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HEAVY ION
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Fluctuations and Isospin Equilibration in Heavy Ion Reactions

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Abstract. The Boltzmann–Uehling–Uhlenbeck (BUU) approach with density fluctuations was used to study the density dependence of the asymmetry term in the nuclear equation of state (EOS). The isospin diffusion and equilibration process was investigated. The isotopic ratios of light particles were not sensitive enough to distinguish between asy-stiff and asy-soft EOS. However, the isospin ratio seems to be more sensitive to the density dependence of the asymmetry term in the nuclear equation of state and it favors a more asy-stiff equation of state.

Keywords: heavy ions, nuclear equation of state, fluctuations, fragmentation, isospin
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1. Introduction

The knowledge of the nuclear mean field potential is very important because the mean field is responsible for nuclear binding, for stabilization of neutron stars against gravitational collapse, and also for determining the dynamics of supernovae explosions. Studies of nuclear collisions provide us with a lot of information about mean field potential and about the nuclear equation of state (EOS). We already have a relatively good understanding of the properties of symmetric nuclear matter but need to learn much more about the density dependence of the asymmetry term in nuclear EOS. This quantity is relevant for studies of nuclei far from the stability line, for fission of asymmetric systems, phase transitions in low-density asymmetric nuclear matter, and many issues in astrophysics. To gain more understanding about asymmetric nuclear matter one may therefore study collisions between heavy

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ions of different N/Z ratio. If in the reaction the projectile and target have different isospin asymmetry, the asymmetry term in nuclear EOS provides diffusive forces which drive the isospin equilibration process. In the case of heavy systems where neutron density is larger than the density of protons, the asymmetry term is attractive for protons and repulsive for neutrons, leading to an enhancement of neutron emission. The magnitude of this enhancement depends on the strength of the asymmetry term and on its dependence on nuclear density [1]. One of the ways to look at this effect is to analyze the properties of bound fragments (projectile-like, target-like, light and intermediate mass). The crucial point in this investigation is to identify those experimental observables which are sensitive to the asymmetry term. Using the Boltzmann–Uehling–Uhlenbeck (BUU) formalism [2–4] we will study the properties of the fragments produced in Sn+Sn reactions at $E/A = 50$ MeV. The density and asymmetry dependence of energy for asymmetric nuclear matter can be written as

$$E(\rho, \delta) = E(\rho, 0) + S(\rho)\, \delta^2 \,, \tag{1}$$

where $\rho = \rho_n + \rho_p$ with ρ_n and ρ_p denoting the neutron and proton densities, and $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$. Some of the suggested forms of $S(\rho/\rho_0)$ (ρ_0 being the equilibrium density of nuclear matter) are shown in Fig. 1. In our calculations we will be using asy-stiff (F1) and asy-soft (F3) forms.

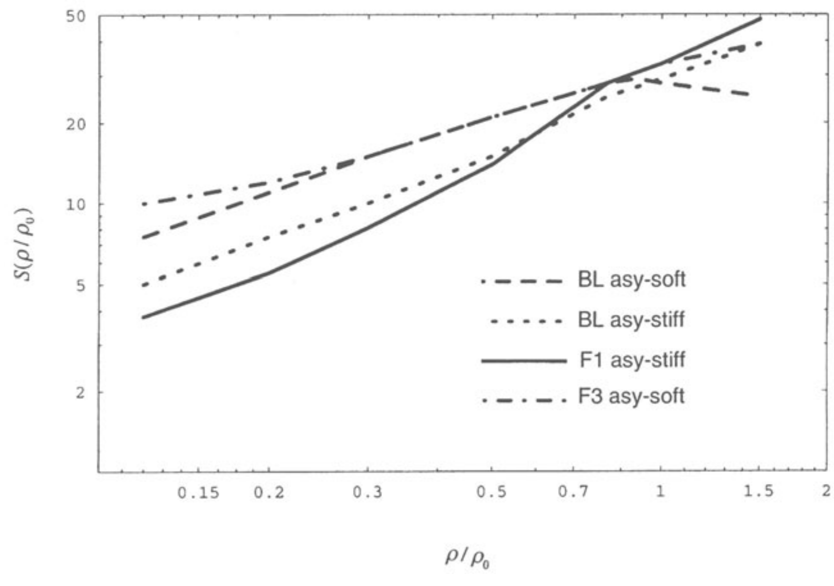

Fig. 1. Asymmetry function $S(\rho/\rho_0)$ (see Eq. (1)) as a function of ρ/ρ_0 (data taken from Bao-An-Li et al., *Phys. Rev. Lett.* **78** (1997) 1644 and M. Colonna et al., *Phys. Rev.* **C57** (1998) 1410)

