Acta Phys. Hung. A 25/1 (2006) 105–115
DOI: 10.1556/APH.25.2006.1.10

**HEAVY ION PHYSICS**

# Erratic Fluctuations in 14.5A GeV/c $^{28}$Si–AgBr Collisions

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*Received 6 August 2005*

**Abstract.** Analysis of multiparticle production data on 14.5A GeV/c $^{28}$Si–AgBr collisions in terms of erraticity is carried out and the results are compared with those obtained from the Monte Carlo simulated data (using event generator HIJING). It is shown that like the multifractal spectrum through $G_q$ moments, erraticity spectrum may also be constructed from the observed power-law behaviour of the erraticity moments. Further, for examining the dominance of statistical fluctuations over the erraticity behaviour, correlation-free Monte Carlo events are simulated and analyzed. A comparison of the experimental and simulation results indicates that the fluctuations observed in the case of experimental data are not only because of the statistical reasons, but may have some dynamical origin.

*Keywords:* relativistic nucleus–nucleus collisions, event-by-event fluctuations, intermittency, erraticity
*PACS:* 25.75.-q, 25,70.pq, 24.60.Ky

## 1. Introduction

Power-law behaviour of the scaled factorial moments, $F_q$, referred to as the intermittency [1] has been extensively used to investigate fluctuations and chaos in multiparticle production in high energy hadronic and heavy-ion ($AA$) collisions [2–6]. These investigations reveal that the presence of large fluctuations in small phase space bins may be rare but not impossible. For studying power-law behaviour, horizontally averaged vertical moments, $F_q^v$ and vertically averaged horizontal moments, $F_q^h$ are calculated. Although the two definitions are complementary and the scaling behaviour, $F_q \propto M^{\Psi_q}$, in either case, is expected to be satisfied, yet none of the two fully account for all the fluctuations that a system may exhibit because of

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the averaging procedures adopted. The former approach would help study fluctuations on event-by-event (e-by-e) basis, however, information regarding the spatial distribution of particles are suppressed. On the other hand, the latter method is useful for investigating the spatial pattern of particles in an event while information on e-by-e fluctuations are lost. Recently, Hwa [7] has pointed out that the values of the vertically averaged horizontal moments, $F_q^{(e)}$, if calculated on e-by-e basis, should exhibit large fluctuations and therefore, a distinct distribution of $F_q^{(e)}$ for a given $q$ and $M$ may be observed for a given sample of events; $q$ is the order of moment and $M$ is the number of equally spaced bins in the pseuodorapidity ($\eta$) space. Such a distribution is envisaged to help disentangle some useful and interesting information about chaotic behaviour of multiparticle production. A few moments of $F_q^{(e)}$ distribution, for example, the normalized moments $C_{p,q}$ are likely to serve the purpose. If $C_{p,q}$ exhibit a power-law behaviour then such a behaviour is referred to as erraticity [7–9]. It may be stressed that erraticity analysis would take into account simultaneously spatial as well as e-by-e fluctuations beyond intermittency. The erraticity analysis performed for the events simulated in the framework of perturbative QCD indicates that the behaviour of multiparticle production is chaotic [10]. Analysis of experimental data on hadronic and heavy-ion collisions at high energies also reveal that erraticity exists in all these cases [8, 9, 11, 1, 13]. However, these studies are not conclusive. It was, therefore, considered worthwhile to examine erraticity behaviour in relativistic $AA$ collisions. Attention is focussed on the behaviour of erraticity exponents and erraticity spectrum which are likely to provide maximum information on self-similar fluctuations [7]. Hence analysis of the experimental data on 14.5A GeV/c $^{28}$Si–AgBr collisions is carried out. Events caused due to the interactions with AgBr group of nuclei are considered for the present investigation for making the observations for the semi-central and central collisions. Multiplicity of relativistic charged particles produced in such events is expected to be rather larger and hence probability of getting the empty phase space bins are reduced.

## 2. Experimental Details

A stack of ILLFORD-G5 emulsion exposed to 14.5A GeV/c silicon ions from AGS, BNL, has been used in the present study. The events due to the AgBr group of targets were selected by using the criterion that the number of heavily ionizing tracks, $n_h$, in an interaction must be $\geq 8$. By applying this criterion, 274 interactions from a random sample of 505 events characterized by $n_h \geq 0$ produced in the interactions of $^{28}$Si ions with emulsion nuclei were considered. The other relevant details regarding scanning procedure, criteria of event selection, classification of tracks, methods of measurements, etc., may be found elsewhere [14, 15].

