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HEAVY ION PHYSICS

Inelastic Multiparticle Collisions in a Parton Cascade

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Abstract. We elaborate on a new 3+1-dimensional Monte Carlo parton cascade solving kinetic Boltzmann processes including — for the first time — inelastic multiplication processes ($gg \leftrightarrow ggg$) in a unified manner. The back reaction channel is treated fully consistently, exactly obeying detailed balance. The algorithm can handle, in principle, any specified initial conditions for the freed partons, the latter being on their kinetic mass shell. First full simulations with minijet initial conditions demonstrate that the inclusion of the inelastic channels leads to a very fast kinetic and chemical equilibration and also to an early creation of pressure.

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1. Introduction, Motivation and Summary

The prime intention for present ultrarelativistic heavy ion collisions at CERN and at Brookhaven lies in the possible experimental identification of a new state of matter, the quark–gluon plasma (QGP). Recent measurements [1] at RHIC of the elliptic flow parameter v_2 for nearly central collisions suggest that — in comparison to fits based on simple ideal hydrodynamical models — the evolving system builds up a sufficiently early pressure and potentially also achieves (local) equilibrium. On the other hand, the system in the reaction is at least initially far from any (quasi-)equilibrium configuration.

To describe the dynamics of ultrarelativistic heavy ion collisions, and to address the crucial question of thermalization and early pressure build-up, we have developed a kinetic parton cascade algorithm [2] inspired by perturbative QCD including for the first time inelastic ('bremsstrahlung') collisions $gg \leftrightarrow ggg$ besides

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the binary elastic collisions. It is the aim to get a more detailed and quantitative understanding of the early dynamical stages of deconfined matter and to test various initial conditions for the liberated gluons, than invoking ad hoc phenomenological, hydrodynamical ansaetze. It is well known, that a parton cascade analysis, incorporating only elastic (and forward directed) $2 \leftrightarrow 2$ collisions described via one-gluon exchange, shows that thermalization and early (quasi-)hydrodynamical behavior (for achieving sufficient elliptic flow) cannot be built up or maintained, but only if a much higher cross section is being employed [3]. Hence, the collective flow phenomena observed at RHIC seem to indicate that the early evolution of deconfined matter cannot be described with standard pQCD inspired interactions, but can only be due to much stronger and yet unknown interactions. This would suggest that a QGP cannot be described via pQCD. On the other hand, the possible importance of the inelastic reactions on overall thermalization was raised in the so called 'bottom up thermalization' picture [4]. It is intuitively clear that gluon multiplication should not only lead to chemical equilibration, but also should lead to a faster kinetic equilibration. This represents one (but not all) important motivation for developing a consistent algorithm to handle also inelastic processes.

In the next section we briefly detail on and show some numerical tests of this new scheme, treating elastic and inelastic multiplication collisions in a *unified* manner [2]. Most importantly, the (multiparticle) back reaction channel ($ggg \to gg$) is treated fully consistently by respecting detailed balance within the *same* algorithm. Incorporating this algorithm in a full 3+1-dimensional Monte Carlo cascade, we achieve a covariant parton cascade which can accurately handle the immense elastic as well as inelastic scattering rates occurring inside the dense (gluonic) system. Furthermore, we can then address the question of the importance of such (still) pQCD inspired reactions on the thermalization and early pressure build for heavy ion collisions at RHIC. The algorithm can incorporate any specified initial conditions for the freed on-shell partons. First results, taking as a conservative point of view minijet initial conditions, will be given in the last section: The exploratory study shows already that indeed the gluon multiplication via bremsstrahlung (and absorption) is of utmost importance, where kinetic equilibration (for the present setting of initial conditions) is achieved on a timescale of about 1 fm/c, whereas the full chemical equilibration occurs on a somewhat slower scale of about 2–3 fm/c.

