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Soliton Propagation of Microwave Modulated Signal through Single-Mode Optical Fiber

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Abstract. Sinusoidally modulated optical signal transmission is experimentally investigated over a 30 km standard single-mode fiber using a range of optical input powers. Experiments and computer simulations showed that fiber-induced self-phase modulation generates a chirp on the signal with an effect opposite to that induced by chromatic dispersion. Calculations were performed to investigate the distortions caused by simultaneous dispersion and non-linearity using a range of fiber parameters. Soliton propagation of the microwave/millimeter wave modulated signal is reported at elevated intensities in lossless and loss compensated cases.

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1. Introduction

Some electro-optical systems transmit radio frequency (RF) modulated light through standard single-mode optical fiber (SMF) for communication purposes. Microwave optical networks are also among the candidates to provide infrastructure-supporting links in the next generation of mobile communication systems [1–3]. Optical microwave networks may offer an economical solution to the increased number of base stations needed both by the increased traffic requirements and by the higher communication frequencies.

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Optical fiber and radio over fiber systems usually operate around the 1550 nm wavelength range where they benefit from low propagation loss but the effects of dispersion and non-linearity can be significant. Chromatic dispersion can cause modulation suppression, linear distortion or even complete canceling of intensity modulation by the RF signal [4–7]. Throughout this paper we shall call the locations where intensity modulation disappears "notches" because of their appearance when we observe the transfer function of the fiber over a relatively wide range of frequencies. (The transfer function is defined as the quotient of the modulated input and detected output signal powers as a function of frequency.)

With the increase of optical power provided by high intensity laser diodes, non-linearity can no longer be neglected. Above a given density of photon flux self-phase modulation (SPM) can become significant [8] and one must consider the distortions caused by SPM during the design process of microwave optical systems. At relatively high average input intensity SPM modifies the transfer function of the fiber [9] and the location of the notch as well [10].

SPM can compensate modulation suppression caused by dispersion [8–11]. As the signal intensity increases the notches caused by chromatic dispersion shift to higher modulation frequencies. In accordance with our previous work [10], calculations show that the effect of dispersion can be compensated by SPM totally, at least in the case of lossless propagation. A critical input power as a function of carrier frequency can also be determined which separates the dispersion dominated and non-linearity dominated regions.

In this paper, we investigate joint effects of dispersion and nonlinear refraction in a region of intensity where non-linearity related distortions become important. We report on measurements and simulations in the modulation frequency range of 50 MHz to 20 GHz. We plot the locations of notches using various fiber parameters. And we also report soliton-like propagation of 10 GHz modulated optical signals. Soliton-like propagation means a propagation which is close to the observed one in case of pulse transmission but some differences, for example spectral formation during the propagation is also noticed.

Transmission of unmodulated RF carriers is discussed here but the results lead to conclusions applicable to the modulated RF signals as well [11].

2. Theory

The propagation of light in a nonlinear, dispersive, single-mode fiber in picosecond regions and larger time-scales is approximated by the nonlinear Schrödinger equation (NLSE) [12]:

$$\frac{\partial a(z,t)}{\partial z} + \frac{1}{v_g}\frac{\partial a(z,t)}{\partial t} + \frac{j\beta_2}{2}\frac{\partial^2 a(z,t)}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 a(z,t)}{\partial t^3} + \frac{\alpha}{2}a(z,t) - j\gamma|a(z,t)|^2 a(z,t) = 0\,, \qquad (1)$$

where $a(z,t)$ is the complex envelope function describing the time and spatial dependence of the signal, v_g is the group-velocity, β_2 is the group-velocity dispersion (GVD), β_3 is the third order term of the Taylor series of the phase constant, j is the imaginary unit and γ is the nonlinear coefficient given by

$$\gamma = \frac{n_2\omega_0}{cA_{\text{eff}}} \tag{2}$$

with n_2 the nonlinear refractive index, ω_0 the angular frequency, A_{eff} the effective fiber core area and c the speed of light in vacuum.

