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Simple Model Calculations of Spin and Quantized Alignment for the $A \sim 60$–90 Superdeformed Mass Region

A.S. Shalaby

Physics Department, Faculty of Science, Cairo University, Beni Suef Branch
Cairo, Egypt; E-mail: Ashalaby15@yahoo.com

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Abstract. We have used a simple model based on the rotational energy formula $E(I, K)$ to study the structure of the superdeformed (SD) mass region 60–90. The higher order inertial parameters A and B of such model were determined by using the Marquardt method of nonlinear least-squares routines to fit the proposed transition energies with their observed values. A good agreement between the calculated and corresponding experimental transition energies of the SD bands is obtained which supports our proposed model. In addition, the frequency dependence of the dynamic, $\theta^{(2)}$, and static, $\theta^{(1)}$, moments of inertia is used to determine the lowest spin (I_f) and the K-value of the considered SD bands; namely, ^{58}Ni(b1), ^{58}Cu, ^{59}Cu(b1), ^{61}Zn, ^{62}Zn, ^{65}Zn, ^{68}Zn, ^{84}Zr, ^{86}Zr(b1), ^{88}Mo(b1, b2, b3) and ^{89}Tc. As a result of the identity exist among some of the considered SD bands, we have studied the incremental alignment and also the angular momentum alignment.

Keywords: superdeformed nuclei, mass region 60–90, spin alignment
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1. Introduction

Since the discovery of superdeformed (SD) rotational band in a fast rotating nucleus ^{152}Dy [1, 2], numerous superdeformed bands have now been observed in the mass regions $A \sim 190$, 150, 130 [3, 4], 80 [4–9] and 60 [10, 11]. The superdeformed mass region $A \sim 60$–90 is of particular interest [12–17] because of the limited number of particles in these nuclei and also as a result of their lightest mass, they exhibit highest rotational frequencies. Most of the superdeformed nuclei in this mass region show similar behaviour of their dynamic moment of inertia, $\theta^{(2)}$, with the rotational frequency, $\hbar\omega$, in that they exhibit a smooth decrease as $\hbar\omega$ increases. The large rise of the dynamical moment of inertia at low angular frequency in the mass region $A \sim 60$–90 has been attributed to the simultaneous alignment of two pairs of

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$g_{9/2}$ protons and neutrons in the presence of pair correlations [18–21]. For the SD bands, gamma-ray energies are unfortunately the only spectroscopic information universally available. Because of the non-observation of the discrete linking transitions between the SD states and the low lying states at normal deformation (ND), the experimental data for the spin of the rotational bands is poor and the only way to obtain the value of the spin is doing theoretically. Several approaches for assigning spins to SD states have been proposed [22–35]. These approaches involve direct and indirect methods for assigning spin to the states in SD bands.

In the direct method, as we saw in our previous paper [35], the energies of the states of a rotational band are expressed as function of spin based on a two-parameter formula. Assuming various values for the spin I_0 of the lowest state in the SD bands, the two parameters can be adjusted to obtain a minimum root-mean-square deviation of the calculated and measured energies. On the other hand, the indirect methods rely mainly on the fitting of the experimental dynamical moment of inertia values with the Harris formula [23,24,31,34,36,37]. The parameters obtained from the fit are then used to calculate the spin. In such a parameterization, the spin may be expressed as an expansion in the rotational frequency, $\hbar\omega$. Such available approaches are usually referred to as the best-fit method (BFM).

The simple model employed here in the present work belongs to the direct methods in which the rotational energy, $E_{\text{rot}}(I, K)$, depends upon the spin, I, and K, which is the projection of the angular momentum along the symmetry axis.

One of the most remarkable properties so far discovered of rotational bands in superdformed nuclei is the extremely close coincidence in the energies of the deexciting γ-ray transitions (or rotational frequencies) between certain pairs of bands in different nuclei [38, 39]. This behaviour may be characterized by a quantized spin alignment [40–43].

In 1990, Stephens et al. [42] have studied the properties of the superdeformed mass region 190 and concluded that such a mass region exhibit nonzero quantized spin alignment when the superdeformed band in ^{192}Hg is taken as a reference band. They defined a quantity called incremental alignment, denoted by Δi. It depends only on the γ-ray energy E_γ (without the need of, or reference to the spins of the individual SD levels) and is related to the so-called alignment, i, through $i = \Delta i + \Delta I$, where ΔI is the spin difference between states in a considered band and those of the reference band (where the considered band "B" = the reference band "A" + αp + βn, where α (β) is the number of extra protons (neutrons)).

The value of i is unknown but ΔI is quantized, if Δi is quantized, i must be quantized. The incremental alignment, Δi, is obtained by subtracting the transition energy of the considered band from the closest transition energy of the reference band and dividing the result by the energy difference between the closest two transitions in the reference band. Stephens et al. [42] found that the alignment values lie between ± 0.5 and the nearly identical transition energies give values very close to zero. Using the same procedure as that utilized by Stephens et al. [42], we will study the alignment between the nearly identical bands appeared among the considered SD bands.

The scope of this work is to estimate the lowest spin $I_{\rm f}$ or the K-value of thirteen SD bands in the mass region 60–90. Also, in this paper, we will calculate the values of the incremental alignment and spin alignment for the identical SD bands existing among the considered bands, to verify Stephens's law of alignment.