In order to compare our findings with the predictions of the QCD inspired models, 14025 events similar to the experimental ones are generated using the Monte Carlo (MC) code, HIJING-1.33 [16]. These events are generated according to the percentage of interaction of the incident beam with different targets in emulsion [17]. These events are also analyzed.

## 3. Method of Analysis

A detailed description about the method of erraticity analysis may be found in Refs. [7–9, 11–13]. However, a brief description is considered necessary and is, therefore, presented here. It may be interesting to note that the single particle distribution in pseudorapidity ($\eta$) space is non-flat. For reducing the effect of non-flatness, the $\eta$ values are, therefore, transformed into the cumulative variable, $X(\eta)$, introduced by Bialas and Gazdzicki [18], where

$$X(\eta) = \frac{\int_{\eta_{\min}}^{\eta} \rho(\eta) d(\eta)}{\int_{\eta_{\min}}^{\eta_{\max}} \rho(\eta) d(\eta)} \,, \tag{1}$$

where $\rho(\eta)$ is single particle rapidity density, and $\eta_{\min}$ and $\eta_{\max}$ represent, respectively, the minimum and maximum values of $\eta$-range considered. The vertically averaged normalized factorial moments are calculated [7, 9] from

$$F_q^h = \frac{1}{N_{\rm ev}} \sum_{i=1}^{N_{\rm ev}} F_q^{(e)}, \tag{2}$$

where $N_{\rm ev}$ is the number of events in a sample and $F_q^{(e)}$ is the event factorial moment describing the spatial pattern of an event and is calculated from

$$F_q^{(e)} = \frac{\langle n(n-1)\ldots(n-q+1)\rangle_e}{\langle n\rangle_e^q} \,, \tag{3}$$

where $q$ is order of the moment and $n$ is the charged particle multiplicity in a particular bin. To quantify e-by-e fluctuations in $F_q^{(e)}$, the normalized moments, $C_{p,q}$ are calculated [11] using

$$C_{p,q} = \langle \Phi_q^p \rangle \,, \qquad \Phi_q = \frac{F_q^{(e)}}{\langle F_q^{(e)} \rangle} \,. \tag{4}$$

The order $q$ is an integer while $p$ takes on any value >0. If $C_{p,q}$ exhibit a power-law behaviour of the type,

$$C_{p,q} \propto M^{(\psi_p(q))} \tag{5}$$

for a given $q$, then such a behaviour is referred to as erraticity [7]; $\psi_p$ is the erraticity exponent. The derivative of $\psi_p$ around $p = 1$,

$$\mu_q = \frac{d}{dp} \psi_p(q) \Big|_{p=1} \tag{6}$$

describes the degree of e-by-e fluctuations, and is referred to as entropy index [10]. Another entropy like quantity $\Sigma_q$ is defined as:

$$\Sigma_q = \langle \Phi_q \ln \Phi_q \rangle \,, \tag{7}$$

and the entropy index $\mu_q$ may also be calculated from $\Sigma_q$ using:

$$\mu_q = \frac{\delta \Sigma_q}{\delta \ln M} \,. \tag{8}$$

Although, the scaling behaviour exhibited by $C_{p,q}$ moments is opposite to that observed in the case of $G_q$ moments [7], yet, like multifractal spectrum, erraticity spectrum $e(\alpha_p)$ may be defined for a given $q$ as:

$$e(\alpha_p) = p\alpha_p - \psi_p \,, \tag{9}$$

where $\alpha_p = d\psi_p/dp$. The function $e(\alpha_p)$ describes certain properties of erraticity more directly than $\psi_p$. It is clear from these definitions that $\alpha_1(q) = \mu_q$ and for this value of $p(=1)$, $e(\alpha_p) = \alpha_p$. On the other hand, for all other values of $p$, $e(\alpha_p)$ will be greater than $\alpha_p$.