2. The Model

The dynamical evolution of collisions between heavy ions can be successfully described by semi-classical microscopic mean field theories. We have solved the BUU equation for the distribution function $f(\vec{r},\vec{p})$ [2–4]:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla}_r f - \vec{\nabla}_r U \vec{\nabla}_p f = I_{\mathrm{coll}}(f)\,, \tag{2}$$

where U is a self-consistent mean field potential and I_{coll} is the Boltzmann collision term. The test-particle method is used to solve this equation [3]. Each test particle is represented by a Gaussian in coordinate and momentum space. In order to reduce possible numerical noise, we use 100 test particles per nucleon. We utilize a Skyrme interaction

$$U(\rho) = A\left(\frac{\rho}{\rho_0}\right) + B\left(\frac{\rho}{\rho_0}\right)^{\sigma} + C\left(\frac{\rho_n - \rho_p}{\rho_0}\right)\tau \tag{3}$$

with $A = -356$ MeV, $B = 303$ MeV, $\sigma = 7/6$, $\tau = 1$ for neutrons and $\tau = -1$ for protons. Two different choices were made for the density dependence of the asymmetry term: $C = 32$ MeV as asy-stiff and $C/\rho_0 = 481.7\,\mathrm{MeV\,fm}^3 - 1638.2\,\mathrm{MeV\,fm}^6\rho$ as the asy-soft case. These choices correspond to F1 and F3 forms, respectively. The influence of the neglected higher order terms can be considered as a fluctuation of the distribution function. We project these fluctuations on coordinate space and consider local density fluctuations which are then evolved by the mean field. In this approach the time evolution of the nuclear density contains the average phase-space trajectory, given by the BUU equation and the fluctuations of the individual trajectories about the average as described by the Boltzmann–Langevin theory. The large number of test particles per nucleon damps the numerical noise. Instead, we introduced a physical noise represented by thermal fluctuations of nuclear density. We have performed multiple calculations for central ($b = 2$ fm) and semi-peripheral ($b = 6$ fm) collisions for ^{112}Sn+^{112}Sn, ^{112}Sn+^{124}Sn, ^{124}Sn+^{112}Sn and ^{124}Sn+^{124}Sn for asy-stiff and asy-soft nuclear EOS, at $E/A = 50$ MeV. If we follow the dynamical evolution of a collision we observe that after an initial compression the colliding system expands and it reaches the spinoidal (volume) instability after 100–150 fm/c. The most unstable modes will lead the system to decomposition into fragments. The dynamical evolution is continued till a "freeze-out" time when the number of fragments is well determined and the nuclear interaction between fragments is negligible. The fragments are identified via a coalescence mechanism in coordinate space. The "freeze-out" time is close to 300 fm/c for central and about 200 fm/c for more peripheral collisions. The excitation energy of the fragments is estimated by calculating the thermal excitation in the local density approximation. The primary fragments are highly excited and will decay via statistical evaporation of light particles. In Fig. 2 we show the charge distributions and relative asymmetries of fragments from a neutron rich ^{124}Sn+^{124}Sn reaction. We may see that for a central collision (rows 1 and 2) we deal with a bulk fragmentation while for a peripheral collision (rows 3 and 4) we observe a neck production of light fragments. In the right-hand

