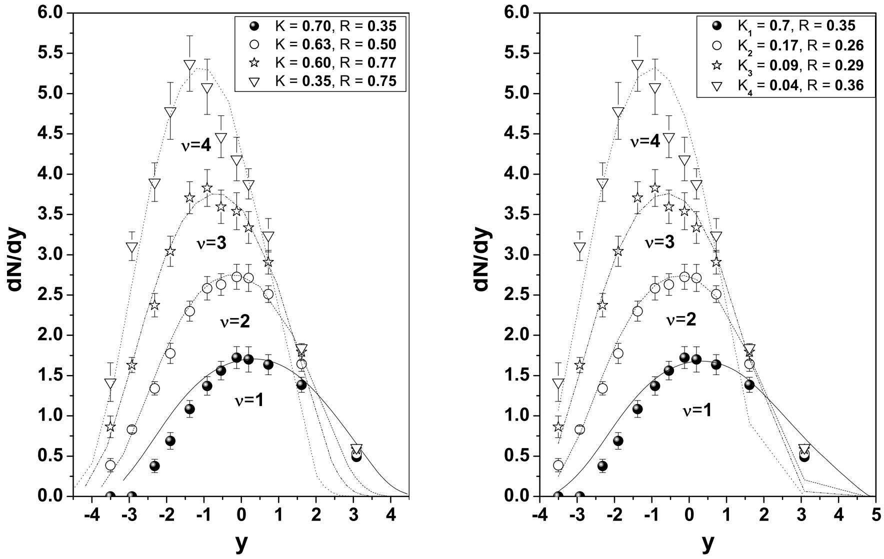

Fig. 5. Left panel: Comparison with pA data for ν [26] using tube model (Eq. (21) and Fig. 3a) for $K \neq R$. Notice that data for $\nu = 1$ are not compatible with data for pp collisions from [27] presented in Fig. 3. Right panel: the same but using sequential model (Eq. (23) and Fig. 3c). K_i is the fraction of the actual projectile energy in the ith collision deposited in central region. The fractions of energy deposited by target nucleons is kept the same and equal $R = R_\nu$

5. Summary and Conclusions

We have demonstrated that large amount of data on multiparticle production can be quite adequately described by using tools from information theory, especially when allowing for its nonextensive realization based on the Tsallis entropy. We have argued that the nonextensivity parameter q entering here can, in addition to the temperature parameter T of the usual statistical approaches, provide us valuable information on dynamical fluctuations present in the hadronizing systems. Such information can be very useful when searching for phase transition phenomena, which should be accompanied by some specific fluctuations of non-monotonic character [23].

As concerns the asymmetric collisions example of pA data, we have demonstrated that they contain additional information to the usual one used in Section 3. The immediate candidates are the rescattering of produced secondaries in the nucleus (effectively increasing values of μ_T and making it y-dependent and in this way limiting rapidity space available on the nuclear side) or diffraction dissociation part of the production (which can take different shape than in the pp collisions), but they can be other possibilities as well [5].

Finally let us comment shortly on the inelasticities in pA collisions obtained. In the tube model they are, in general, increasing (or, at least, non-decreasing) with A or with ν, cf. Figs. 3, 4 and 5. Notice that for $K \neq R$ case the effective inelasticity, which is of interest for any statistical model approach (i.e. the part of the total energy available for production of secondaries in the central region of reaction) is $K_{\text{eff}} = \sqrt{K \cdot R}$. In the sequential model this inelasticity is clearly decreasing with the consecutive collisions (in agreement with what was found in [30]). However, the more precise statement could be only done with a data taken at much higher energies. Notice that we did not present fits to pAr and pXe data [27] in this case because we would have to introduce here another piece of additional information represented by distribution of number of collisions, $P(\nu)$, which is a model dependent quantity. The high energy counterpart of data [26] would be therefore most welcome. The point is, however, that in addition one should also have data taken for pn inelastic collisions (possibly with the help of deuterons) as it is possible that at least a part of the inconsistency between the pp data from [27] (cf. Fig. 4) and the $\nu = 1$ data from [26] in Fig. 5 is because the latter contain also contribution from pn reactions.[n]

We close with the statement that the results presented here are very encouraging and call for further systematic effort to describe the existing data in terms of (T, q) for different configurations and energies in order to find possible regularities in their system and energy dependencies and possible correlations between them.