## 4. Results and Discussion

Pseudorapidity values of the relativistic charged particles produced in each interaction in the $\eta$-range, $\eta_0 \pm 3.0$ are transformed into the cumulative variable $X(\eta)$, where $\eta_0$ is the centre-of-mass hadron–nucleus rapidity. Values of $C_{p,q}$ are calculated for different values of $M$, $p$ and $q$, and the variations of $\ln C_{p,q}$ with $\ln M$ are shown in Fig. 1; the error bars are shown for the alternate sets of data to avoid overlapping. Values of $\Sigma_q$ for various $q$ values are evaluated using Eqs. (4), (7). In Fig. 2, dependence of $\Sigma_q$ on $\ln M$ are shown for $q = 2$, 3 and 4. It is evident from Figs. 1 and 2 that the values of $\ln C^p_q$ and $\Sigma_q$ for the experimental and HIJING data are nearly the same. The slight difference in the experimental and HIJING predicted values, as is evident in the figures, might arise due to the differences in $\langle n_s \rangle$ values and the shapes of the $n_s$ distributions observed for the experimental and HIJING events; $n_s$ being the multiplicity of relativistic charged particles produced in an event. The values of $\langle n_s \rangle$ for the experimental and HIJING data samples are found to be respectively $27.64 \pm 1.04$ and $33.88 \pm 0.07$, while the $n_s$ distribution corresponding to the HIJING events is observed to have peak at relatively lower values of $n_s$ and have a longer tail in comparison to the experimental $n_s$ distribution. It has been reported [8,9,19] that at lower energies erratic fluctuations are independent of the collision energy, phase space cut and mass of the beam/target nuclei. However, the magnitude of the moments $C_{p,q}$ and $\Sigma_q$ decrease with increasing multiplicity and exhibit a slight dependence on the shape of the $n_s$ distribution [20].

Values of entropy index, $\mu_q (= d\psi_p(q)/dp|_{p=1})$ are calculated for $q = 2$, 3 and 4. These values are determined by evaluating $\psi_p$ at $p = 0.9$ and 1.1. Dependence of $\mu_q$

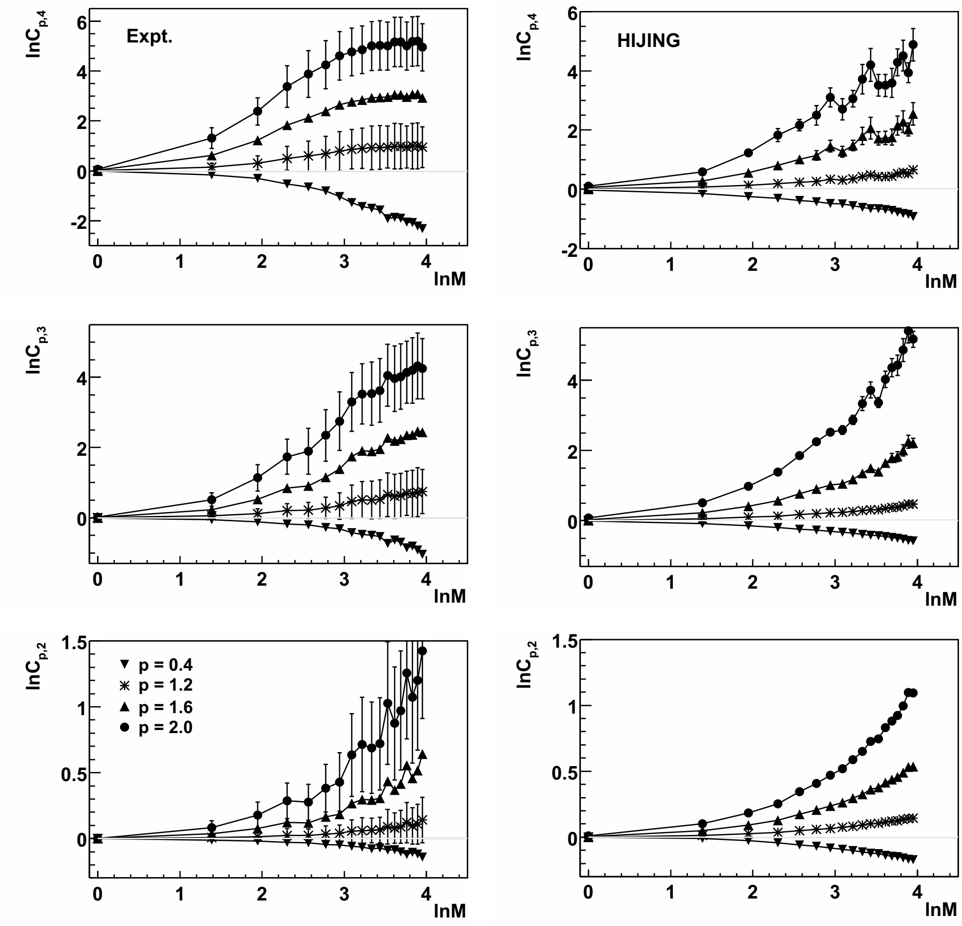


**Fig. 1.** Variation of $\ln C_{p,q}$ with $\ln M$

on $q$ is displayed in Fig. 3. It is evident in the figure that the trend of variation $\mu_q$ with $q$ observed experimentally are nicely reproduced by the HIJING data. Since higher values of $\mu_q$ corresponds to smaller entropy and more chaoticity [10, 21], it may be concluded that the experimental data used in the present study clearly exhibits the chaotic nature of multiparticle production in relativistic $AA$ collisions.