A theory used to determine the lowest spin, $I_{\rm f}$ or the K-value of the considered superdeformed rotational bands in the $A \sim 60$–90 mass region with the present simple approach is given in Section 2. This section presents also the expressions needed to calculate the incremental alignment and the angular momentum alignment. In Section 3, all the data on the thirteen SD bands observed in the $A \sim 60$–90 region are analyzed by making use of such an approach. The spins of all these SD bands are determined and the results seem reasonable. With the lowest spin values thus assigned, the energy spectra of these SD bands were calculated and the results turned out unexpectedly well. This section also analyzes the values obtained for the incremental alignment and the spin alignment for some of the considered SD bands (identical bands) of the $A \sim 60$–90 mass region. The conclusion is given in Section 4.

2. Theory

In the present simple model used to determine the lowest spin values of the SD rotational bands, the superband was suggested to be of purely rotational nature [44]. In this model, the expression for the rotational energy $E_{\rm rot}(I,K)$ is given in terms of the rotational angular momentum as:

$$E_{\rm rot}(I,K) = (\hbar^2/2\varphi)\,[I(I+1)-K^2]\,, \tag{1}$$

where φ is the moment of inertia.

This equation can be written more generally as a function of $[I(I+1)-K^2]$ as:

$$E_{\rm rot}(I,K) = A\,[I(I+1)-K^2] + B\,[I(I+1)-K^2]^2 + C\,[I(I+1)-K^2]^3 + \,\ldots\,, \tag{2}$$

where $A = (\hbar^2/2\varphi)$, B and C correspond to higher order inertial parameters.

The total energy for superdeformed band is given by:

$$E_S(I,K) = E_0 + A\,[I(I+1)-K^2] + B\,[I(I+1)-K^2]^2 + C\,[I(I+1)-K^2]^3 + \ldots\,, \tag{3}$$

where E_0 is the band head energy of the superdeformed band.

Making use of Eq. (3), the transition energy from level $I+2$ to level I is found to has the form

$$\begin{aligned} E_\gamma(I) &= E_S(I+2 \to I) \\ &= A\,[4I+6] + B\,[8I^3 + 36I^2 + I(60-8K^2) - 12K^2 + 36] + \,\ldots\,. \end{aligned} \tag{4}$$

For an SD cascade

$$I_0 + 2n \to I_0 + 2n - 2 \to \ldots \to I_0 + 4 \to I_0 + 2 \to I_0\,, \tag{5}$$

the observed transition energies $E_\gamma(I_0+2n)$, $E_\gamma(I_0+2n-2),\ldots,E_\gamma(I_0+4)$ and $E_\gamma(I_0+2)$ can be least-squares fit by Eq. (4) with fitting parameters A and B.

A most useful concept to study the SD band requires essentially a successful assignment of the K-value and/or the angular momentum of the band. In the considered mass region 60–90, the angular momentum is estimated in our previous works [34, 35] using other different theoretical approaches and the results seem to be relatively good.

In this work, the rotational frequency, $\hbar\omega(I) = [E_\gamma(I) + E_\gamma(I+2)]/4126$, the dependence of the static moment of inertia, $\theta^{(1)}(I-1) = (2I-1)\times 1126/E_\gamma(I)$, and the dynamic moment of inertia, $\theta^{(2)}(I) = 4126/[E_\gamma(I+2) - E_\gamma(I)]$, are used very carefully to determine the lowest spin, I_f, and also the band head spin, I_0.

The expressions used to calculate the incremental alignment and spin alignment are given as [42]:

$$\Delta i = \frac{\Delta E_\gamma(\text{reference band}) - \Delta E_\gamma(\text{considered band})}{\Delta E_\gamma^{(2)}(\text{reference band}) - \Delta E_\gamma^{(1)}(\text{reference band})}\,, \tag{6}$$

$$\Delta I = i(\text{considered band}) - i(\text{reference band})\,, \tag{7}$$

$$i(\omega) = \Delta i + \Delta I\,, \tag{8}$$

where Δi, ΔI and $i(\omega)$ are referred to the incremental alignment, spin differences and spin alignment, respectively.

3. Results and Discussion

In the superdeformed mass region 60–90, ΔE_γ increases with I so both $\theta^{(1)}$ and $\theta^{(2)}$ must decrease with I, where $\theta^{(1)}$ in most cases must be greater than $\theta^{(2)}$. To determine the correct value of the lowest spin I_f, we draw the dynamic and static moments of inertia against the rotational frequency, $\hbar\omega$, for different values of spin. The results are illustrated in Figs. 1–13, where the assigned value of the lowest spin is representd by a minus sign. One can see from these figures that there is a critical spin below which the normal behaviour of $\theta^{(1)}$ and $\theta^{(2)}$ is reversed and the relation between them becomes hard to understand. This critical spin is to be regarded as the baseline spin or the lowest spin of the superdeformed band. It is well known that for many bands such as the ground state β- and γ-bands, the value of K is equal to the value of the lowest spin, I_f. In the same manner, we can propose that the K-value for the SD band has the same value of the lowest spin or the baseline spin of that band. Using the values of K that assigned before, the transition energies in the thirteen SD bands observed in $A \sim 60$–90 mass region have been least-squares fit by Eq. (4) and the results are encouraging. We have obtained the values of the coefficients A, B by using the Levenberg–Marquardt method [45], to fit the proposed transition energies with their observed values. This fitting is succeedingly done for the SD bands, ^{58}Ni(b1), ^{58}Cu, ^{59}Cu(b1), ^{61}Zn, ^{62}Zn, ^{65}Zn, ^{68}Zn, ^{84}Zr,

