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HEAVY ION PHYSICS

Improved Description of Bose–Einstein Correlation Function

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Abstract. The L3 data on Bose–Einstein correlations of equally charged pion pairs produced in hadronic Z decays are analyzed in terms of various parametrizations. Preliminary results are presented here.

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1. Introduction

In particle and nuclear physics intensity interferometry provides a direct experimental method for the determination of sizes, shapes and lifetimes of particle emitting sources (for recent reviews see [1, 2]). In particular, boson interferometry provides a powerful tool for the investigation of the space-time structure of particle production processes, since Bose–Einstein correlations (BEC) of two identical bosons reflect both geometrical and dynamical properties of the particle radiating source.

Originally, the method of Bose–Einstein correlations was invented by the radio astronomers Hanbury Brown and Twiss (HBT), who applied it to determine the angular diameters of main sequence stars [3, 4]. The first experimental evidence for Bose–Einstein correlations in high energy physics dates back to 1959 [5]. The results were interpreted as BEC by G. and S. Goldhaber, Lee and Pais (GGLP) [6]. Angular distributions of pions could be described more exactly by applying Bose–Einstein statistics instead of a classical statistical model. This effect is frequently referred as either the HBT, or GGLP effect, or simply Bose–Einstein correlations.

The L3 experiment provides a good opportunity to perform detailed investigation of Bose–Einstein correlations in e^+e^- annihilation at a center of mass energy of $\sqrt{s} = 91$ GeV. After "standard" event and track selection we concentrate on 2-jet events detected using the Durham algorithm [7] with resolution parameter $y_{\mathrm{cut}} = 0.006$. Finally, approximately 13 million pion pairs are analyzed. The results presented here are preliminary.

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2. Shape of the BEC Function

The two-particle Bose–Einstein correlation function is defined as:

$$C_2(p_1,p_2) = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)}\,, \tag{1}$$

where $\rho_2(p_1,p_2)$ is the two-particle invariant momentum distribution, $\rho_1(p_i)$ is the single-particle invariant momentum distribution and p_i stands for the four-momentum of particle i. Since it is difficult to create the product of the single-particle distribution it is replaced by a "reference sample", the two-particle density that would occur in the absence of BE interference.

If long-range correlations can be neglected or corrected for, and if the short-range correlations are dominated by Bose–Einstein correlations, this two-particle BEC function is related to the Fourier-transformed source distribution. If we assume that $f(x)$ is the density distribution of the source of the pions then the correlation function is found to be

$$C_2(p_1,p_2) = 1 + |\tilde{f}(Q)|^2, \tag{2}$$

where Q is the invariant four-momentum difference, $Q = -(p_1 - p_2)^2$ and $\tilde{f}(Q)$ is the Fourier transform of $f(x)$

$$\tilde{f}(Q) = \int \mathrm{d}x \exp(iQx) f(x)\,. \tag{3}$$

2.1. Gaussian distributed source

The simplest assumption is that the source has a Gaussian distribution, in which case the Fourier-transformed source function is determined as $\tilde{f}(Q) = \exp\left(-\frac{1}{2}Q^2\right)$ which yields

$$C_2(Q) = \gamma\left[1 + \lambda \exp\left(-R^2Q^2\right)\right](1 + \delta Q)\,, \tag{4}$$

where the parameter γ is a constant of normalization, λ is an incoherence factor, which measures the strength of the correlation, R is a scale parameter which measures the length of homogeneity and $(1 + \delta Q)$ is introduced to parametrize possible long-range correlations.

A fit of this correlation function results in an unacceptably low confidence level from which one concludes that the shape of the source deviates from a Gaussian. The fit is particularly bad at low Q values, as shown in Fig. 1.

2.2. Lévy distributed source

The Central Limit Theorem states that, under certain conditions, the sum of a large number of random variables behaves as a Gaussian distribution. The generalization of the central limit theorem gives the classification of the stable distributions. The study of these stable distributions was begun by Paul Lévy in the 1920's. For recent results see [8].