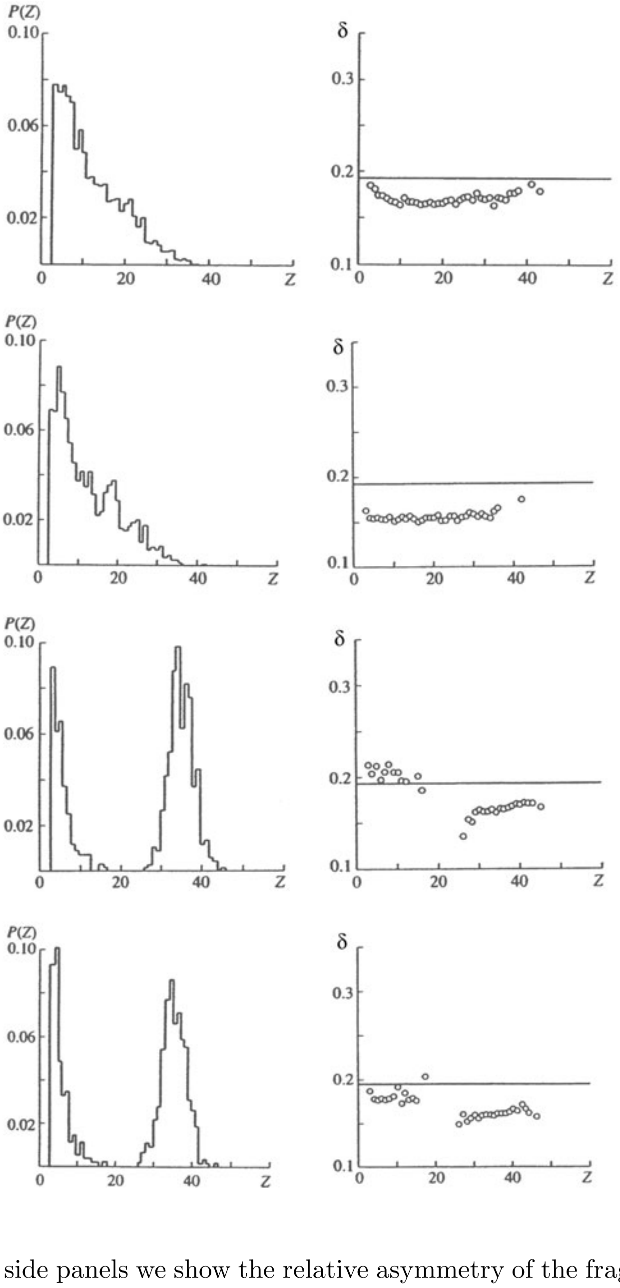

Fig. 2. Charge and asymmetry distributions for ^{124}Sn+^{124}Sn reactions. The first panel in each row shows the charge distribution of primary fragments in the freeze-out configuration; the second gives the relative asymmetry as function of the fragment charge. The first row corresponds to $b = 2$ fm asy-stiff; the second to $b = 2$ fm asy-soft; the third shows $b = 6$ fm asy-stiff and the fourth to $b = 6$ fm asy-soft

side panels we show the relative asymmetry of the fragments $\delta = (N - Z)/(N + Z)$ for an asy-stiff (rows 1 and 3) and asy-soft (rows 2 and 4) equation of state. We see lower values of δ for the case of an asy-soft EOS since it strongly enhances emission of neutrons, thus lowering the asymmetry. Although δ seems to be sensitive to the choice of EOS, this quantity cannot be directly compared with the experimental data due to subsequent evaporation from the primary fragments.

3. Isoscaling

In the search for more sensitive observables the isotopic yields $Y(N, Z)$ of light fragments (with N neutrons and Z protons) have been calculated for both EOS. Unfortunately the differences caused by a different asymmetry term are rather small and become even smaller after the secondary emission through which the primary fragments de-excite. A more sensitive way to compare isotopic distribution is to use the isotopic ratio $R_{21}(N, Z) = Y_2(N, Z)/Y_1(N, Z)$, where the isotopic yields $Y_1(N, Z)$ and $Y_2(N, Z)$ come from 2 different reactions. It has been shown recently [5] that the isotopic yields for two systems produced at the same temperature satisfy an isoscaling relation; namely, the isotopic ratio

$$R_{21}(N, Z) = Ce^{\alpha N + \beta Z}\,, \tag{4}$$

where C is a normalization constant and α and β are the scaling parameters. If we choose reaction 2 to be more neutron rich, we expect α to be positive and β to be negative. The advantage of using the isotopic ratio is, for example that the binding energy factors which are similar for both reactions are cancelled, leaving the terms related to neutron and proton chemical potentials [1]. We have calculated R_{21} for ^{124}Sn+^{124}Sn and ^{112}Sn+^{112}Sn reactions for both asy-stiff and asy-soft EOS. Since these reactions defer by number of neutrons only, we looked at the behavior of the scaling parameter α. Similar to Ref. [1], we found that α is larger for the stiff EOS, being slightly closer to the experimental data. In Ref. [6] a quantity sensitive to the isospin diffusion, an isospin ratio, or an imbalance parameter R_i is defined

$$R_i = \frac{2x - x_{124+124} - x_{112+112}}{x_{124+124} - x_{112+112}}\,, \tag{5}$$

where x is an isospin sensitive observable, preferably linear in asymmetry, for example the scaling parameter α.