The last term of Eq. (1), accounting for SPM, can have a significant role if the absolute value square of the complex envelope function: $|a(z,t)|^2$, is sufficiently large to render the magnitude of the SPM term comparable to the magnitude of the GVD term (3rd term in (1)). This is well known in the case of pulse propagation [12]. In agreement with previous studies [8–11], we found this is valid in the case of RF modulated signal propagation as well. In case of nonlinear pulse propagation in anomalous dispersive media, the broadening factor on a given length of fiber is less than if dispersion alone is considered. In the case of RF modulated signal propagation the nonlinear term appears to decrease the effect of β_2, resulting in an increase of the frequency at which the RF notch appears [8–11].

A qualitative explanation of the sharp dip (notch) in the transfer function can be given as follows. When the optical carrier is intensity modulated by a sinusoid of angular frequency ω_m the two first-order modulation sidebands propagate with different speeds; thus they arrive to the photo-detector, placed at the output of the fiber, with unequal phases. Consequently, the photo-detector produces two interfering signals resulting in a modulation transfer function that is less than unity, as expressed below in (3), indicating that modulation is suppressed. The form of this transfer function depends slightly on the design of the modulator. In the case of an ideal amplitude modulator, for example, its form is (see e.g. [1], Chapter 1)

$$K(L,\omega_m) = \cos\left(\frac{\beta_2 L}{2}\omega_m\right) \tag{3}$$

with ω_m being the modulating (RF) angular frequency and L the propagation path length. We can see that modulation is completely suppressed if the argument of the cosine is an odd multiple of $\pi/2$. In the case of anomalous dispersion the first notch appears when the argument is $-\pi/2$:

$$L_n^l = \Theta(\beta_2)\frac{\pi}{\beta_2\omega_m^2}\,, \tag{4}$$

where subscript n refers to notch and superscript l to the linear case. The $\Theta(\beta_2)$ function is 1 if β_2 is positive or zero and -1 otherwise. Expression (4) is valid if the fiber is linear. If SPM has a significant contribution to the signal evolution, notches will appear at locations different from those that could be calculated from (4).

3. Measurement

Figure 1 illustrates our experimental setup. The frequency of the RF signal is varied between 50 MHz and 20 GHz in 800 steps. These frequencies modulate a 1 mW, 1550.8 nm continuous-wave (CW) laser using a Mach–Zehnder (MZ) modulator with 25% modulation depth. This type of MZ modulator (HP 83422A) has a relatively large 6.5 dB attenuation reducing its optical output power to 0.22 mW.

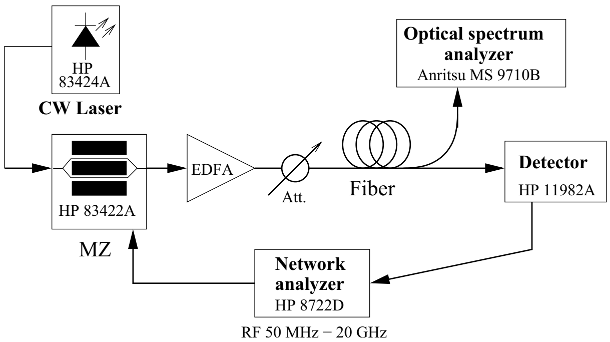

Fig. 1. Experimental setup to demonstrate the effect of SPM

The RF signal propagates through 30 km of SMF and at the end of it a network analyzer registers the field parameters. An optical spectrum analyzer monitors the frequency and optical power during the experiments. We used attenuators and an erbium-doped fiber amplifier (EDFA) before the fiber input to measure the transfer function of the fiber at various average input intensities. The gain of the EDFA is 20.6 dB raising the 0.22 mW input to 25.5 mW when no attenuation is used before the fiber input. An attenuator was applied to reduce the optical intensity by about 4 dB before injecting it into the fiber, and some additional attenuation was used before the detector to protect it from damagingly high intensities.

We used a standard single-mode optical fiber with 0.22 dB/km attenuation and 16.8 ps/(nm km) chromatic dispersion at 1550 nm. The effective core area of the fiber, necessary for calculating the nonlinear coefficient (2), was 75 μm^2.