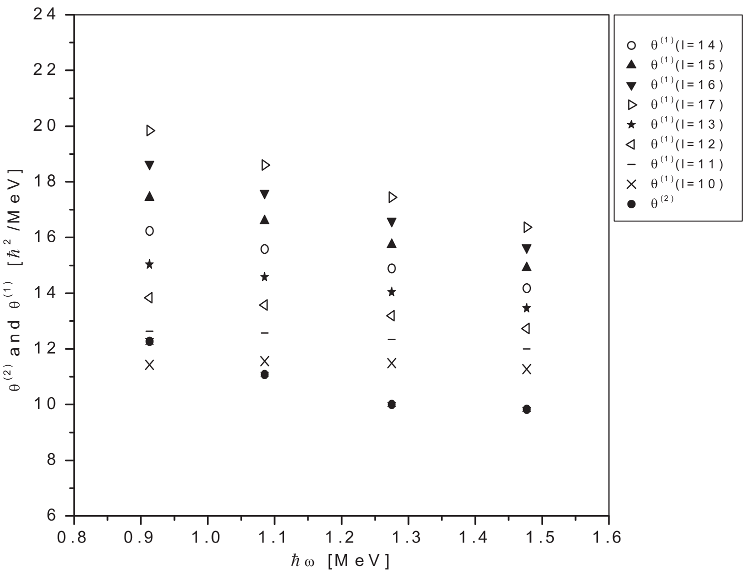

Fig. 1. Plots of the experimental dynamic, $\theta^{(2)}$, and static, $\theta^{(1)}$, moments of inertia versus rotational frequency, $\hbar\omega$, for the superdeformed (SD) band ^{58}Ni(b1)

Table 1. Spin assignments of the thirteen SD bands in the $A \sim 60$–90 mass region, E_γ corresponds to the $I_0 + 2 \rightarrow I_0$ transitions

SD bands	E_γ (keV)	Parameters		K-value	Band head spin (I_0)			
		A (keV)	B $\times 10^{-3}$	or lowest spin (I_f)	Present work	Ref. [34]	Ref. [35]	Exp. [46]
^{58}Ni(b1)	1663	27.33	10.9	11	13	14	8	15
^{58}Cu	830	21.36	20.7	6	8	–	5	9
^{59}Cu(b1)	1599	25.67	8.97	23/2	27/2	–	17/2	–
^{61}Zn	1432	18.10	3.31	31/2	35/2	–	19/2	25/2
^{62}Zn	1993	20.45	2.50	20	22	19	18	–
^{65}Zn	1341	13.62	3.55	37/2	41/2	–	27/2	–
^{68}Zn	1506	18.94	2.27	16	18	14	13	–
^{84}Zr	1526	14.15	0.85	23	25	27	21	–
^{86}Zr(b1)	1518	13.06	0.80	25	27	29	21	–
^{88}Mo(b1)	1238	10.23	1.43	25	27	30	17	–
^{88}Mo(b2)	1458	14.02	1.12	22	24	22	19	–
^{88}Mo(b3)	1260	12.59	1.19	21	23	25	18	–
^{89}Tc	1147	11.13	1.16	43/2	47/2	–	31/2	–

^{86}Zr(b1), ^{88}Mo(b1, b2, b3) and ^{89}Tc, in the $A \sim 60$–90 mass region, where b1, b2 and b3 refer to band1, band2 and band3, respectively.

The lowest spin assignments for each SD bands and also the corresponding fitting parameters are given in Table 1. It also includes the available experimental data [46] for the band head spin and our previous results for the band head spin [34, 35]. It is clear from this comparison that our present results for the band head spin are seem to relatively satisfy the available experimental data compared with that obtained in our previous works [34, 35].

The results for the transition energies in the thirteen superdeformed bands observed in $A \sim 60$–90 mass region are given in Tables 2–8, where the experimental data for the transition energies (labelled Exp^{a}) are taken from Ref. [46] and the calculated transition energies (labelled Cal^{b}) are done at the two fitting parameters (A and B) given in Table 1.

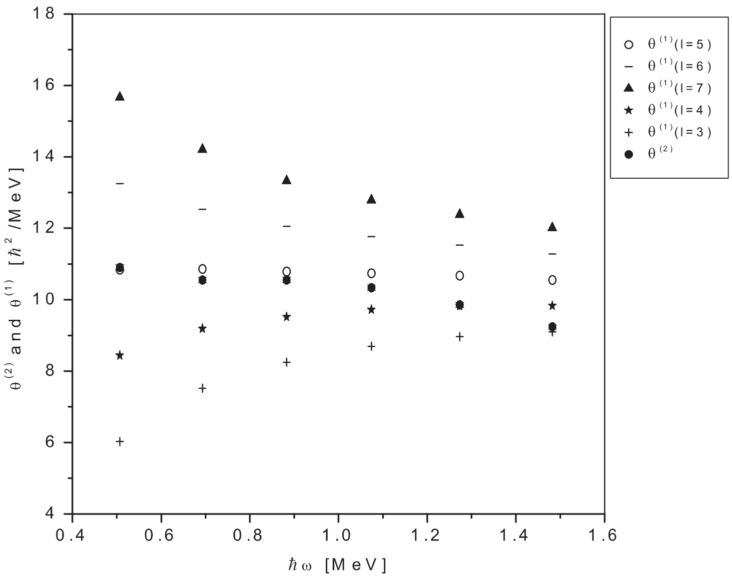

Fig. 2. The same as Fig. 1 but for the SD band ^{58}Cu

A good agreement between the calculated energy values of the SD–gamma transition and the corresponding experimental values is observed as shown in Tables 2–8. Therefore, we can arrive to a conclusion that the previous assumption for the K-value estimation by means of the frequency dependence of $\theta^{(1)}$ and $\theta^{(2)}$ is to a great extent acceptable. Also, our results for the transition energies are much better in reproducing the observed transition energies than that obtained previously [34, 35]. This reflects the suitability of using the present simple model rather than that employed in our previous ones [34, 35] in describing the SD bands in the mass region 60–90.