2. Treatment of Elastic and Inelastic Multiparticle Collisions in a Unified Scheme

In developing the algorithm special emphasis is put on obeying the principle of detailed balance among the gain and loss contributions. The standard incorporation of $2 \leftrightarrow 2$ scattering processes in a transport description is based on the geometric interpretation of the cross section [5]. For large particle densities, however, such an implementation leads to considerable problems to generate a causal collision sequence among the various partons, resulting in a severe reduction of the collision

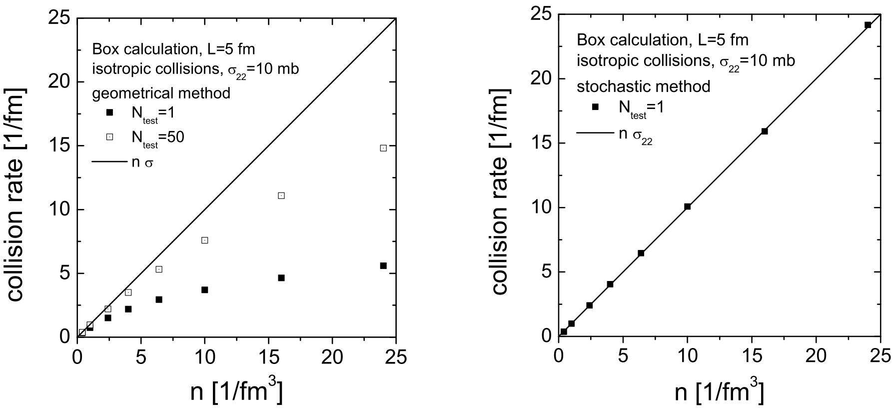

Fig. 1. 2 by 2 elastic collision rate in a simulation of propagating particles in a box versus the ideal rates $1/(n\sigma)$. The total cross section is fixed to 10 mb. In the left (right) figure the geometrical (stochastic) method is employed

rate compared to the one dictated by the Boltzmann equation. This numerical artifact is demonstrated in left panel of Fig. 1. In principle, this can be cured by the test particle method specifying each physical particle a specified number of test particles (see left panel of Fig. 1 and also see [5]). However, for such a situation of an ultra-dense plasma, where the mean free path is in the order or comes below the interaction length $\sqrt{\sigma/\pi}$, the stochastic method (or 'particle in cell' method, see e.g. [6]) is better suited to solve the Boltzmann equation directly via transition rates in sufficiently small spatial cells, see right panel of Fig. 1. The actual collision rate being realized in the simulation fully equals the ideal one of the Boltzmann equation which shall be simulated. One can go to arbitrary high densities. Moreover, this method can, in principle, be generalized to (any) multiparticle scattering processes.

As outlined in the Introduction, we want to incorporate the direct bremsstrahlung process $gg \leftrightarrow ggg$. Detailing on the back reaction, we define a transition probability in a time interval with $0 \leq P_{32} \ll 1$ for a given triplet of gluons with specific momenta in a sufficiently small local cell:

$$P_{32} = \frac{\Delta N_{\mathrm{coll}}^{3\to 2}}{\Delta N_1 \Delta N_2 \Delta N_3} = \frac{1}{8E_1E_2E_3}\frac{I_{32}}{N_{\mathrm{test}}^2}\frac{\Delta t}{(\Delta^3 x)^2}\,. \qquad (1)$$

Here I_{32} is defined as the to be calculated phase space integral

$$\frac{1}{2!}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'}|\mathcal{M}_{123\to 1'2'}|^2(2\pi)^4\delta^{(4)}(p_1+p_2+p_3-p_1'-p_2')\,.$$

$\Delta^3 x$ denotes the volume of the specific cell and N_{test} represents the number of test particles. (The incorporation of test particles can also be applied, in addition, to achieve a better statistics [2].) The entering matrix element one obtains from the