4. Simulation

A second order (symmetric) split-step Fourier (SSF) method [12] with a sinusoidally modulated input field is used to solve (1). The modulated output of the MZ modulator is

$$a_{\mathrm{out}}(z=0,t) = a_{\mathrm{laser}}(t)\sqrt{d(t)}\exp\left[j\Delta\phi(t)\right], \tag{5}$$

where

$$d(t) = 1 + m(\sin(\omega_m t)) \tag{6}$$

is the power transfer function, m is the modulation depth, ω_m is the modulation angular frequency and $\Delta\phi(t)$ is the phase difference between the two branches of the modulator which can be set to zero if an ideal amplitude modulator is to be simulated. The input field in the calculated time window provided by the CW laser, like in the measurements, is $a_{\text{laser}}(t)$, the square root of the laser intensity. The phase difference in (5) can be given by

$$\Delta\phi(t) = C_1 - C_2 \left(\sin(\omega_m t) - \frac{1}{2} \right), \tag{7}$$

where C_1 and C_2 are constants representative of the modulator, chosen to be 0.06 and 0.073, respectively, to fit the simulated signal to the experimental values.

The fiber parameters used in the modeling were the same as those in the measurements (loss: 0.22 dB/km, chromatic dispersion: 16.8 ps/(nm km), effective core area: 75 μm^2). Silica based fibers usually have a nonlinear refractive index as large as 2.6×10^{-20} m^2/W and third order chromatic dispersion in the range of 0.05–0.1 $\times 10^3$ ps/(nm^{2}km) in the vicinity of 1550 nm [12]. All the simulations were prepared using the above parameters. Results were obtained by varying the input intensity of the CW laser and the modulation frequency. We also made calculations varying the optical power level and the fiber length using a constant 10 GHz RF modulation. In order to detect the RF component of the signal alone, a narrow band (100 MHz) Gaussian filter centered at 10 GHz was applied.

5. Results and Discussion

5.1. Measurements and calculations

Figure 2 shows the measured and calculated detected RF power normalized to the modulated input power as a function of the modulation frequency for three different optical input powers. One can see that as the optical power is increased the notches appear at higher modulation frequencies. We note that this trend is valid for the notch positions as well (see Fig. 3 below). The RF power corresponding to the 25.5 mW signal has a positive slope between 50 MHz and 7 GHz and remains above the input RF average intensity up to 8 GHz. As further explained below, this occurs because the Kerr non-linearity converts the optical power (central peak in the spectrum) to RF power (sidebands). We note that according to our simulations if a different type of modulator is used, e.g. an ideal amplitude modulator or a MZ modulator with smaller extinction ratio, the rise in RF power will occur at a much smaller RF signal input intensity, at about 10 mW.

The measured fiber responses beyond 30 km propagation length are in excellent agreement with calculated results. These correspondences make it possible to extend our investigations of parameter dependencies into regions, which can not, or are very difficult to be measured. In what follows we make some numerical investigations into these regions.

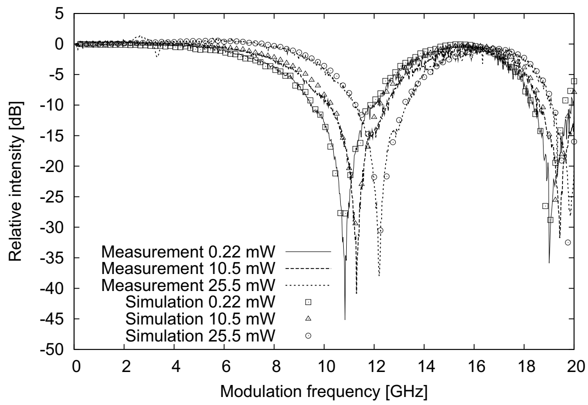

Fig. 2. Measurement and simulation results at three different average input intensities. 10 GHz modulation frequency was applied and 30 km fiber was used. Experimental and simulation results are in close agreement

5.2. Notch positions

From an engineering point of view, it might be important to know the notch location and their dependence on the optical input power. These simulations can be seen in Fig. 3, where the average intensity of the RF signal is plotted as a function of fiber length, retaining the parameters used previously. As one can see, when the input power is increased notches appear at longer distances. Here we also plotted the fiber