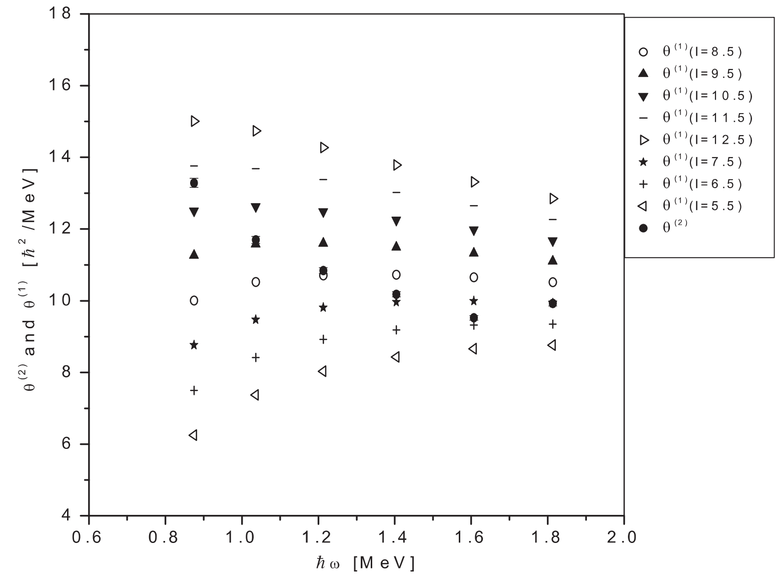

Fig. 3. The same as Fig. 1 but for the SD band ^{59}Cu(b1)