In order to disentangle further information about the spatial and e-by-e fluctuations, the values of erraticity exponents, $\psi_p$, are determined by plotting $\ln C_{p,q}$ against $\ln M$ and doing the fits in the linear region ($M = 5$–$20$). Using these values of $\psi_p$, the values of $\alpha_p(= d\psi_p/dp)$ are estimated and the erraticity spectrum, $e(\alpha_p)$ is obtained. Dependence of $\psi_p$ on $p$ are shown in Fig. 4. It is seen in the figure

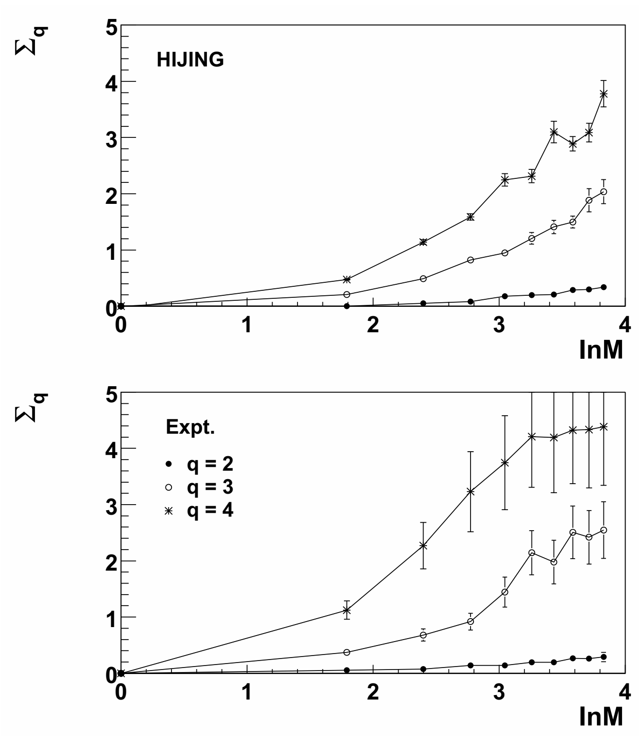


**Fig. 2.** Variation of $\Sigma_q$ with $\ln M$

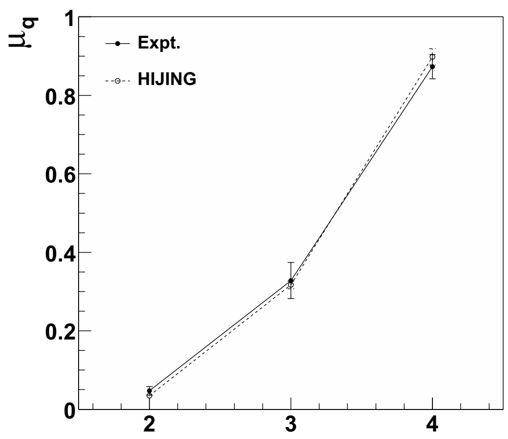


**Fig. 3.** $\mu_q$ vs. $q$ plots for the experimental and HIJING data

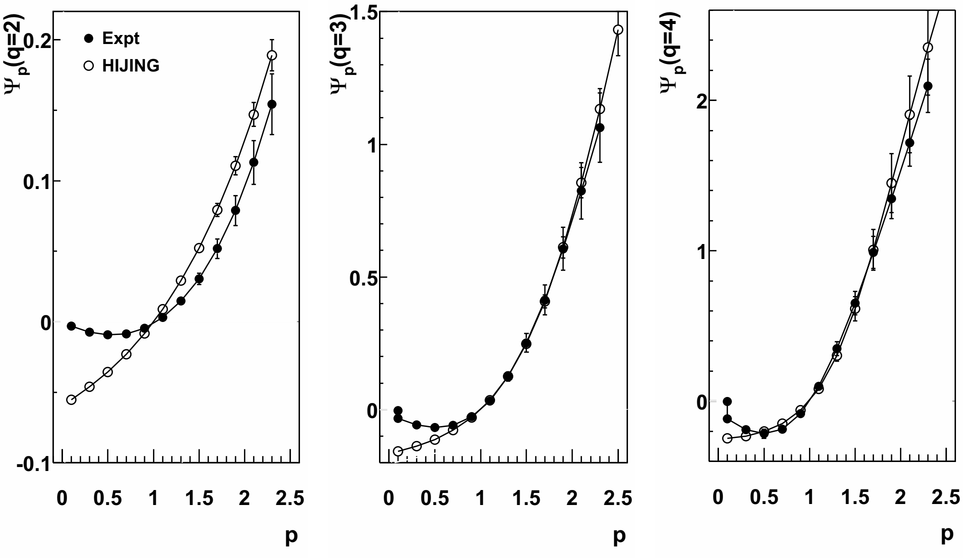


**Fig. 4.** Variations of erraticity exponents, $\psi_p$ with $p$. The lines are to guide the eyes

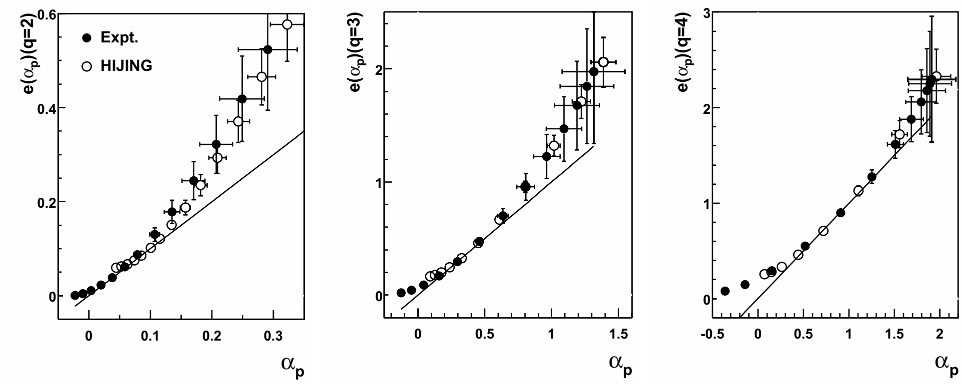


**Fig. 5.** Erraticity spectra, $e(\alpha_p)$ for $q = 2$, 3 and 4. The straight lines correspond to $e(\alpha_p) = \alpha_p$

that the experimental values of $\psi_p$ are quite close to the corresponding values obtained for HIJING-MC events. It has been reported [22] that the values of $\psi_p$ vary linearly with $p$ in the case of 147 GeV/c hadron–proton collisions. However, in the present study, a linear dependence of $\psi_p$ on $p$ is observed for the values of $p > 1.5$, which, incidentally, is in fine accord with the predictions of random-cascading $\alpha$ model [19]. The erraticity spectra, $e(\alpha_p)$ vs. $\alpha_p$, for $q = 2$, 3 and 4 are displayed in

Fig. 5. The straight lines in the figure correspond to $e(\alpha_p) = \alpha_p$. It is interesting to notice in Fig. 5 that the lines are tangent to the respective curves at points for $p \simeq 1$. At this point, in each case, the parameter $e(\alpha_p)$ acquires a minimum value equal to $\alpha_p$. This would, therefore, make entropy index, $\mu_q$ to satisfy $\mu_q = \alpha_1(q)$. However, for other values of $p$, $e(\alpha_p)$ is comparatively higher than $\alpha_p$. It is also evident from Figs. 4, 5 that experimental and HIJING data exhibits the erraticity behaviour more or less on the same scale.

It has been pointed out [8, 9, 12] that the observed erratic fluctuations in high energy hadronic and heavy-ion collisions are mostly due to the statistical reasons. For investigating the dominance of statistical fluctuations over the erraticity behaviour, therefore, correlation-free MC events are obtained in the framework of independent emission hypothesis (IEH) model, which is based on the following assumptions [14, 15]:

i) Multiplicity distribution of the simulated data sample should be similar to the experimental one,