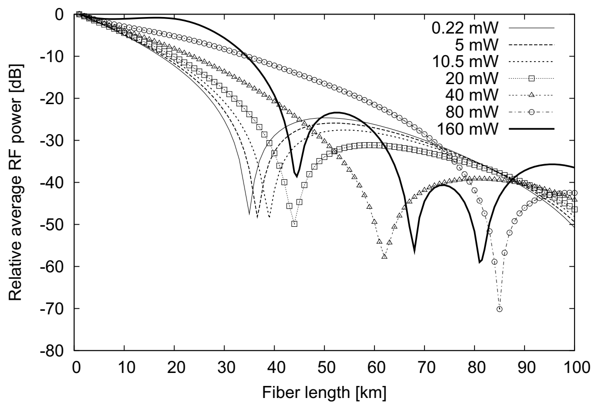

Fig. 3. Simulation of the normalized RF power as a function of fiber length with different input intensities. The plot for the 160 mW input signal (thick solid line) shows an irregular behavior compared to lower intensity signals

response to an unusually high intensity RF signal, showing a behavior unexpected from that displayed at smaller RF intensities, namely, that modulation suppression occurs much earlier than before.

The dependency of notch position on input power can not be expressed by basic functions. In order to show these locations as caused by simultaneous chromatic dispersion and SPM, we performed computations with different nonlinear refractive indices and different dispersion parameters as well. These are shown in Figs. 4(a) and (b). Figure 4(a) shows the notch location as a function of input intensity, using the nonlinear refractive index as a parameter at a fixed dispersion (16.8 ps/(nm km)), while Fig. 4(b) shows the corresponding results for three dispersion values, using a fixed nonlinear refractive index (2.6×10^{-20} m^2/W).

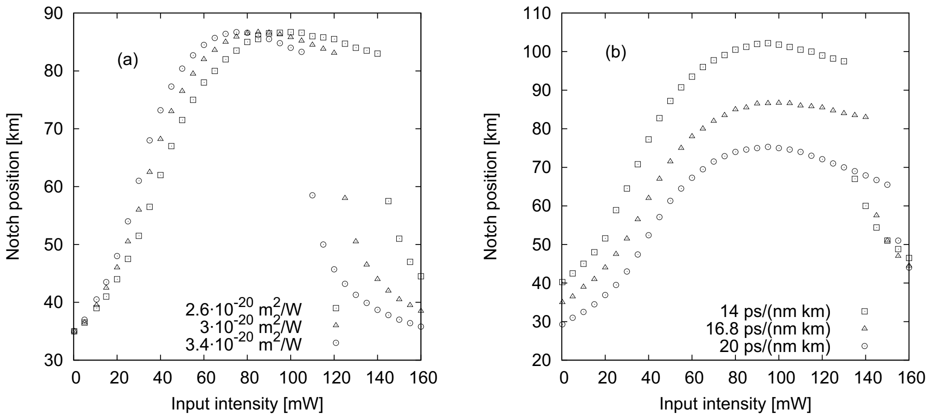

Fig. 4. Notch position as a function of average input intensity for different nonlinear refractive indices (a) and different chromatic dispersion values (b) of SMF. Dispersion was fixed at 16.8 ps/(nm km) in case of (a) and nonlinear refractive index at 2.6×10^{-20} m^2/W in case of (b)

Notice that both in Figs. 4(a) and (b) at power levels below a certain threshold the notch position shifts to longer fiber lengths as the input power increases. Above a certain input power however, the notch positions drop abruptly, i.e. they reappear at a much reduced fiber length. As expected, keeping the chromatic dispersion fixed, when non-linearity is increased (larger nonlinear refractive index) dispersion caused suppression occurs at larger distances. However, at a given power level, there is a change in this behavior and higher non-linearity causes a drop in notch position at a smaller input power. A similar relationship holds when the magnitude of chromatic dispersion is increased while holding the nonlinear refractive index fixed. In this case, when the chromatic dispersion is decreased notches appear at longer path lengths, but drop at smaller input powers.

Considering the shape of the plotted functions in Figs. 4(a) and (b) three different regions can be observed, namely:

- exponentially increasing region,
- region of downturn,
- region of abrupt drop.