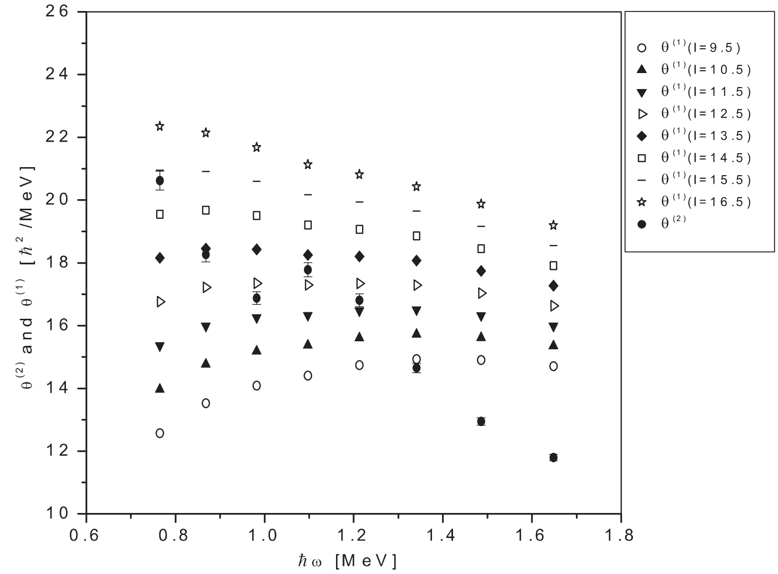

Fig. 4. The same as Fig. 1 but for the SD band ^{61}Zn

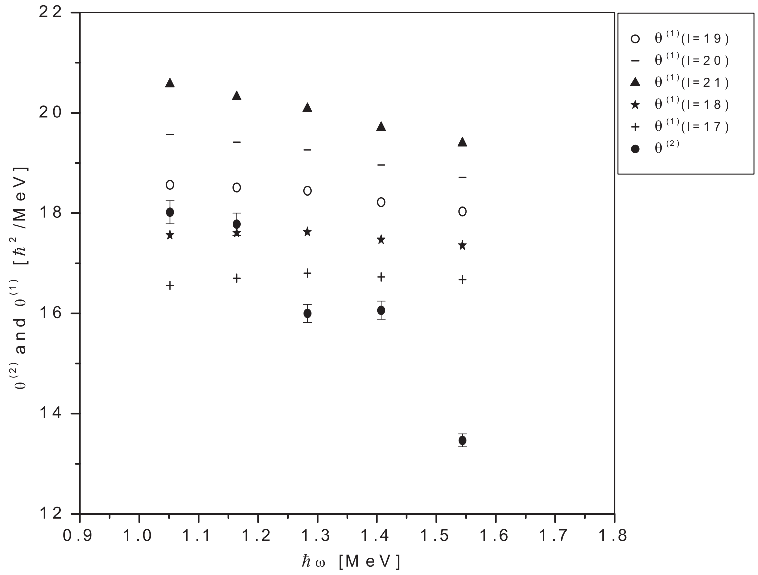

Fig. 5. The same as Fig. 1 but for the SD band ^{62}Zn

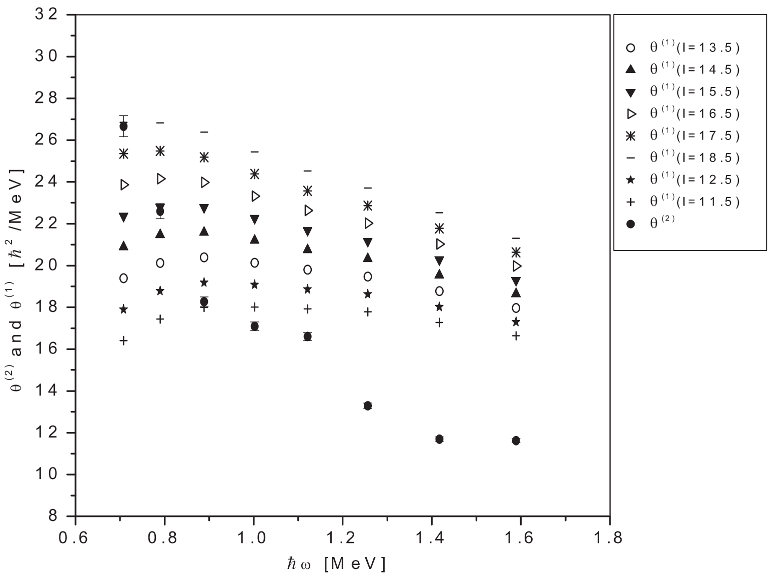

Fig. 6. The same as Fig. 1 but for the SD band ^{65}Zn

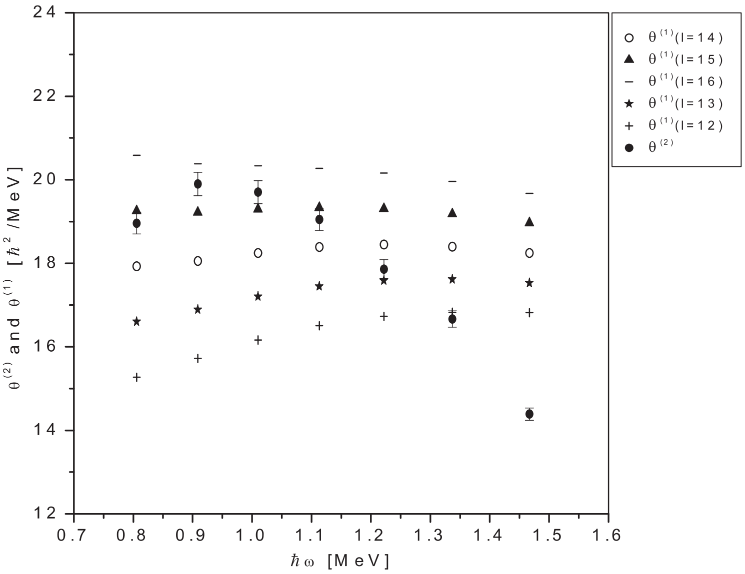

Fig. 7. The same as Fig. 1 but for the SD band ^{68}Zn

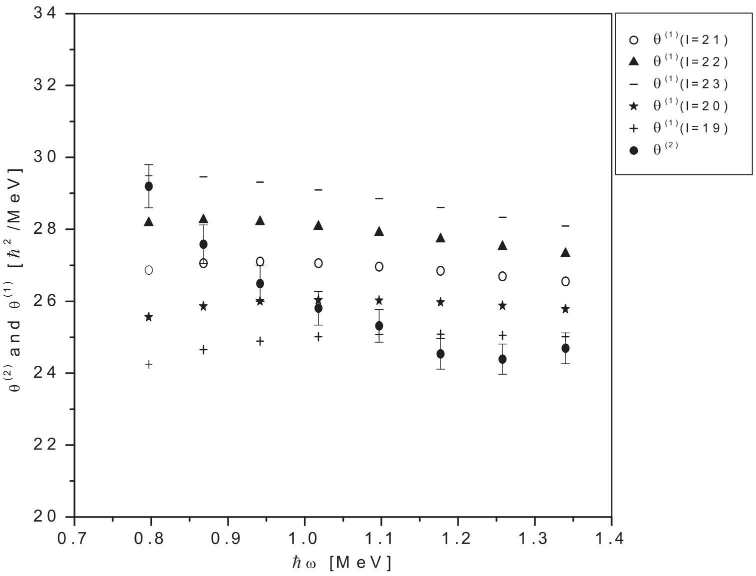

Fig. 8. The same as Fig. 1 but for the SD band ^{84}Zr

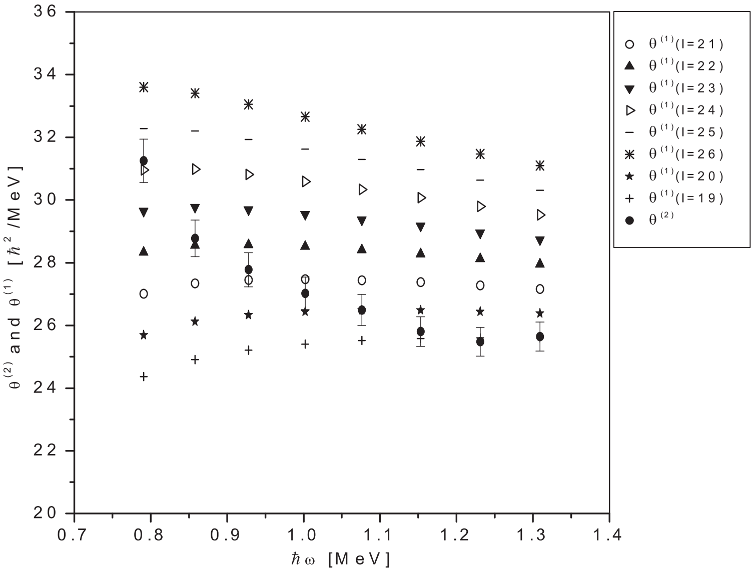

Fig. 9. The same as Fig. 1 but for the SD band ^{86}Zr(b1)