For the two symmetric systems ^{124}Sn+^{124}Sn and ^{112}Sn+^{112}Sn the value of R_i equals 1 and -1, respectively. If there is no isospin diffusion, pre-equilibrium emission from the projectile should be approximately equal for ^{124}Sn+^{124}Sn and for ^{124}Sn+^{112}Sn reactions. The similar thing should happen when ^{112}Sn is a projectile. $R_i(\alpha)$ essentially removes the sensitivity to pre-equilibrium emission and emphasizes the differences in isospin diffusion. In Ref. [6] the experimentally extracted scaling parameter α was used as x in Eq. (5). It yields $R_i = 0.5$ for ^{124}Sn+^{112}Sn reaction and $R_i = -0.5$ for ^{112}Sn+^{124}Sn. $R_i = 0$ corresponds to a complete isospin equilibration. If one assumes that the isoscaling described in Eq. (4) reflects the emission of particles from projectile-like fragments, one may show that [5]

$$\alpha \sim (\delta_2 - \delta_1)\left(1 - \frac{\delta_2 + \delta_1}{2}\right)\,, \tag{6}$$

where δ_1 and δ_2 are relative asymmetries of the projectile-like fragments in reactions 1 and 2, respectively. It leads to $R_i(\alpha) \approx R_i(\delta)$. The last relation allows a direct comparison of the BUU calculations with experimentally extracted values of R_i by using the asymmetry δ of the projectile-like fragments as the isospin sensitive parameter in Eq. (5).

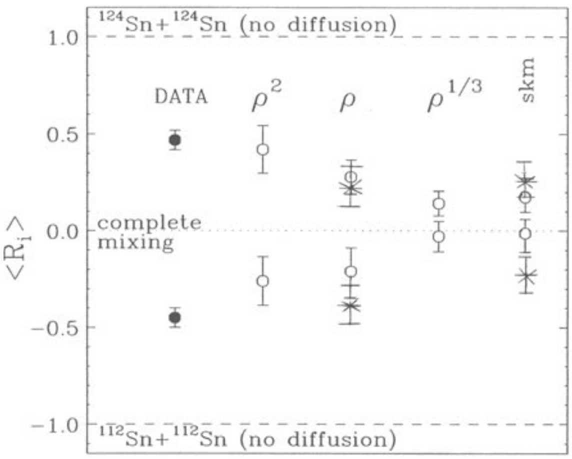

Fig. 3. Average measured [6] (solid circles) and calculated (open circles [6] for MSU BUU code, and stars for the present calculation) values of R_i for decreasing (from left to right) asy-stiffness of the nuclear EOS

We have run multiple trajectories for ^{112}Sn+^{112}Sn, ^{112}Sn+^{124}Sn, ^{124}Sn+^{112}Sn and ^{124}Sn+^{124}Sn at $E/A = 50$ MeV and $b = 6$ fm using the asy-stiff and the asy-soft EOS. For each reaction we calculated the average value of δ for the projectile-like fragments at the freeze-out time and then calculated the values of the isospin ratio R_i. In the upper part of Fig. 3 we show R_i for ^{124}Sn+^{112}Sn and in the lower part for ^{112}Sn+^{124}Sn. The solid circles represent the experimental data while the open circles give the results of BUU simulations using the MSU BUU code with four different dependences of the asymmetry term on nuclear density, with the stiffness decreasing from left to right. The second and the fourth cases correspond to our asy-stiff and asy-soft EOS and the findings of the present calculations are shown as stars. The results of both calculations seem to favor a more stiff equation of state.

4. Summary

We have used a BUU model with density fluctuations to investigate the density dependence of the asymmetry term in the nuclear equation of state. We studied central and peripheral collisions of neutron rich and neutron poor isotopes of tin using an asy-stiff and asy-soft nuclear EOS. We have seen some sensitivity of the relative asymmetry of light, intermediate mass, projectile- and target-like fragments to the asy-stiffness of the nuclear EOS. However, due to the secondary decay of the excited primary fragments, we cannot make a valuable comparison with experimental data.

The quantity which is more sensitive to the asymmetry properties of the mean field potential — the isospin ratio — seems to favor a stiff equation of state. It is also important to emphasize that the measured and calculated values of the isospin ratio show that the colliding system does not reach a full isospin equilibration. It encourages further study of the isospin diffusion process.

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