At a given modulation frequency (ω_m), using the above fiber and signal parameters, these regions can be separated as follows. The exponentially increasing region reaches until the effect of non-linearity, defined by γP_0 where P_0 is the optical input power, has just equalized the dispersive one ($\beta_2 \omega_m^2$), i.e. until

$$\frac{\gamma P_0}{\beta_2 \, \omega_m^2} \leq 1 \,, \tag{8}$$

where the numerator is practically equal to the coefficient of the envelope function in the last term of Eq. (1) with $P_0 = |a(z = 0, t)|^2$, and the denominator is related to the GVD term, the second in (1).

Further behaviors can be assigned also to the loss because in loss compensated case notches do not appear after a critical intensity at all (see next subsection).

The second region, where the average RF signal intensity as a function of input intensity turns downward, non-linearity exceeds the magnitude of the chromatic dispersion at the fiber input. The notch position, however, does not appear at a longer length but at a shorter one. This is because optical power is attenuated progressively during the propagation and SPM related changes in spectrum can not happen so effectively.

In practice, this region is not significant because this behavior occurs only at very high intensities. However, if the fiber is dispersion compensated or dispersion shifted, this region might become important at reduced chromatic dispersion values.

Abrupt drop happens when the non-linearity and GVD related term exhibit the following relation

$$\frac{\gamma P_0}{\beta_2 \, \omega_m^2} \approx \frac{3}{4}\pi \approx 2.3 \,. \tag{9}$$

Above this region we call the propagation of microwave modulated signal as irregular.

This irregular behavior caused by the intense non-linearity can be observed in Fig. 3 at the plotted average intensity of the 160 mW signal. Figure 4 shows however that the abrupt drops still happen regularly but further behaviors (average intensity as a function of propagating length, position of the second, third notch, etc.) are impossible to predict analytically.

We must note that the evaluated expressions depend slightly on the used modulator as well. The place of dispersion caused suppression of RF signal belong to a system outlined in the previous sections.

5.3. Lossless propagation

Although propagation in a lossless fiber has only theoretical significance it does have far reaching practical connotations. The attenuation of a signal in optical telecommunication systems and also in fiber-radio systems can be compensated along the transmission medium. A possible method to amplify the signal without the use of doped amplifier media is called Raman amplification. Through a phenomenon called induced Raman scattering the fiber is capable to amplify the signal using an approximately 13.5 THz blue-shifted high power pump. Since loss can thus be compensated, we prepared simulations neglecting loss from the previous calculations. We used various input powers using the original fiber parameters: 16.8 ps/(nm km) chromatic dispersion and 2.6×10^{-20} m^2/W nonlinear refractive index.

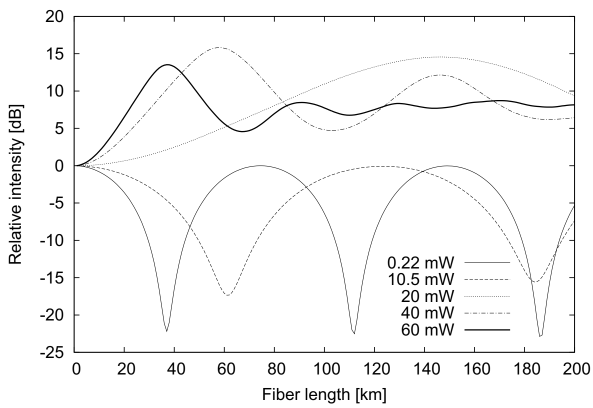

Fig. 5. Relative intensity of a 10 GHz sine modulated RF signal as a function of distance in a lossless fiber. As the input intensity increases the plots display the dominance of SPM over GVD

As illustrated in Fig. 5, one can see that above a critical input power the effect of dispersion is virtually completely compensated by non-linearity. As a consequence, there are no notches at all in this domain. At 10 mW input power the effect of dispersion in the form of modulation suppression is still apparent, but at 20 mW input the signal does not show this behavior. Between these two values therefore, a critical intensity exists where dispersion and Kerr non-linearity cancel each other.