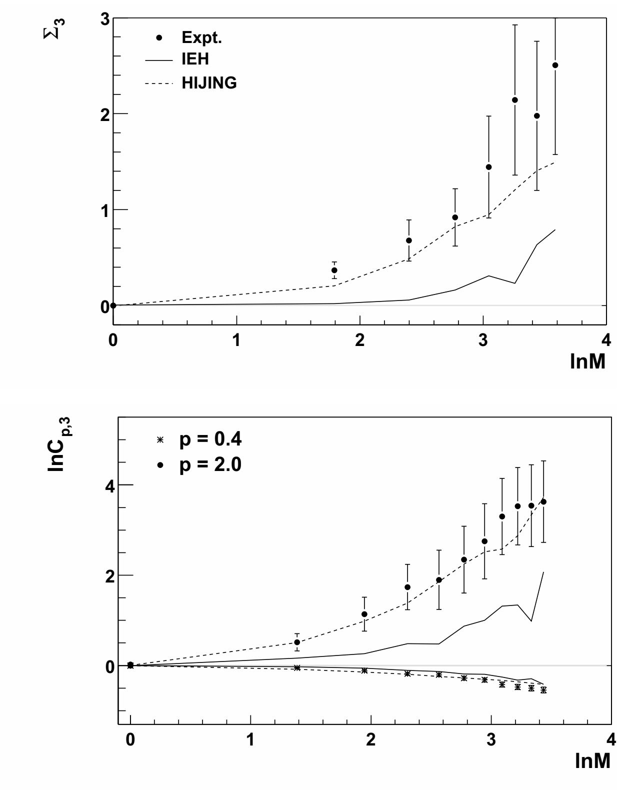


**Fig. 6.** Variations of $\Sigma_q$ and $\ln C_{p,q}$ with $\ln M$ for the experimental HIJING and IEH data

ii) $X(\eta)$ values of all the charged particles produced in each event should be uniformly distributed in the range 0−1 and

iii) there should be no correlation amongst the emitted particles.

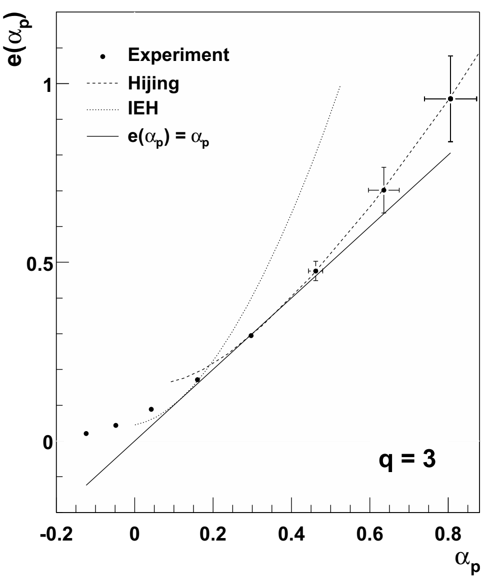


**Fig. 7.** Erraticity spectra for $q = 3$. The straight line (solid) corresponds to $e(\alpha_p) = \alpha_p$

By adopting these criteria, a sample having the same number (274) of events as in the experimental data, are generated and analyzed. Variations of $\ln C_{p,q}$ and $\Sigma_q$ with $\ln M$ for the experimental, HIJING and IEH simulated data are displayed in Fig. 6. The erraticity spectra for these data sets are displayed in Fig. 7. It is noted from these figures that the trends of variations of $\ln C_{p,q}$ and $\Sigma_q$ with $\ln M$ and the shape of the erraticity spectra are almost similar for the experimental and HIJING event samples. The slight difference in the the values of the two parameters for large $M$ values has already been discussed. It is, however, interesting to notice in Fig. 6 that the values of the two moments, $C_{p,q}$ and $\Sigma_q$, corresponding to the IEH data are significantly smaller as compared to those obtained from the experimental data. The erraticity spectra from the IEH event sample too show up on significantly different scale in comparison to the spectra corresponding to the experimental/HIJING data. These observations, therefore tend to suggest that the erratic fluctuations observed in the present study are not only due to the statistical reasons, but might have some contribution due to some dynamical reasons too.

## 5. Conclusions

Based on the present investigations, following conclusions may be arrived at:

1. The observed power-law behaviour of the normalized moments, $C_{p,q}$, indicates that erratic fluctuations exhibited by the experimental data are somewhat larger as compared to those obtained from the MC simulations (correlation-free events).
2. The observed trend of $\mu_q$ dependence on $q$ agrees fairly well with the predictions of QCD based models — HIJING — and indicates chaotic nature of particle production in $AA$ collisions.
3. Similar to the multifractal spectrum, erraticity spectrum may also be obtained which may help disentangle useful information regarding the entropy and(or) chaoticity in particle production phenomenon.
4. Comparison of various findings based on the experimental, HIJING and IEH-MC data suggest that the observed fluctuations in the case of experimental data might have some dynamical origin.

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