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Notes

a. The complete list of references concerning *IT* relevant to our discussion and providing the necessary background can be found in [4, 5]. The value of the Boltzmann constant is set to unity, $k = 1$.

b. Notice that using the entropic measure $S = \sum_i [p_i \ln p_i \mp (1 \pm p_i) \ln (1 \pm p_i)]$ (which, however, has nothing to do with *IT*) would result instead in Bose–Einstein and Fermi–Dirac formulas: $p_i = (1/Z) \cdot [\exp[\beta(\varepsilon_i - \mu)] \mp 1]^{-1}$, where β and μ are obtained solving two constraint equations given, respectively, by energy and number of particles conservation [6]. It must be also stressed that the final functional form of p_i depends also on the functional form of the constraint functions $R_k(x_i)$. For example, $R(x) \propto \ln(x)$ and $\ln(1-x)$ type constrains lead to $p_i \propto x_i^{\alpha}(1-x_i)^{\beta}$ distributions.

c. It is straightforward to check that the Shannon entropy decreases from the most broad geometrical distribution towards the most narrow Poissonian distribution.

d. The most drastic situation was with the multi-Regge model in which, in addition to the basic model assumptions, two purely phenomenological ingredients have been introduced in order to get agreement with experiment: i) the energy s was used in the scaled (s/s_0) form (with s_0 being a free parameter, this works the same way as inelasticity) and ii) the so-called "residual function" factor $e^{\beta\cdot t}$ was postulated ($t = -(p_i - p_j)^2$ and β being a free parameter) to cut the transverse part of the phase space. Therefore s_0 and β were *the only relevant* parameters.

e. For our purpose this is sufficient and there is no need to use more sophisticated approaches exploring escort probabilities formalism, see [12] (for the most recent discussion of different constraints and their meaning see [13], and references therein).

f. Strictly speaking in [15] it was shown only for fluctuations of $1/\Lambda$ given by gamma distribution. However, it was soon after generalized to other form of fluctuations and the word *superstatistics* has been coined to describe this new phenomenon [16]. Another generalization of this idea can be found in [17].

g. Notice that the y space is limited to $y \in (-Y_M, Y_M)$, where (with $\mu_T = \sqrt{m^2 + \langle p_T\rangle^2}$ being the mean transverse mass and $M' = M - (N-2)\mu_T$ accessible kinetic energy) the limits are given by

$$Y_M = \ln\left\{M'/(2\mu_T)\left[1 + \left(1 - 4\mu_T^2/M'^2\right)^{1/2}\right]\right\}.$$

h. It was jus a coincidence that at ISR energies this condition was satisfied. But because such a behavior of N as function of energy was only some transient phenomenon, there will never be scaling of this type at higher energies, notwithstanding all opinions to the contrary heard from time to time.

i. In [18] we have fitted data on $\bar{\text{p}}$p with $q < 1$ being the only parameter and $M = \sqrt{s}$. The $q < 1$ was cutting off the available phase space playing effectively the role of inelasticity K. This result has provided the cosmic ray physicist community the justification of the empirical formula they used, namely that $f(x) \propto (1 - a\cdot x)^n$, where x is the Feynman variable, $x = 2E/\sqrt{s}$, and where a and n are free parameters. They turned out to be both given by the parameter q only [18].

j. In what follows only interactions resulting in the production of particles in the central region of reaction are of interest to us, elastic and diffractive dissociation collisions will not be considered.

k. Although this regularity has been observed only for charged secondaries, we shall assume here its validity both for the total number of produced particles as well as for the fixed number of collisions ν.

l. Notice that such a procedure is possible only under the assumption of independent collisions.

m. We would like to stress at this point that our approach to pA using IT concepts and tube model differs from that of [28] because we have only one parameter, inelasticity $K = R$, with both the normalization, shift of the momenta and "partition temperature" being fixed by it, whereas in [28] they are all free parameters.

n. We shall not pursue further this problem, which in our opinion can be investigated in the spirit of IT only when the same experiment will provide data both for different and well defined values of ν and for $\langle\nu\rangle$, i.e. averaged over different ν (such a possibility was apparently under consideration by NA49 Collaboration [31]). We are also aware of potentially interesting new data from RHIC on dAu collisions [32] and recent attempts of their description [33] and we plan to address this issue using IT approach elsewhere.

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