$2 \to 3$ element via a standard prefactor governed by a detailed balance relation. The latter is given by

$$|\mathcal{M}_{gg\to ggg}|^2 = \left(\frac{9g^4}{2}\frac{s^2}{(\mathbf{q}_\perp^2 + m_D^2)^2}\right)\left(\frac{12g^2\mathbf{q}_\perp^2}{\mathbf{k}_\perp^2[(\mathbf{k}_\perp - \mathbf{q}_\perp)^2 + m_D^2]}\right)\theta(k_\perp\Lambda_g - \cosh y)\,, \tag{2}$$

where $g^2 = 4\pi\alpha_s$. $\mathbf{q}_\perp$ and $\mathbf{k}_\perp$ are the perpendicular component of the momentum transfer and that of the momentum of the radiated gluon in the c.m. frame of the collision, respectively. y denotes the rapidity of the radiated gluon. We thus consider $gg \to ggg$ in leading order of pQCD and employ an effective Landau–Pomeranchuk–Migdal suppression with Λ_g denoting the gluon mean free path, which is given by the inverse of the total gluon collision rate $\Lambda_g = 1/R_g$, and also employ a standard screening mass m_D for the infrared sector of the scattering amplitude.

To demonstrate the applicability of the method, we show in the following the formation of a quark–gluon plasma within a fixed box and study the way of kinetic and chemical equilibration for different parton species. The initial conditions of the partons entering the cascade are given by multiple minijet production from the binary nucleon–nucleon scattering in a nucleus–nucleus collision, where we have chosen a conservatively large transverse momentum cutoff of $p_t > p_0 = 2$ GeV/c [7], according to the differential jet cross section:

$$\frac{d\sigma_{\rm jet}}{dp_T^2 dy_1 dy_2} = K \sum_{a,b} x_1 f_a(x_1, p_T^2)\, x_2 f_b(x_2, p_T^2) \frac{d\sigma_{ab}}{d\hat{t}}\,. \tag{3}$$

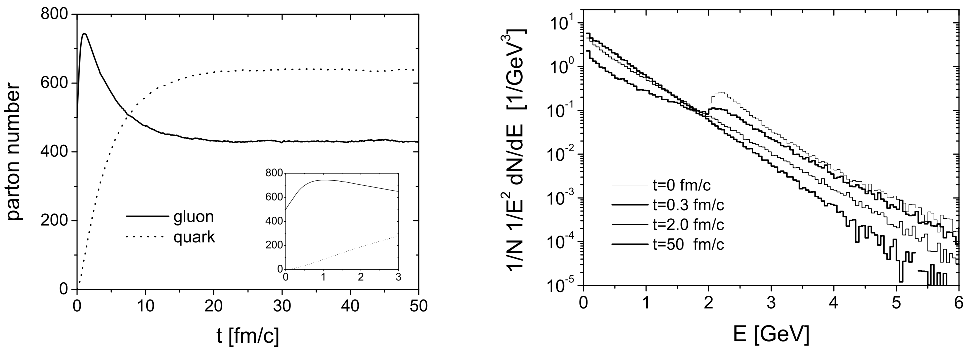

Fig. 2. Left panel: Time evolution of the gluon and quark numbers for a typical box calculation. The gluon number starts at 500. Right panel: Energy spectra at different times within the box calculation. At $t = 0$ only minijets with $p_t > 2$ GeV are populated. Energy degradation to lower momenta proceeds rapidly by gluon emission. At $t \approx 0.5$–1 fm/c full kinetic as well as chemical equilibrium among the gluons is reached. The quark degrees of freedom do equilibrate on a much slower timescale, roughly one order of magnitude larger

p_T denotes the transverse momentum and y_1 and y_2 are the rapidities of the produced partons. The minijets are considered in the central rapidity interval $(-0.5 : 0.5)$ at RHIC energy of $\sqrt{s} = 200$ GeV. The initial partons are distributed uniformly in a cubic box of size 3 fm, which approximately corresponds to the central region of heavy ion collisions at some early initial time.

Figure 2 shows the time evolution of the gluon and quark number. Gluon equilibration undergoes two stages: at first the gluon number increases rapidly and then smoothly evolves to its final equilibrium value together with the quark number at a much lower rate. The early stage of gluon production, on a timescale of 0.5 fm/c, also leads to an immediate kinetic equilibration of the momentum distribution (see also the right panel of Fig. 2), as well as to a rather abrupt lowering of the temperature by soft gluon emission until equal balance among gain and loss contributions in the transitions is reached. The later slower evolution is then governed by chemical equilibration of the quark degrees of freedom. The final temperature is identical to the slope parameter of the late energy spectra depicted in the right panel of Fig. 2.