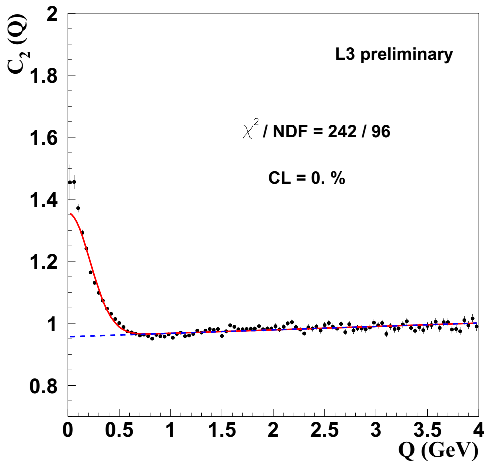

Fig. 1. The Bose–Einstein correlation function C_2. The full line corresponds to the fit by Eq. (4), while the dashed line stands for the long-range momentum correlations

According to the generalized Central Limit Theorem, if particle production is a result of a multifold probabilistic process such as the branching of gluons into gluons into gluons etc., then the class of possible limiting probability distributions coincides with the class of Lévy distributions [8]. The characteristic function of symmetric stable distributions is [9]

$$\tilde{f}(Q) = \exp\left(iQ\delta - \frac{|RQ|^{\alpha}}{2}\right) . \tag{5}$$

Here we utilize the notation of Nolan [8]. The index of stability, α, satisfies the inequality $0 < \alpha \leq 2$. The case $\alpha = 2$ corresponds to a Gaussian source distribution. For more details see [8].

Thus the Bose–Einstein correlation function for Lévy stable distributions has the following, relatively simple form:

$$C_2(Q) = \gamma\left[1 + \lambda \exp\left(-(RQ)^{\alpha}\right)\right](1 + \delta Q) . \tag{6}$$

After fitting the data with Eq. (6) it is clear that the correlation function is far from Gaussian: $\alpha \approx 1.3$. However, the confidence level is still unacceptably low. The curve of this fit is shown in Fig. 2.

Since there is no particle production before the onset of the collision, a more appropriate form of the source distribution for the time component is the asymmetric stable distribution. In this case, one obtains the following result for the correlation function:

$$C_2(Q) = \gamma\left[1 + \lambda \cos\left[(R_a Q)^{\alpha}\right] \exp\left(-(RQ)^{\alpha}\right)\right](1 + \delta Q), \tag{7}$$

where R_a is an additional parameter.

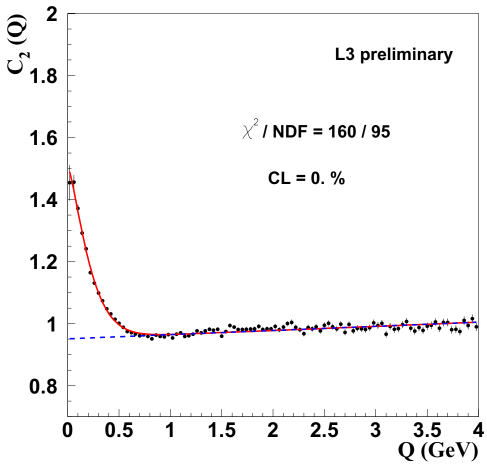

Fig. 2. The Bose–Einstein correlation function C_2. The full line corresponds to the fit by Eq. (4), while the dashed line stands for the long-range momentum correlations

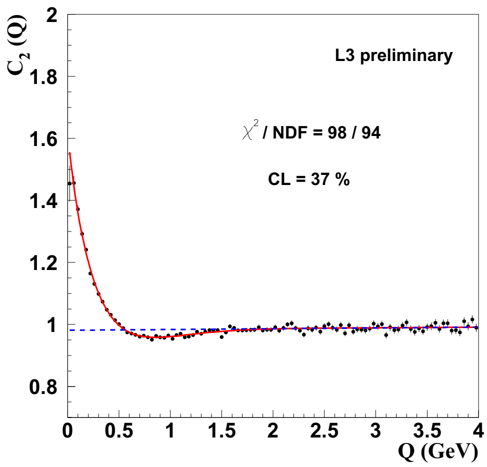

Fig. 3. The Bose–Einstein correlation function C_2. The full line corresponds to the fit by Eq. (4), while the dashed line stands for the long-range momentum correlations

The fit of Eq. (7) to the data is statistically acceptable. The data points are well described by the fit curve which is shown in Fig. 3. Note that for Q between 0.5 GeV and 1.5 GeV the fitted curve goes below one. This is caused by the cosine term, which comes from the asymmetric Lévy assumption, in Eq. (7). The fitted value of the index of stability, α is found to be approximately 0.8.

3. Conclusions

The assumption that the source has a Gaussian shape is too simple. A good description of the Bose–Einstein correlation function is achieved using Lévy stable distributions as the source function.

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