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Entanglement in the Process of Stimulated Brillouin Scattering of Light

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Abstract. The process of stimulated Brillouin scattering is described by the model of two-dimensional oscillator. The phenomenon of entanglement which appears in the photon–phonon modes after the interaction with matter is discussed.

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Spontaneous Brillouin scattering which is considered to be a scattering of light at the acoustic waves caused by thermal fluctuations was investigated in 1922. Physically stimulated Brillouin scattering (SBS) is analogous to another type of stimulated scattering of light the stimulated Raman scattering (SRS) of light in which the laser light is scattered by the vibrational mode of the medium. The main difference between these two types of scatterings is the different frequency shift (till about several hundred cm^{-1} for SRS and $\sim 0.01\text{cm}^{-1}$ for SBS). Quantum mechanical description of SBS can be realized with the help of the Hamiltonian quadratic in photon and phonon creation and annihilation operators in which the electromagnetic laser wave is considered to be classical. Such a Hamiltonian was used [1, 2] for the description of SRS. The purpose of our work is to discuss the phenomenon of entanglement which appears in the photon–phonon modes after the interaction with medium.

The simplest phenomenological Hamiltonian which can be used for the description of SBS can be written as [1]

$$\hat{H} = \hbar\omega_S \hat{a}_s^\dagger \hat{a}_s + \hbar\omega_k \hat{a}_k^\dagger \hat{a}_k + \hbar\lambda \left[e^{-i\omega_L t} \hat{a}_s^\dagger \hat{a}_k^\dagger + e^{i\omega_L t} \hat{a}_k \hat{a}_s \right] , \tag{1}$$

where $\hat{a}_s$ and ω_s are the annihilation operator and the frequency of the Stokes photon, $\hat{a}_k$ and ω_k are the annihilation operator and the frequency of the acoustic

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phonon, ω_L is the laser frequency, and λ is the coupling constant. Laser field amplitude is involved in λ. The laser field is considered as classical and its frequency is determined by the condition $\omega_L = \omega_k + \omega_S$. The damping and depletion of the laser light wave are neglected.

If the photons and phonons of the medium were in the ground state at the initial moment of time, then after the interaction with laser field the quadrature dispersions take the form

$$\begin{aligned}
\sigma_{p_s^2} &= \sigma_{p_k^2} = \sigma_{q_s^2} = \sigma_{q_k^2} = \frac{1}{2}\cosh(2\lambda\, t), \quad \sigma_{p_s q_s} = \sigma_{p_k q_k} = 0\,, \\
\sigma_{p_s q_k} &= \sigma_{q_s p_k} = -\frac{1}{2}\sinh(2\lambda\, t)\cos(\omega_L\, t)\,, \\
\sigma_{p_s p_k} &= \sigma_{p_k p_s} = -\sigma_{q_s q_k} = -\sigma_{q_k q_s} = \frac{1}{2}\sinh(2\lambda\, t)\sin(\omega_L\, t)\,.
\end{aligned} \tag{2}$$

The dispersions of the photon and phonon quadratures have no oscillations and are functions of the coupling constant. The cross-variances have an oscillating behaviour. One of the intermode covariances is not equal to zero, what means that a statistical dependence of mode appeared after interaction.

If the photons were in ground state and phonons were in the thermodynamic equilibrium state at the initial moment of time, then at the time t the dispersions of the photon and phonon quadrature components will be of the form

$$\begin{aligned}
\sigma_{p_s^2} &= \sigma_{q_s^2} = \frac{1}{2}(1+\coth\theta)\sinh^2(\lambda t) + \frac{1}{2} \\
\sigma_{p_k^2} &= \sigma_{q_k^2} = frac12(1+\coth\theta)\sinh^2(\lambda t) + \frac{1}{2}\coth\theta, \\
\sigma_{p_s p_k} &= \sigma_{p_k p_s} = -\sigma_{q_k q_s} = -\sigma_{q_s q_k} = \frac{1}{4}\sinh(2\lambda\, t)\sin(\omega_L\, t)\,(1+\coth(\theta))\ , \\
\sigma_{p_s q_k} &= \sigma_{p_k p_s} = -\frac{1}{4}\sinh(2\lambda\, t)\cos(\omega_L\, t)\,(1+\coth(\theta))\ , \\
\sigma_{p_s q_s} &= \sigma_{p_k q_k} = 0\,, \quad \theta = \frac{\hbar\omega_k}{2kT}\,.
\end{aligned} \tag{3}$$

The dispersions and intermode covarianses are functions of the coupling constant and the temperature of the medium in this case. If the medium was so prepared, that the phonons were in squeezed states at the initial moment of time, then the dispersions of photon and phonon quadratures at the time t will be

$$\begin{aligned}
\sigma_{p_s^2} &= \frac{1}{2\eta}\,\left[\eta\cosh^2(\lambda\, t) + \sinh^2(\lambda\, t)\left(\eta^2\sin^2(\omega_s\, t) + \cos^2(\omega_s\, t)\right)\right] \qquad (4) \\
\sigma_{p_s\, p_k} &= \frac{1}{4\eta}\,\sinh(2\lambda\, t)\left[\eta\sin(\omega_L\, t) + \eta^2\sin(\omega_s\, t)\cos(\omega_k\, t) + \cos(\omega_s\, t)\sin(\omega_k\, t)\right] \\
\sigma_{p_s\, q_k} &= \frac{1}{4\eta}\,\sinh^2(\lambda\, t)\sin(2\omega_s\, t)\left(1-\eta^2\right) \\
\sigma_{p_s\, q_k} &= -\frac{1}{4\eta}\,\sinh(2\lambda\, t)\left[\eta\cos(\omega_L\, t) - \eta^2\sin(\omega_s\, t)\sin(\omega_k\, t) + \cos(\omega_s\, t)\cos(\omega_k\, t)\right]
\end{aligned}$$