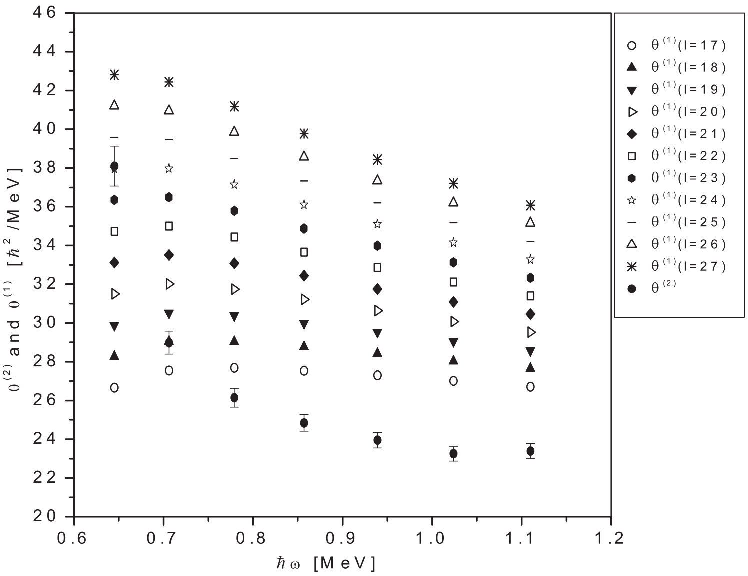

Fig. 10. The same as Fig. 1 but for the SD band ^{88}Mo(b1)

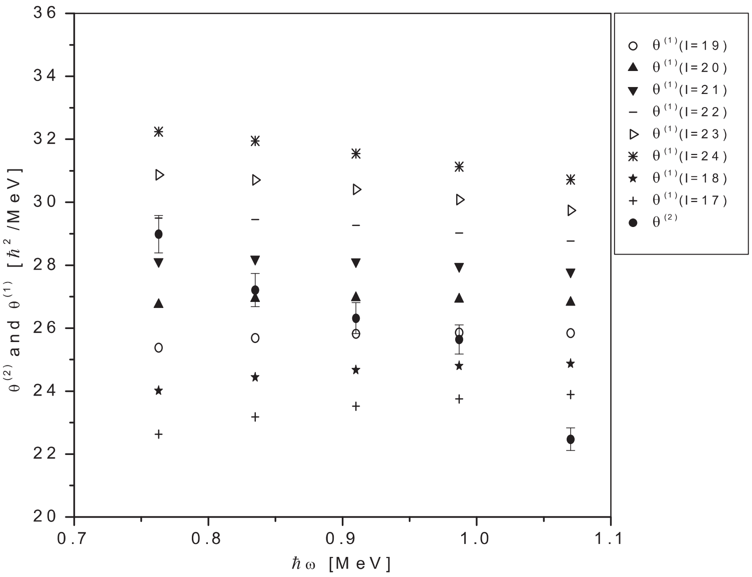

Fig. 11. The same as Fig. 1 but for the SD band ^{88}Mo(b2)

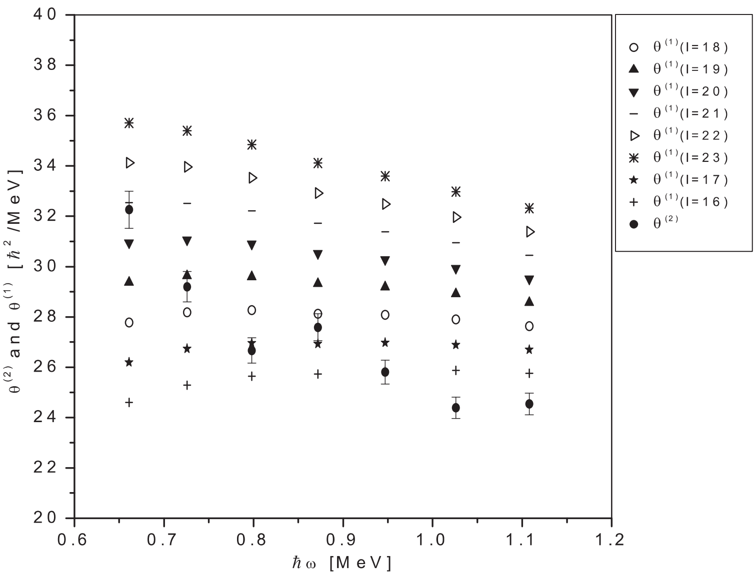

Fig. 12. The same as Fig. 1 but for the SD band ^{88}Mo(b3)