Above this critical intensity, SPM exceeds dispersion and as it can be seen in Fig. 6 the average intensity of the RF signal becomes higher at certain places than the input. We plotted in Fig. 6 a 400 km propagation with 20 km steps. The input signal was a 10 GHz, 20 mW sine modulated signal. Soliton-like recurrence can be observed in the changes of signal shape during this length. The periodicity of these oscillations can be observed in Fig. 5 too, where in case of a 60 mW input, the average RF signal oscillates several times along a 200 km path, although the

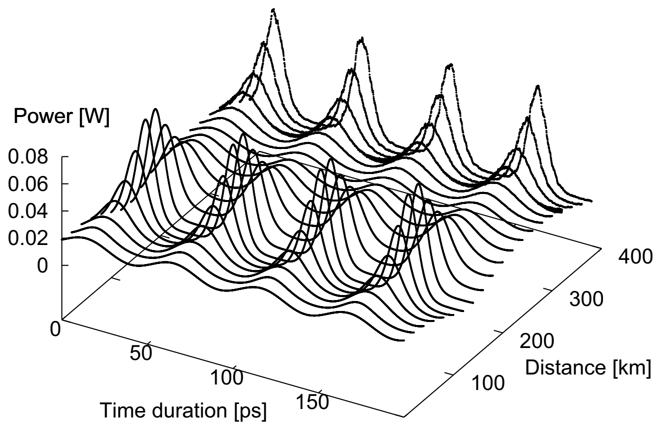

Fig. 6. The propagation of 10 GHz sine modulated signal with 25% modulation depth and 20 mW input power. RF signal peak intensities become higher than input ones after 100–140 km propagation in accordance with the increase of the average RF intensity in Fig. 5

amplitude of these oscillations are decreasing. Smaller intensity signals oscillate less than higher ones during the same propagation length.

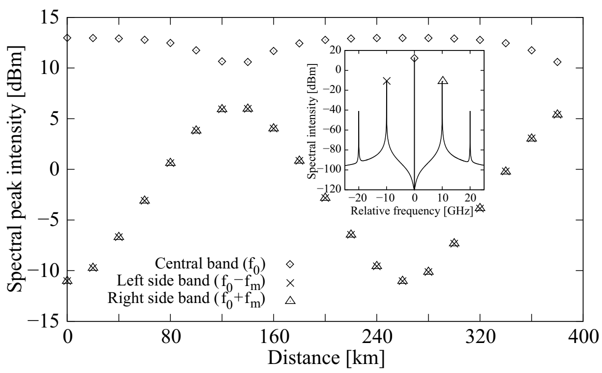

Fig. 7. Oscillatory behavior of the peak intensities in the spectrum during the propagation. f_0 stands for the frequency of the perfect CW light source and f_m is the modulation frequency. The small inset is intended to show the input spectral shape and the corresponding notation of peaks

This kind of propagation can happen because part of the optical power transfers to the RF signal due to SPM (see Fig. 7). Figure 7 shows how the peaks of spectrum is changing as a function of propagation path length. We plotted here only the central peak (f_0) and the two side bands ($f_0 \pm f_m$ where f_m is the modulation frequency). The center peak has an input spectral intensity around 13 dBm. Side bands start from smaller intensities and strong oscillations can be observed which coincide with the oscillations of the peak intensities of signal in the temporal space (Fig. 6). Where central peak intensity is decreasing the two side bands are increasing in intensity and vice versa. This kind of special "crass-talk" effect gives rise to the observation of soliton propagation of microwave/millimeter-wave signals.

6. Conclusion

We investigated experimentally and numerically the joint effects of non-linearity and chromatic dispersion in a fiber-radio system modulated with 10 GHz modulation frequency. We calculated the average RF power along the transmission path and the position of notches as a function of input optical intensity.

Using modulation frequencies higher than 10 GHz, notches will appear at shorter propagation path lengths as can be seen from expressions (8) and (9). This can be avoided using fibers with smaller dispersion value in the given wavelength range.

We showed that non-linearity could compensate the effect of dispersion in a lossless fiber, or in a lossy one where loss is eliminated by Raman amplification. We have given a detailed description of the phenomenon causing the shifting or total disappearance of dispersion generated suppression. We showed that SPM and GVD modifies the temporal and spectral shape of the modulated signal; energy transfer happens from the central band to the sidebands increasing the average intensity of the RF signal. We reported also the soliton propagation of the RF signal in connection with these spectral and temporal changes. Soliton propagation may be observed at optical powers of a few milliwatts (10–20 mW) in a loss compensated system using a chirp-free modulator.

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Note

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