3. Thermalization in a Real Cascade Operating at RHIC

For implementing this scheme into a 3-D operating cascade, the important task is to develop a dynamical mesh of (expanding) cells in order to handle the extreme initial situation [2]. For the initial out-of-equilibrium conditions we take minijets (3) with a cutoff of 2 GeV and which are distributed in space-time via the corresponding overlap function. The so produced number of initial gluons is about 900 with a $dN_g/dy \approx 200$ at midrapidity. This is a rather low value, but we keep to this conservative estimate for our first exploratory study. In the following we also restrict ourselves only to the dynamics of gluons. The coupling constant, the screening mass as well as the LPM cutoff are calculated dynamically.

In Fig. 3 we show the transverse momentum spectrum at (spatial) midrapidity $(\Delta\eta = 1)$ at different times for partons of a central cylinder of radius $R \leq 1.5$ and, respectively, of the total transversal region. From $t = 0$ on first only gluonic minijets with $p_t > 2$ GeV are populated. Energy degradation to lower momenta proceeds rapidly by gluon emission within the first fm/c. Maintenance of (quasi-)kinetic and (later) chemical equilibrium is given up to 4 fm/c, where longitudinal and transversal (quasi-hydrodynamical) work is done resulting in a continuous lowering of the temperature. This can be seen by the continuous steepening of the exponential slopes of the spectrum with progressing times. It turns out that kinetic momentum equilibration occurs at times of about 1 fm/c, whereas full chemical equilibration occurs on a smaller scale of about 3 fm/c. For the complete transversal region, of course, a remedy of the initial non-equilibrium high momentum tail remains stemming from the escaping minijets of the outer region.

In Fig. 4 we plot the evolution of the effective temperature $T := E/(3N)$ of the expanding system in the central cylinder. Indeed the further cooling after 1 fm/c

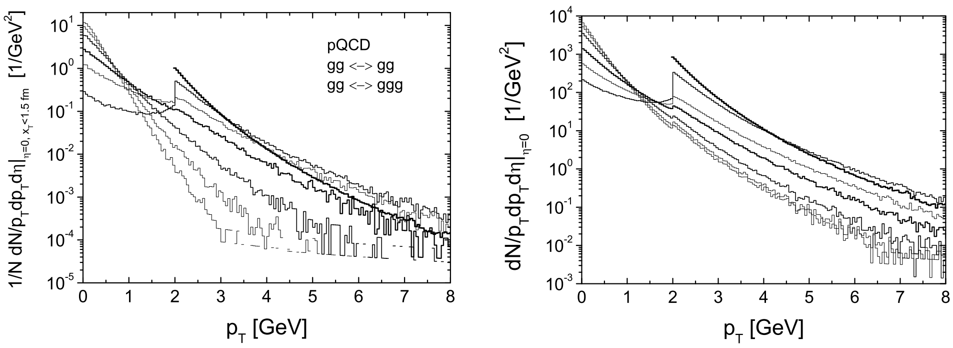

Fig. 3. Left panel: Transverse momentum spectrum at (spatial) midrapidity ($\Delta\eta = 1$) at different times ($t = 0.2$, 0.5, 1, 2, 3 and 4 fm/c from second upper to lowest line) for a real, central fully 3-D ultrarelativistic heavy ion collision. Only the partons residing in a central cylinder of radius $R \leq 1.5$ fm are depicted. From $t = 0$ on first only gluonic minijets with $p_t > 2$ GeV are populated (most-upper, bold-folded line). Energy degradation to lower momenta proceeds rapidly by gluon emission within the first fm/c. Maintenance of (quasi-)kinetic and (later) chemical equilibrium is given up to 4 fm/c, where longitudinal and transversal (quasi-hydrodynamical) work is done resulting in a continuous lowering of the temperature. Right panel: Like the left panel, but now all partons in the transverse direction are counted for. The evolution of the spectra at lower momenta is qualitatively similar to that of the central cylinder. The initial non-equilibrium high momentum tail following a standard power-law for minijet production is partly surviving, stemming from the escaping minijets of the outer region