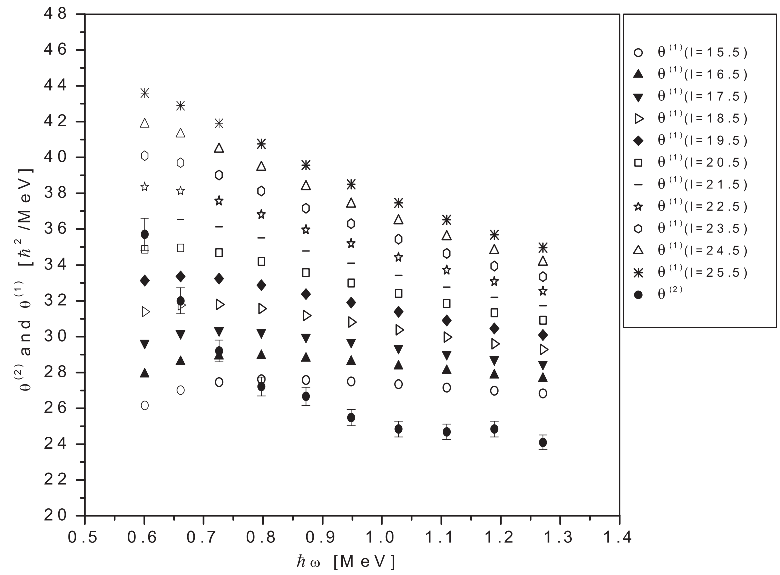

Fig. 13. The same as Fig. 1 but for the SD band ^{89}Tc

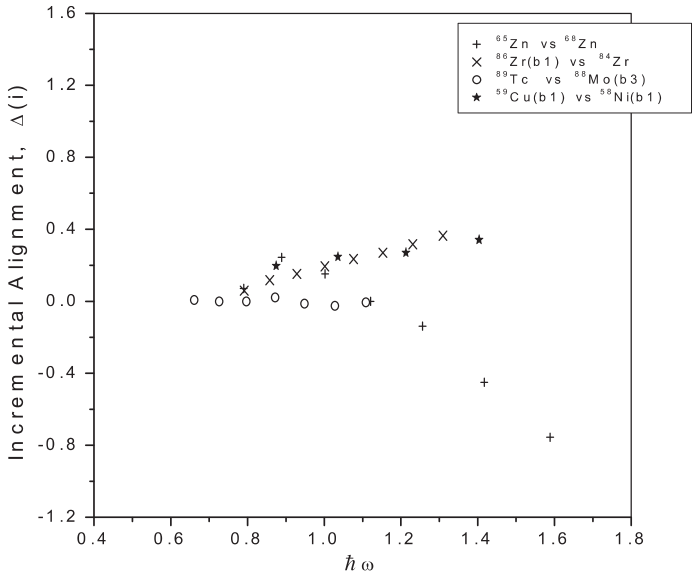

Fig. 14. Plot of the incremental alignment Δi as a function of rotational frequency $\hbar\omega$

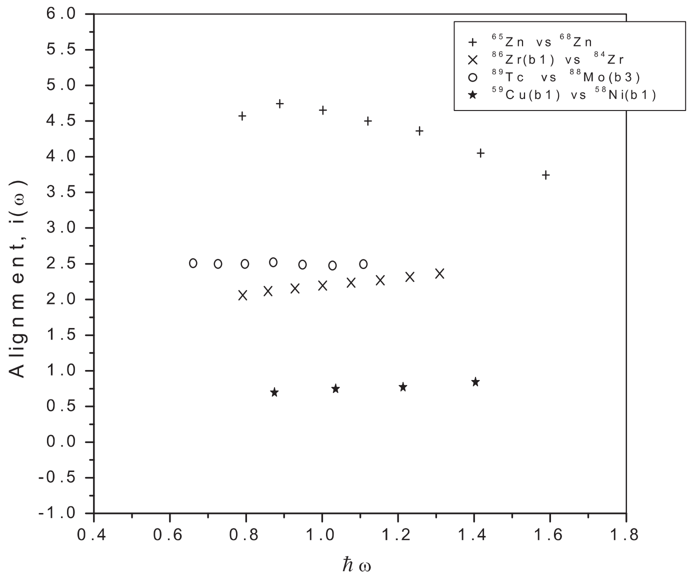

Fig. 15. Plot of the spin alignment $i(\omega)$ as a function of rotational frequency $\hbar\omega$

In general, the determination of the spins of the SD bands enables us to compare the values of the incremental alignments Δi and the alignments $i(\omega)$ of the identical bands [42]. In Tables 2–8, it is very interesting to note that when the difference in the mass number $\Delta A = 1$, the spin differences ΔI in the identical bands are just 1/2 in the case of ^{59}Cu(b1) vs. ^{58}Ni(b1) while it equals 2.5 in the case of ^{89}Tc vs. ^{88}Mo(b3), where the SD bands ^{58}Ni(b1) and ^{88}Mo(b3) are taken as reference bands and ^{59}Cu(b1) and ^{89}Tc are taken as considered bands.

Table 2. Calculation of the transition energies in ^{58}Ni(b1) and ^{58}Cu

^{58}Ni(b1)			^{58}Cu		
$E_\gamma(I)$ (keV)		Assigned I	$E_\gamma(I)$ (keV)		Assigned I
Exp[a]	Cal[b]	I	Exp[a]	Cal[b]	I
1663	1669.996	13	830	898.111	8
1989	1989.955	15	1197	1167.210	10
2350	2344.603	17	1576	1482.038	12
2750	2738.146	19	1955	1850.545	14
3157	3174.789	21	2342	2280.685	16
			2748	2780.411	18
			3181	3357.676	20

Table 3. Calculation of the transition energies in ^{62}Zn and ^{88}Mo(b2)

^{62}Zn			^{88}Mo(b2)		
$E_\gamma(I)$ (keV)		Assigned I	$E_\gamma(I)$ (keV)		Assigned I
Exp[a]	Cal[b]	I	Exp[a]	Cal[b]	I
1993	1994.218	22	1458	1457.914	24
2215	2213.918	24	1596	1598.285	26
2440	2445.859	26	1743	1744.549	28
2690	2691.000	28	1895	1897.134	30
2939	2950.300	30	2051	2056.468	32
3236	3224.721	32	2229	2222.981	34

Table 4. Calculation of the transition energies in ^{68}Zn and ^{88}Mo(b1)

^{68}Zn			^{88}Mo(b1)		
$E_\gamma(I)$ (keV)		Assigned I	$E_\gamma(I)$ (keV)		Assigned I
Exp[a]	Cal[b]	I	Exp[a]	Cal[b]	I
1506	1521.424	18	1238	1228.016	27
1717	1709.498	20	1343	1355.365	29
1918	1906.947	22	1481	1491.088	31
2121	2114.642	24	1634	1635.735	33
2331	2333.456	26	1795	1789.853	35
2555	2564.260	28	1962	1953.994	37
2795	2807.928	30	2134	2128.704	39
3073	3065.331	32	2305	2314.535	41

In these two cases, Fig. 14 shows that the difference in Δi is around 0.2 in the first case and around zero in the second case. Figure 15 shows that the alignment i is around 0.5 in the first case and 2.5 in the second case. However, if $\Delta A = 2$, the spin differences ΔI in the identical bands are just 2 in the case of ^{86}Zr(b1) vs. ^{84}Zr, where ^{84}Zr is taken as reference band and ^{86}Zr(b1) is taken as considered band. Figure 14 shows that Δi is around 0.2, while the alignment i is around 2 as indicated by Fig. 15. In the case of $\Delta A = 3$, the spin differences ΔI in the identical bands are just 4.5 in the case of ^{65}Zn vs. ^{68}Zn, where ^{68}Zn is taken as a reference band and ^{65}Zn is taken as a considered band. Figure 14 shows that Δi is close to ± 0.5, while the alignment i lies between 7/2 and 9/2 as showing by Fig. 15.