$$\begin{aligned}
\sigma_{p_k^2} &= \frac{1}{2\eta}\left[\eta\sinh^2(\lambda\,t)+\cosh^2(\lambda\,t)\left(\eta^2\cos^2(\omega_k\,t)+\sin^2(\omega_k\,t)\right)\right]\\
\sigma_{p_k\,q_s} &= -\frac{1}{4\eta}\sinh(2\lambda\,t)\left[\eta\cos(\omega_L\,t)-\sin(\omega_s\,t)\sin(\omega_k\,t)+\eta^2\cos(\omega_s\,t)\cos(\omega_k\,t)\right]\\
\sigma_{p_k\,q_k} &= -\frac{1}{4\eta}\cosh^2(\lambda\,t)\sin(2\omega_k\,t)\left[1-\eta^2\right]\\
\sigma_{q_s^2} &= \frac{1}{2\eta}\left[\eta\cosh^2(\lambda\,t)+\sinh^2(\lambda\,t)\left(\eta^2\cos^2(\omega_s\,t)+\sin^2(\omega_s\,t)\right)\right]\\
\sigma_{q_s\,q_k} &= -\frac{1}{4\eta}\sinh(2\lambda\,t)\left[\eta\sin(\omega_L\,t)+\sin(\omega_s\,t)\cos(\omega_k\,t)+\eta^2\cos(\omega_s\,t)\sin(\omega_k\,t)\right]\\
\sigma_{q_k^2} &= \frac{1}{2\eta}\left[\eta\sinh^2(\lambda\,t)+\cosh^2(\lambda\,t)\left(\eta^2\sin^2(\omega_k\,t)+\cos^2(\omega_k\,t)\right)\right]\,,
\end{aligned}$$

where η is the squeezing parameter. The dispersions of photon and phonon quadratures are functions of the coupling constant and the squeezing parameter and have an oscillating behaviour. The intermode covariances are not equal to zero, so the statistical dependence of the modes takes place in this case, too. The dependence of the intermode covariances on the squeezing parameter, shows the possibility of controlling the properties of the statistical dependence of the modes. Investigations of the statistical properties and the possibility of obtaining nonclassical states in SRS processes and in other types of the stimulated scatterings of light by preparing the medium in nonclassical states were suggested in Refs. [3]–[8].

Entangled states are the states which are constructed as a superposition of states, each of which has the wave function expressed as a product of wave functions depending on different degrees of freedom. Different measures of entanglement were suggested last years. In [9] the following measure of entanglement was suggested

$$F=\sigma^2_{q_sq_k}+\sigma^2_{p_sp_k}+\sigma^2_{q_sp_k}+\sigma^2_{p_sq_k}\,. \tag{5}$$

We will use this measure to discuss the entanglement in the stimulated Brillouin scattering process. In cases (2), (3), (4) the measures of entanglement of the modes after the interaction of laser field with the medium are expressed as

$$F_1=\frac{1}{2}\sinh^2(2\lambda\,t)\,,\quad F_2=\frac{1}{8}\sinh^2(2\lambda\,t)\,(1+\coth(\theta))^2\,,$$

$$F_3=\frac{1}{16}\sinh^2(2\lambda\,t)\left(\left(1+\frac{1}{\eta}\right)^2\left(\eta^2+1\right)\right)\,,$$

respectively.

In [10] the measure of the entanglement was defined as the distance between the system density matrix and the tensor product of its partial traces over the subsystem degrees of freedom. For Gaussian states, the measure of entanglement reads

$$e_G=\frac{1}{4\sqrt{\det\sigma(t)}}+\frac{1}{4\sqrt{\det\tilde{\sigma}}}-\frac{2}{\sqrt{\det(\sigma(t)+\tilde{\sigma})}}\,, \tag{6}$$

where we present the inverse of the dispersion matrix in block form to define the matrix $\tilde{\sigma}$, i.e.,

$$\sigma(t)^{-1} = \begin{pmatrix} B & C \\ C^T & D \end{pmatrix}, \qquad \tilde{\sigma} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix},$$

$$\sigma_1^{-1} = B - CD^{-1}C^T, \qquad \sigma_2^{-1} = D - C^T B^{-1} C.$$

In cases (2), (3) the measures of entanglement of photon and phonon modes after the interaction of laser field with the medium are

$$e_{G1} = \frac{\cos^2(\omega_L t)\sinh^2(2\lambda t)(3\cos^2(\omega_L t)\sinh^2(2\lambda t) + 2)}{(1+\cos^2(\omega_L t)\sinh^2(2\lambda t))(4 + 3\cos^2(\omega_L t)\sinh^2(2\lambda t))},$$

$$e_{G2} = \frac{\cos^2(\omega_L t)\sinh^2(2\lambda t)(3\cos^2(\omega_L t)\sinh^2(2\lambda t) + 2\tau(\theta))}{(\tau(\theta)+\cos^2(\omega_L t)\sinh^2(2\lambda t))(4\tau(\theta) + 3\cos^2(\omega_L t)\sinh^2(2\lambda t))},$$

where $\tau(\theta) = 2(\coth(2\theta) + 1)^{-1}$.

To conclude, we obtained the result that in SBS there exists the phenomenon of entanglement. We evaluated the entanglement measure by two methods. In both methods, the measure of entanglement depends on the medium parameters (for example, temperature) and coupling constant.

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