proceeds on a noticeable faster scale than suggested by one-dimensional Bjorken expansion. This is mainly due to the ongoing production of gluons. At the end of the evolution at $t = 4$ fm/c about $dN_g/dy \approx 400$ are at midrapidity, i.e. the gluon number has doubled. If we compare the amount of transversal energy at midrapidity with experimental factor, we are roughly below by at least a factor of 2 to 3. This means, that in the initial conditions too few gluons have been assumed. Indeed, there exist parameterizations that the initial gluon number at midrapidity should be $dN_g/dy \approx 800$, i.e. a factor of 4 more than assumed here. In turn, an incorporation of such a gluon number as initial conditions would roughly shorten both the kinetic and chemical equilibration by a factor of 4, so that a full thermalization should be achieved on an accordingly smaller timescale! We leave this for a future detailed investigation.

In the right panel of Fig. 4 the angular distribution of the scattering processes $gg \to gg$ and $gg \to ggg$ for a representative situation during the evolution are shown. Whereas the elastic scattering is forward peaked, this is not the case for the inelastic reaction including a LPM cutoff. Especially the emitted gluons show a flat and non-forward angle distribution. This underlines why the inelastic processes are

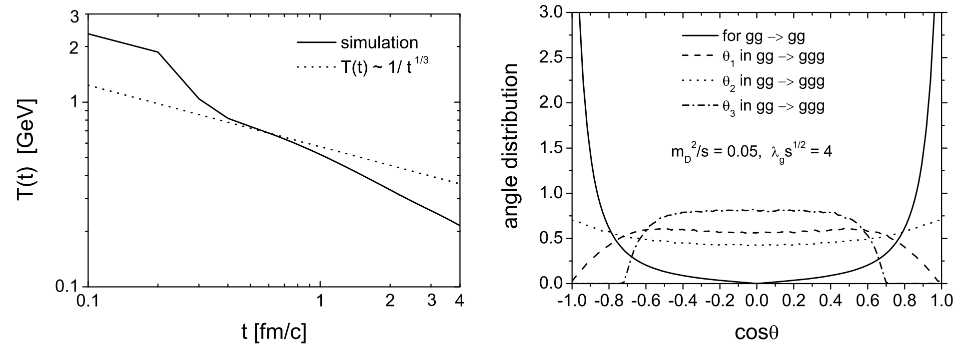

Fig. 4. Left panel: The evolution of the effective temperature $T := E/(3N)$ of the expanding system in the central cylinder of the left panel of Fig. 3 is shown. After kinetic equilibration on a timescale of 1 fm/c, the further cooling proceeds on a noticeable faster scale than suggested by one-dimensional ideal hydrodynamical expansion. This is mainly due to an ongoing production of low momentum gluons, as the system is chemically saturated only at times of about 3 fm/c. Right panel: The angular distribution of the scattering processes $gg \to gg$ and $gg \to ggg$ for a representative situation during the evolution. Whereas the elastic scattering is clearly forward peaked, this is not the case for the inelastic reaction including a LPM cutoff. Especially the emitted gluons show a flat and non-forward angle distribution

so important not only for accounting for chemical equilibration, but also for kinetic equilibration. It is these radiations that actually brings about early thermalization to the QGP.

In the future a lot of details have to be explored: thermalization (also of the light and heavy quark degrees of freedom) has to be investigated with various initial conditions like minijets, with a detailed comparison to data, or the color glass condensate, serving as input for the so called bottom up picture [4]. How likely is the latter picture for true coupling constants and not parametrically small ones? Furthermore we will study the impact parameter dependence of the transverse energy in order to understand elliptic and transverse flow at RHIC within this new scheme of a kinetic parton cascade including pQCD inelastic interactions, again by comparing to experimental data. Can the inelastic interactions generate the seen elliptic flow v_2? How good works (ideal) hydrodynamics? Also the partonic jet-quenching picture can be analyzed in 3-D details.

Acknowledgments

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