Table 5. Calculation of the transition energies in ^{84}Zr and ^{86}Zr(b1)

^{84}Zr			^{86}Zr(b1)		
$E_\gamma(I)$ (keV)		Assigned I	$E_\gamma(I)$ (keV)		Assigned I
Exp^a	Cal^b	I	Exp^a	Cal^b	I
1526	1531.478	25	1518	1522.550	27
1663	1668.354	27	1646	1652.343	29
1808	1809.876	29	1785	1786.803	31
1959	1956.369	31	1929	1926.236	33
2114	2108.161	33	2077	2070.949	35
2272	2265.576	35	2228	2221.247	37
2435	2428.941	37	2383	2377.437	39
2599	2598.581	39	2540	2539.823	41
2761	2774.824	41	2696	2708.714	43

Table 6. Calculation of the transition energies in ^{88}Mo(b3)

^{88}Mo(b3)		
$E_\gamma(I)$ (keV)		Assigned I
Exp^a	Cal^b	I
1260	1260.713	23
1384	1389.447	25
1521	1524.257	27
1671	1665.600	29
1816	1813.935	31
1971	1969.720	33
2135	2133.415	35
2298	2305.477	37

These results are in agreement with the relation between the incremental alignment Δi and the alignment i, where $i(\omega) = \Delta i + \Delta I$ and provide evidence supporting the assumption of quantized alignments in units of $(\hbar/2)$ previously suggested by Stephens et al. [42, 43]. Also, our results for the alignments leads to the so-called natural alignments, where odd-mass nuclei such as ^{59}Cu(b1), ^{89}Tc and ^{65}Zn have half integer ($\frac{1}{2}\hbar$, $\frac{3}{2}\hbar$, $\frac{5}{2}\hbar$, $\frac{7}{2}\hbar$, ...) alignments while even-mass nucleus like ^{86}Zr(b1) has integer $\sim 2\hbar$ alignment.

Table 7. Calculation of the transition energies in ^{61}Zn and ^{65}Zn

^{61}Zn			^{65}Zn		
$E_\gamma(I)$ (keV)		Assigned I	$E_\gamma(I)$ (keV)		Assigned I
Exp[a]	Cal[b]	I	Exp[a]	Cal[b]	I
1432	1436.430	17.5	1341	1310.194	20.5
1626	1632.079	19.5	1491	1491.921	22.5
1845	1841.062	21.5	1668	1689.984	24.5
2082	2064.649	23.5	1887	1905.743	26.5
2307	2304.109	25.5	2121	2140.561	28.5
2545	2560.713	27.5	2362	2395.797	30.5
2818	2835.730	29.5	2663	2672.814	32.5
3127	3130.431	31.5	3005	2972.973	34.5
3466	3446.084	33.5	3349	3297.635	36.5

Table 8. Calculation of the transition energies in ^{59}Cu(b1) and ^{89}Tc

^{59}Cu(b1)			^{89}Tc		
$E_\gamma(I)$ (keV)		Assigned I	$E_\gamma(I)$ (keV)		Assigned I
Exp[a]	Cal[b]	I	Exp[a]	Cal[b]	I
1599	1615.143	13.5	1147	1150.603	23.5
1900	1908.596	15.5	1259	1268.620	25.5
2242	2231.337	17.5	1384	1392.629	27.5
2611	2586.812	19.5	1521	1523.075	29.5
3004	2978.466	21.5	1668	1660.402	31.5
3424	3409.745	23.5	1818	1805.052	33.5
3827	3884.094	25.5	1975	1957.472	35.5
			2136	2118.103	37.5
			2298	2287.391	39.5
			2459	2465.778	41.5
			2625	2653.710	43.5

4. Conclusion

We have obtained the values of the lowest spins I_f, the values of the band head spins I_0, and the K-values of the thirteen superdeformed bands, namely, ^{58}Ni(b1), ^{58}Cu, ^{59} Cu(b1), ^{61}Zn, ^{62}Zn, ^{65}Zn, ^{68}Zn, ^{84}Zr, ^{86}Zr(b1), ^{88} Mo(b1, b2, b3) and ^{89}Tc, by studying the frequency dependence of the dynamic, $\theta^{(2)}$ and static, $\theta^{(1)}$

moments of inertia. Using a simple model in which the rotational energy $E_{\rm rot}(I,K)$ depends upon the spins K and I, the calculated transition energies are found to be well reproduced their observed values rather than that obtained by using other approaches [34, 35]. Our good results for the spins and transition energies reflect that the fitting parameters (A and B) obtained here for the considered SD bands are acceptable and the simple model employed here is more appropriate to use than others. From spin determination of the identical bands, it appears that for many, but not all of these superdeformed rotational bands, the alignments tend to be quantized in units of $\frac{1}{2}\hbar$ or $1\hbar$. The agreement between the lowest spin and the assumption of quantized alignment in unit of $\frac{1}{2}\hbar$ illustrates the validity of the proposed simple model.

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